## COMMON CORE STATE $-\frac{I}{\Sigma}$ STANDARDS

## A Crosswalk to the Michigan High School Content Expectations

## Introduction

In June 2010, the Michigan State Board of Education adopted the Common Core State Standards (CCSS) as the state K-I2 content standards for Mathematics and English Language Arts. The complete CCSS standards document can be found at www.michigan.gov/k-12 by clicking the Common Core State Standards Initiative link.

Districts are encouraged to begin this transition to instruction of the new standards as soon as possible to prepare all students for career and college. New assessments based on the Common Core State Standards will be implemented in 2014-2015. More information about Michigan's involvement in the CCSS initiative and development of common assessments can be found at www.michigan.gov/k-12 by clicking the Common Core State Standards Initiative link.

The CCSS for Mathematics are divided into two sets of standards: the Standards for Mathematical Practices and the Standards for Mathematical Content. This document is intended to show the alignment of Michigan's current mathematics High School Content Expectations (HSCE) to the Standards for Mathematical Content to assist with the transition to instruction and assessment based on the CCSS.

It is anticipated that this initial work will be supported by clarification documents developed at the local and state level, including documents from national organizations and other groups. This document is intended as a conversation starter for educators within and across grades. While curriculum revisions will be guided by local curriculum experts, ultimately the alignment will be implemented at the classroom level. Educators will need to unfold these standards in order to compare them to current classroom practice and identify adjustments to instruction and materials that support the depth of understanding implicit in these new standards.

The crosswalk between the High School Content Expectations and the Standards for Mathematical Content is organized by Michigan Strands and Standards. There is not an attempt to show one-to-one correspondence between expectations and standards because, for the most part, there is none at this level. The alignment occurs when looking across Michigan topics and CCSS clusters.

## Mathematical Practices

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These standards appear in every grade level and are listed below:

## Mathematical Practices

I. Make sense of problems, and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments, and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for, and make use of, structure.
8. Look for, and express regularity in, repeated reasoning.

## Organization of the Common Core State Standards

The high school CCSS Common Core State Standards themselves are organized into six Conceptual Categories, then into Domains (large groups that progress across grades) and finallyby Clusters (groups of related standards, similar to the Topics in the High School Content Expectations). In the example provided, the Conceptual Category is "Number and Quantity" (N) and the Domain is "The Real Number System" (RN). The Cluster is defined by the statement "Extend the properties of exponents to rational exponents" and includes two standards.


Extend the properties of exponents to rational exponents.

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1 / 3}$ to be the cube root of 5 because we want $\left(5^{1 / 3}\right)^{3}=5^{(1 / 3) 3}$ to hold, so $\left(5^{1 / 3}\right)^{3}$ must equal 5 .
2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

## Progressions of CCSS for 8th Grade and the High School Conceptual Categories

| 8th Grade |  |
| :---: | :---: |
|  |  |

Expressions and Equations

- Work with radicals and integer exponents
- Understand the connections between

Proportional relationships, lines, and linear equations.

- Analyze and solve linear equations and pairs of simultaneous linear equations

Functions

- Define, evaluate, and compare functions
- Use functions to model relationships between quantities.

| Algebra |
| :---: |
| Seeing Structure in Expressions |

Seeing Structure in Expressions

- Interpret the structure of expressions
- Write expressions in equivalent forms to solve problems

Arithmetic with Polynomials and Rational Functions

- Perform arithmetic operations on polynomials
- Understand the relationship between zeros and factors of polynomials
- Use polynomial identities to solve problems
- Rewrite rational expressions


## Creating Equations

- Create equations that describe numbers or relationships

Reasoning with Equations and Inequalities

- Understand solving equations as a process of

Reasoning and explain the reasoning

- Solve equations and inequalities in one variable
- Solve systems of equations
- Represent and solve equations and inequalities graphically

HIGH SCHOOL

Interpreting Functions

- Understand the concept of a function and use function notation
- Interpret functions that arise in applications in terms of the context
- Analyze functions using different representations
Building Functions
- Build a function that models a relationship between two quantities
- Build new functions from existing functions

Linear, Quadratic, and Exponential Models

- Construct and compare linear and exponential models and solve problems
- Interpret expressions for functions in terms of the situation they model
Trigonometric Functions
- Extend the domain of trigonometric functions using the unit circle
- Model periodic phenomena with trigonometric functions
- Prove and apply trigonometric identities

Expressing Geometric Properties with Equations

- Translate between the geometric description and the equation for a conic section
- Use coordinates to prove simple geometric theorems algebraically


## Progressions of CCSS for 8th Grade and the High School Conceptual Categories (continued)

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8. Look for, and express regularity in, repeated reasoning.
8th Grade

Expressions and Equations

- Work with radicals and integer


## HIGH SCHOOL

## Number and Quantity

The Real Number System

- Extend the properties of exponents to rational exponents
- Use properties of rational and irrational numbers.


## The Complex Number System

- Perform arithmetic operations with complex numbers
- Represent complex numbers and their operations on the complex plane
- Use complex numbers in polynomial identities and equations


## Vector and Matrix Quantities

- Represent and model with vector quantities.
- Perform operations on vectors.
- Perform operations on matrices and use matrices in applications


## 8th Grade

## Statistics and

 Probability- Investigate patterns of association in bivariate data.


## HIGH SCHOOL

## Statistics and Probability

## Interpreting Categorical and Quantitative Data

- Summarize, represent, and interpret data on a single count or measurement variable
- Summarize, represent, and interpret data on two categorical and quantitative variables
- Interpret linear models

Making Inferences and Justifying Conclusions

- Understand and evaluate random processes underlying statistical experiments
- Make inferences and justify conclusions from sample surveys, experiments and observational studies


## Conditional Probability and the Rules of Probability

- Understand independence and conditional probability and use them to interpret data
- Use the rules of probability to compute probabilities of compound events in a uniform probability model


## Using Probability to Make Decisions

- Calculate expected values and use them to solve problems
- Use probability to evaluate outcomes of decisions

Progressions of CCSS for 8th Grade and the High School Conceptual Categories (continued)

| 8th Grade |
| :--- |
| Geometry |
| - Understand congruence and similarity using physical |
| models, transparencies, or geometry software |
| - Understand and apply the Pythagorean |
| Theorem. |
| - Solve real-world and mathematical problems involving |
| volume of cylinders, cones, and spheres. |

## 

## HIGH SCHOOL

## Geometry

Congruence

- Experiment with transformations in the plane
- Understand congruence in terms of rigid motions
- Prove geometric theorems
- Make geometric constructions

Similarity, Right Triangles, and Trigonometry

- Understand similarity in terms of similarity transformations
- Prove theorems involving similarity
- Define trigonometric ratios and solve problems involving right triangles
- Apply trigonometry to general triangles


## Circles

- Understand and apply theorems about circles
- Find arc lengths and areas of sectors of circles


## Geometric Measurement and Dimension

- Explain volume formulas and use them to solve problems
- Visualize relationships between two-dimensional and three-dimensional objects


## Modeling with Geometry

- Apply geometric concepts in modeling situation


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Standard LI: REASONING ABOUT NUMBERS, SYSTEMS, AND QUANTITATIVE SITUATIONS

## Number Systems and Number Sense

LI.I.I Know the different properties that hold in different number systems, be able to recognize that the applicable properties change in the transition from the positive integers to all integers, the rational numbers, and the real numbers.
LI.I. 2 Explain why the multiplicative inverse of a number has the same sign as the number, while the additive inverse of a number has the opposite sign.
LI.I. 3 Explain how the properties of associativity, commutativity, and distributivity, as well as identity and inverse elements, are used in arithmetic and algebraic calculations.
LI.I.6 Explain the importance of the irrational numbers $\sqrt{ } 2$ and $\sqrt{ } 3$ in basic right triangle trigonometry, and the importance of $\pi$ because of its role in circle relationships.

## CCSS Cluster Statements and Standards

Use properties of rational and irrational numbers.
N.RN. 3 Explain why the sum or product of rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

## Perform arithmetic operations with complex numbers.

N.CN. 2 Use the relation $i^{2}=-\mid$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

## Rewrite rational expressions.

A.APR. 7 (+)Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

## Extend the domain of trigonometric functions using the unit circle.

F.TF. 2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
F.TF. 3 (+)Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi / 3, \pi / 4$ and $\pi / 6$, and tangent for $\times, \pi+\times$, and $2 \pi$ $-x$ in terms of their values for $x$, where $x$ is any real number.

Standard LI: REASONING ABOUT NUMBERS, SYSTEMS, AND QUANTITATIVE SITUATIONS

## Representations and Relationships

LI.2.I Use mathematical symbols to represent quantitative relationships and situations.
LI.2.3 Use vectors to represent quantities that have magnitude and direction, interpret direction and magnitude of a vector numerically, and calculate the sum and difference of two vectors.
LI.2.4 Organize and summarize a data set in a table, plot, chart, or spreadsheet; find patterns in a display of data; understand and critique data displays in the media.

Reason quantitatively and use units to solve problems.
N.Q.I Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
Represent and model with vector quantities.
N.VM.I (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, |v|, \||v|, v).
N.VM. $2(+)$ Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
N.VM. 3 (+) Solve problems involving velocity and other quantities that can be represented by vectors.

## Perform operations on vectors.

N.VM. 4 (+) Add and subtract vectors.
N.VM.4a (+) Add vectors end-to-end, componentwise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
N.VM.4b (+) Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
N.VM.4c (+) Understand vector subtraction $\mathbf{v} \mathbf{- w}$ as $v+(-\mathbf{w})$, where $(-\mathbf{w})$ is the additive inverse of $\mathbf{w}$, with the same magnitude as $w$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
N.VM. 5 (+) Multiply a vector by a scalar.

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Summarize, represent, and interpret data on a single count or measurement variable.
S.ID.I Represent data with plots on the real number line (dot plots, histograms, and box plots).
S.ID. 2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.
S.IC. 3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
S.IC. 6 Evaluate reports based on data.

MP. 2 Reason abstractly and quantitatively. (Mathematical Practice)

Standard LI: REASONING ABOUT NUMBERS, SYSTEMS, AND QUANTITATIVE SITUATIONS

## Counting and Probabilistic Reasoning

LI.3.I: Describe, explain, and apply various counting techniques; relate combinations to Pascal's triangle; know when to use each technique.
LI.3.2 Define and interpret commonly used expressions of probability.
LI.3.3 Recognize and explain common probability misconceptions such as "hot streaks" and "being due."

## CCSS Cluster Statements and Standards

Use polynomial identities to solve problems.
A.APR. 5 (+) Know and apply that the Binomial Theorem gives the expansion of $(x+y)^{n}$ in powers of $x$ and $y$ for a positive integer $n$, where $x$ and $y$ are any numbers, with coefficients determined, for example, by Pascal's Triangle. (The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.)

## Understand and evaluate random processes underlying statistical experiments.

S.IC. 2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5 . Would a result of 5 tails in a row cause you to question the model?
Make inferences and justify conclusions from sample surveys, experiments, and observational studies.
S.IC. 4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
Understand independence and conditional probability and use them to interpret data.
S.CP. 5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.
Use probability to evaluate outcomes of decisions.
S.MD. 7 (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

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Standard L2 CALCULATIONS, ALGORITHMS, AND ESTIMATION

## Calculation Using Real and Complex Numbers

L2.I.2: Calculate fluently with numerical expressions involving exponents; use the rules of exponents; evaluate numerical expressions involving rational and negative exponents; transition easily between roots and exponents.

L2.I.3 Explain the exponential relationship between a number and its base 10 logarithm and use it to relate rules of logarithms to those of exponents in expressions involving numbers.

L2.I.4 Know that the complex number $i$ is one of two solutions to $x^{2}=-1$.

L2.I.5 Add, subtract, and multiply complex numbers; use conjugates to simplify quotients of complex numbers.

## CCSS Cluster Statements and Standards

## Extend the properties of exponents to rational

 exponents.N.RN. 2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.
Perform arithmetic operations with complex numbers.
N.CN.I Know there is a complex number I, such that $i^{2}=-I$, and every complex number has the form $\mathrm{a}+\mathrm{bi}$ with a and b real.
N.CN. 2 Use the relation $i^{2}=-\mid$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
N.CN. $3(+)$ Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

Represent complex numbers and their operations on the complex plane.
N.CN. $5(+)$ Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, $(-1+\sqrt{3} i) 3=8$ because $(-1+\sqrt{ } 3 i)$ has modulus 2 and argument $120^{\circ}$.
N.CN. $6(+)$ Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

Use complex numbers in polynomial identities and equations.
N.CN. 7 Solve quadratic equations with real coefficients that have complex solutions.
N.CN. 8 (+) Extend polynomial identities to the complex numbers. For example, rewrite $x^{2}+4$ as $(x+2 i)(x-2 i)$.
N.CN. 9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Standard L2 CALCULATIONS, ALGORITHMS, AND ESTIMATION

## Calculation Using Real and Complex Numbers (continued)

## Sequences and Iteration

L2.2. I Find the nth term in arithmetic, geometric, or other simple sequences.

L2.2.2 Compute sums of finite arithmetic and geometric sequences.

L2.2.3 Use iterative processes in such examples as computing compound interest or applying approximation procedures.

## CCSS Cluster Statements and Standards

Solve equations and inequalities in one variable.
A.REI.4b Solve quadratic equations by inspection (e.g., for $x^{2}=49$ ), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a $\pm$ bi for real numbers $a$ and $b$.

## Build new functions from existing functions

F.BF. 5 (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
Construct and compare linear, quadratic, and exponential models and solve problems.
F.LE. 4 For exponential models, express as a logarithm the solution to $a b^{(c t)}=d$ where $a, c$, and $d$ are numbers and the base $b$ is 2,10 , or e; evaluate the logarithm using technology.

## Write expressions in equivalent forms to solve problems.

A.SSE. 4 Derive the formula for the sum of a finite geometric series (when the common ratio is not I), and use the formula to solve problems. For example, calculate mortgage payments.

Build a function that models a relationship between two quantities.
F.BF. 2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Construct and compare linear, quadratic, and exponential models and solve problems.
F.LE. 2 Construct linear and exponential functions that include arithmetic and geometric sequences given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

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standard L2 CALCULATIONS, ALGORITHMS, AND ESTIMATION

Measurement Units, Calculations, and Scales
L2.3.I Convert units of measurement within and between systems; explain how arithmetic operations on measurements both affect units, and carry units through calculations correctly.

L2.3.2 Describe and interpret logarithmic relationships in such contexts as the Richter scale, the pH scale, and decibel measurements; solve applied problems.

## Understanding Error

L2.4.I Determine what degree of accuracy is reasonable for measurements in a given situation; express accuracy through use of significant digits, error tolerance, or percent of error; describe how errors in measurements are magnified by computation; recognize accumulated error in applied situations.

L2.4.2 Describe and explain round-off error, rounding, and truncating.

L2.4.3 Know the meaning of and interpret statistical significance, margin of error, and confidence level.

## CCSS Cluster Statements and Standards

Reason quantitatively and use units to solve problems.
N.Q.I Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
N.Q. 2 Define appropriate quantities for the purpose of descriptive modeling.
N.Q. 3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Reason quantitatively and use units to solve problems.
N.Q. 3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

## Understand and evaluate random processes underlying statistical experiments.

S.IC.I Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
S.IC. 2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0 . 5 . Would a result of 5 tails in a row cause you to question the model?

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.
S.IC. 4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
S.IC. 5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

MP. 6 Attend to precision. (Mathematical Practice)

Standard L3 MATHEAMTICAL REASONING,
LOGIC, AND PROOF

## Mathematical Reasoning

L3.I.I Distinguish between inductive and deductive reasoning, identifying and providing examples of each.

L3.1.2 Differentiate between statistical arguments (statements verified empirically using examples or data) and logical arguments based on the rules of logic.

L3.I.3 Define and explain the roles of axioms (postulates), definitions, theorems, counterexamples, and proofs in the logical structure of mathematics. Identify and give examples of each.

## Language and Laws of Logic

L3.2. I Know and use the terms of basic logic.
L3.2.2 Language and Laws of Logic: Use the connectives " $n o t$, ," ' and," ' "or," and "if..., then," in mathematical and everyday settings. Know the truth table of each connective and how to logically negate statements involving these connectives.

L3.2.3 Language and Laws of Logic: Use the quantifiers "there exists" and "all" in mathematical and everyday settings and know how to logically negate statements involving them.

## CCSS Cluster Statements and Standards

Understand and evaluate random processes underlying statistical experiments.
S.IC.I Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.
S.IC. 3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
S.IC. 6 Evaluate reports based on data.

MP. 3 Construct viable arguments and critique the reasoning of others. (Mathematical Practice)

Understand independence and conditional probability and use them to interpret data.
S.CP.I Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").

MP. 3 Construct viable arguments and critique the reasoning of others. (Mathematical Practice)

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Standard AI EXPRESSIONS, EQUATIONS, AND INEQUALITIES

## Construction, Interpretation, and Manipulation of Expressions

AI.I.I Give a verbal description of an expression that is presented in symbolic form, write an algebraic expression from a verbal description, and evaluate expressions given values of the variables.
AI.I. 2 Construction, Interpretation, and Manipulation of Expressions: Know the definitions and properties of exponents and roots transition fluently between them, and apply them in algebraic expressions.
AI.I. 3 Factor algebraic expressions using, for example, greatest common factor, grouping, and the special product identities.

AI.I. 4 Add, subtract, multiply, and simplify polynomials and rational expressions.

AI.I. 5 Divide a polynomial by a monomial.
AI.I. 6 Transform exponential and logarithmic expressions into equivalent forms using the properties of exponents and logarithms, including the inverse relationship between exponents and logarithms.

## CCSS Cluster Statements and Standards

Extend the properties of exponents to rational exponents.
N.RN.I Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{(1 / 3)}$ to be the cube root of 5 because we want $\left(5^{(1 / 3))^{3}}=5^{(1 / 3) 3}\right.$ to hold, so $5^{(1 / 3) 3}$ must equal 5.
Use complex numbers in polynomial identities and equations
N.CN. $8(+)$ Extend polynomial identities to the complex numbers. For example, rewrite $x^{2}+4$ as $(x+2 i)$ $(x-2 i)$.
Interpret the structure of expressions.
A.SSE.I Interpret expressions that represent a quantity in terms of its context.
A.SSE.I a Interpret parts of an expression, such as terms, factors, and coefficients.
A.SSE. 2 Use the structure of an expression to identify ways to rewrite it. For example, see $x^{4}-y^{4}$ as $\left(x^{2}\right)^{2}-\left(y^{2}\right)^{2}$, thus recognizing it as a difference of squares that can be factored as $\left(x^{2}-y^{2}\right)\left(x^{2}+y^{2}\right)$.

Write expressions in equivalent forms to solve problems.
A.SSE.3c Use the properties of exponents to transform expressions for exponential functions. For example the expression $1.15^{t}$ can be rewritten as $\left(1.15^{(1112)}\right) 12^{t} \approx 1.012^{12 t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15\%.
Perform arithmetic operations on polynomials.
A.APR.I Understand that polynomials form a system analogous to the integers; namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Standard AI EXPRESSIONS, EQUATIONS,
AND INEQUALITIES

## CCSS Cluster Statements and Standards

## Rewrite rational expressions.

A.APR. 6 Rewrite simple rational expressions in different forms; write $a(x) / b(x)$ in the form $q(x)+$ $r(x) / b(x)$, where $a(x), b(x), q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
A.APR. 7 (+)Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

## Build new functions from existing functions

F.BF. 5 (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
Construct and compare linear, quadratic, and exponential models and solve problems.
F.LE. 4 For exponential models, express as a logarithm the solution to abct = d where $\mathrm{a}, \mathrm{c}$, and d are numbers and the base $b$ is 2,10 , or e; evaluate the logarithm using technology.

## Use complex numbers in polynomial identities and equations.

N.CN. 7 Solve quadratic equations with real coefficients that have complex solutions.

## Write expressions in equivalent forms to solve problems.

A.SSE.3a Factor a quadratic expression to reveal the zeros of the function it defines.
A.SSE.3c Use the properties of exponents to transform expressions for exponential functions. For example the expression $1.15^{t}$ can be rewritten as $\left(1.15^{(11 / 2)}\right) 12^{t} \approx 1.012^{12 t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15\%.

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## Solutions of Equations and Inequalities

Al.2.I Write equations and inequalities with one or two variables to represent mathematical or applied situations, and solve.

A I.2.2 Associate a given equation with a function whose zeros are the solutions of the equation.

AI.2.3 Solve linear and quadratic equations and inequalities including systems of up to three linear equations with three unknowns. Justify steps in the solution, and apply the quadratic formula appropriately.

AI.2.4 Solve absolute value equations and inequalities, and justify steps in the solution.
Al.2.5 Solve polynomial equations and equations involving rational expressions, and justify steps in the solution.

## Mathematical Practices

I. Make sense of problems, and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments, and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for, and make use of, structure.
8. Look for, and express regularity in, repeated reasoning.

Standard AI EXPRESSIONS, EQUATIONS, AND INEQUALITIES

Al.2.6 Solve power equations and equations including radical expressions, justify steps in the solution, and explain how extraneous solutions may arise.

AI.2.7 Solve exponential and logarithmic equations, and justify steps in the solution.

AI.2.8 Solve an equation involving several variables (with numerical or letter coefficients) for a designated variable. Justify steps in the solution.

AI.2.9 Know common formulas and apply appropriately in contextual situations.

AI.2.10 Use special values of the inverse trigonometric functions to solve trigonometric equations over specific intervals.

## CCSS Cluster Statements and Standards

Understand the relationship between zeros and factors of polynomials.
A.APR. 3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Create equations that describe numbers or relationship.
A.CED.I Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
A.CED. 2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
A.CED. 4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law $V=\mathbb{R}$ to highlight resistance $R$.

Understand solving equations as a process of reasoning and explain the reasoning.
A.REI.I Explain each step in solving a simple equation; from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
A.REI. 2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Solve equations and inequalities in one variable.
A.REI. 3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
A.REI. 4 Solve quadratic equations in one variable.

Standard AI EXPRESSIONS, EQUATIONS, AND INEQUALITIES

CCSS Cluster Statements and Standards
A.REI.4b Solve quadratic equations by inspection (e.g., for $x^{2}=49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a $\pm$ bi for real numbers $a$ and $b$.

## Solve systems of equations.

A.REI. 5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
A.REI. 6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
A.REI. 7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y=-3^{x}$ and the circle $x^{2}+y^{2}=3$.
Represent and solve equations and inequalities graphically.
A.REI. 12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

## Model periodic phenomena with trigonometric functions

F.TF. 7 (+)Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

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## Standard A2 FUNCTIONS

## CCSS Cluster Statements and Standards

## Definitions, Representations, and Attributes of Functions

A2.I.I Determine whether a relationship (given in contextual, symbolic, tabular, or graphical form) is a function and identify its domain and range.

A2.I.2 Read, interpret, and use function notation and evaluate a function at a value in its domain.

A2.I.3 Represent functions in symbols, graphs, tables, diagrams, or words and translate among representations.

A2.I.4 Recognize that functions may be defined by different expressions over different intervals of their domains; such functions are piecewise-defined.

A2.I. 5 Recognize that functions may be defined recursively. Compute values of and graph simple recursively defined functions.

A2.I.6 Identify the zeros of a function, the intervals where the values of a function are positive or negative, and describe the behavior of a function as $x$ approaches positive or negative infinity, given the symbolic and graphical representations.

A2.I. 7 Identify and interpret the key features of a function from its graph or its formula(e).

Write expressions in equivalent forms to solve problems.
A.SSE.3a Factor a quadratic expression to reveal the zeros of the function it defines.

Represent and solve equations and inequalities graphically.
A.REI.I IExplain why the $x$-coordinates of the points where the graphs of the equations $y=f(x)$ and $y=g(x)$ intersect are the solutions of the equation $f(x)=g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, or logarithmic functions.

## Understand the concept of a function and use function notation

F.IF.I Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$. The graph of $f$ is the graph of the equation $y=f(x)$.
F.IF. 2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
F.IF. 3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0)=f(1)=1$, $f(n+1)=f(n)+f(n-1)$ for $n \geq 1$.

Interpret functions that arise in applications in terms of the context.
F.IF. 4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

Standard A2 FUNCTIONS
Definition, Representation, and Attributes of functions (continued)

## Operations and Transformations

A2.2.I Combine functions by addition, subtraction, multiplication, and division.

A2.2.2 Operations and Transformations: Apply given transformations to basic functions and represent symbolically.

A2.2.3 Operations and Transformations: Recognize whether a function (given in tabular or graphical form) has an inverse and recognize simple inverse pairs.

## CCSS Cluster Statements and Standards

F.IF. 5 Relate the domain of a function to its graph and, to the quantitative relationship it describes. where applicable. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble $n$ engines in a factory, then the positive integers would be an appropriate domain for the function.

## Analyze functions using different representations.

F.IF.7b Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
F.IF.7c Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
F.IF. 8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
F.IF. 9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.
Build a function that models a relationship between two quantities.
F.BF.I Write a function that describes a relationship between two quantities.
F.BF.Ia Determine an explicit expression, a recursive process, or steps for calculation from a context.

Interpret expressions for functions in terms of the situation they model
F.LE. 5 Interpret the parameters in a linear or exponential function in terms of a context.

## Perform arithmetic operations on polynomials.

A.APR.I Understand that polynomials form a system analogous to the integers; namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

## Mathematical Practices

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## Standard A2 FUNCTIONS

## CCSS Cluster Statements and Standards

## Build a function that models a relationship between

 two quantitiesF.BF.Ib Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

## Build new functions from existing functions.

F.BF. 3 Identify the effect on the graph of replacing $f(x)$ by $f(x)+k, k f(x), f(k x)$, and $f(x+k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and respective algebraic expressions.
F.BF. 4 Find inverse functions.
F.BF.4c (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.

Experiment with transformations in the plane.
G.CO. 2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

## Representations of Functions

A2.3.I Identify a function as a member of a family of functions based on its symbolic or graphical representation; recognize that different families of functions have different asymptotic behavior.

A2.3.2 Describe the tabular pattern associated with functions having constant rate of change (linear), or variable rates of change.

A2.3.3 Write the general symbolic forms that characterize each family of functions.

Interpret functions that arise in applications in terms of the context.
F.IF. 4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
F.IF. 6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
Analyze functions using different representations.
F.IF. 9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

## Build a function that models a relationship between two quantities

F.BF.I Write a function that describes a relationship between two quantities.

Construct and compare linear, quadratic, and exponential models and solve problems.
F.LE.I I Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.
F.LE.Ib. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
F.LE. 3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

## Models of Real-world Situations Using Families of

 FunctionsA2.4.I Identify the family of function best suited for modeling a given real-world situation.
A2.4.2 Adapt the general symbolic form of a function to one that fits the specification of a given situation by using the information to replace arbitrary constants with numbers.
A2.4.3 Using the adapted general symbolic form, draw reasonable conclusions about the situation being modeled.

Reason quantitatively and use units to solve problems.
N.Q. 2 Define appropriate quantities for the purpose of descriptive modeling.

## Interpret the structure of expressions

A.SSE.Ib Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(I+r)^{n}$ as the product of $P$ and a factor not depending on $P$.
Create equations that describe numbers or relationship.
A.CED. 3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

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## CCSS Cluster Statements and Standards

Build a function that models a relationship between two quantities.
F.BF.I Write a function that describes a relationship between two quantities.
F.BF. 2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations and translate between the two forms.

Construct and compare linear, quadratic, and exponential models and solve problems.
F.LE.I Distinguish between situations that can be modeled with linear functions and with exponential functions.

Model periodic phenomena with trigonometric functions.
F.TF. 5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

Summarize, represent, and interpret data on two categorical and quantitative variables.
S.ID.6a Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic and exponential models.

Standard A3 FAMILIES OF FUNCTIONS
Lines and Linear Functions
A3.I.I Lines and Linear Functions:Write the symbolic forms of linear functions (standard, point-slope, and slope-intercept) given appropriate information, and convert between forms.

A3.I.2 Graph lines (including those of the form $x$ $=h$ and $y=k$ ) given appropriate information.
A3.I.3 Relate the coefficients in a linear function to the slope and $x$ - and $y$-intercepts of its graph.

A3.I.4 Find an equation of the line parallel or perpendicular to the given line, through a given point; understand and use the facts that non-vertical parallel lines have equal slopes, and that non-vertical perpendicular lines have slopes that multiply to give - 1 .

## CCSS Cluster Statements and Standards

Represent and solve equations and inequalities graphically.
A.REI.IO Understand that the graph of an equation in two variables is the set of all its solutions plotted on the coordinate plane, often forming a curve (which could be a line).

## Analyze functions using different representations.

F.IF. 7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases, and using technology for more complicated cases.
F.IF.7a Graph linear and quadratic functions and show intercepts, maxima, and minima.
F.IF. 8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

Build a function that models a relationship between two quantities.
F.BF.I Write a function that describes a relationship between two quantities.

Construct and compare linear, quadratic, and exponential models and solve problems
F.LE.2. Construct linear and exponential functions, including arithmetic and geometric sequences given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Interpret expressions for functions of the situation they model.
F.LE. 5 Interpret the parameters in a linear or exponential function in terms of a context.

Use coordinates to prove simple geometric theorems algebraically.
G.GPE. 5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

## Mathematical Practices

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## Standard A3 FAMILIES OF FUNCTIONS

## CCSS Cluster Statements and Standards

## Exponential and Logarithmic Functions

A3.2.I Write the symbolic form and sketch the graph of an exponential function given appropriate information.

A3.2.2 Interpret the symbolic forms and recognize the graphs of exponential and logarithmic functions; recognize the logarithmic function as the inverse of the exponential function.

A3.2.3 Apply properties of exponential and logarithmic functions.

A3.2.4 Understand and use the fact that the base of an exponential function determines whether the function increases or decreases and understand how the base affects the rate of growth or decay.
A.3.2.5 Relate exponential and logarithmic functions to real phenomena, including half-life and doubling time.

Use the properties of exponents to transform expressions for exponential function
A.SSE.3c s. For example the expression $1.15^{t}$ can be rewritten as $\left(1.15^{(1 / 12)}\right)^{12 t} \approx 1.012^{12 t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is $15 \%$.

## Analyze functions using different representations.

F.IF.7e Graph exponential and logarithmic functions, showing intercepts and end behavior, as well as trigonometric functions, showing period, midline, and amplitude.
F.IF. 8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
F.IF.8b Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y=$ $(1.02)^{t}, y=(0.97)^{t}, y=(1.01)^{12 t}, y=(1.2)^{t / 10}$, and classify them as representing exponential growth and decay.

Build a function that models a relationship between two quantities.
F.BF.I Write a function that describes a relationship between two quantities.

## Build new functions from existing functions.

F.BF. 4 Find inverse functions.
F.BF. 5 (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Construct and compare linear and exponential models and solve problems.
F.LE.Ia Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.*
F.LE.Ic Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
F.LE. 2 Construct linear and exponential functions, including arithmetic and geometric sequences given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

## Standard A3 FAMILIES OF FUNCTIONS

(continued)

## Quadratic Functions

A3.3. I Write the symbolic form and sketch the graph of a quadratic function given appropriate information.

A3.3.2 Identify the elements of a parabola (vertex, axis of symmetry, direction of opening) given its symbolic form or its graph, and relate these elements to the coefficient(s) of the symbolic form of the function.

A3.3.3 Convert quadratic functions from standard to vertex form by completing the square.
A3.3.4 Relate the number of real solutions of a quadratic equation to the graph of the associated quadratic function.
A3.3.5 Express quadratic functions in vertex form to identify their maxima or minima, and in factored form to identify their zeros.

## CCSS Cluster Statements and Standards

F.LE. 3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or as a polynomial function.

Interpret expressions for functions of the situation they model.
F.LE. 5 Interpret the parameters in a linear or exponential function in terms of a context.

Write expressions in equivalent forms to solve problems.
A.SSE.3a Factor a quadratic expression to reveal the zeros of the function it defines.
A.SSE.3b Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
Solve equations and inequalities in one variable.
A.REI.4a Use the method of completing the square to transform any quadratic equation in $x$ into an equation of the form $(x-p)^{2}=q$ that has the same solutions. Derive the quadratic formula from this form.

## Represent and solve equations and inequalities graphically.

A.REI.IO Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

## Analyze functions using different representations.

F.IF. 7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases, or using technology for more complicated cases.
F.IF.7a Graph linear and quadratic functions and show intercepts, maxima, and minima.
F.IF. 8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
F.IF.8a Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

## Mathematical Practices

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## Standard A3 FAMILIES OF FUNCTIONS

Quadratic Functions (continued)

## Power Functions

A3.4.I Write the symbolic form and sketch the graph of power functions.

## Polynomial Functions

A3.5.I Polynomial Functions: Write the symbolic form and sketch the graph of simple polynomial functions.

A3.5.2 Understand the effects of degree, leading coefficient, and number of real zeros on the graphs of polynomial functions of degrees greater than 2 .

A 3.5.3 Determine the maximum possible number of zeros of a polynomial function, and understand the relationship between the $x$-intercepts of the graph and the factored form of the function

## CCSS Cluster Statements and Standards

Build a function that models a relationship between two quantities.
F.BF.IWrite a function that describes a relationship between two quantities.

Interpret expressions for functions of the situation they model.
F.LE. 5 Construct and compare linear, quadratic, and exponential models and solve problems. Interpret the parameters in a linear or exponential function in terms of a context.

Represent and solve equations and inequalities graphically.
A.REI.IO Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

## Analyze functions using different representations.

F.IF. 7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

Build a function that models a relationship between two quantities.
F.BF.I Write a function that describes a relationship between two quantities.

Use complex numbers in polynomial identities and equations.
N.CN. 8 (+) Extend polynomial identities to the complex numbers. For example, rewrite $x^{2}+4$ as $(x+2 i)(x-2 i)$.

Write expressions in equivalent forms to solve problems
A.SSE.3a Factor a quadratic expression to reveal the zeros of the function it defines.

Understand the relationship between zeros and factors of polynomial.
A.APR. 2 Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number $a$, the remainder on division by $x-a$ is $p(a)$, so $p(a)=0$ if and only if $(x-a)$ is a factor of $p(x)$.
A.APR. 3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Represent and solve equations and inequalities graphically.
A.REI.IO Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

Interpret functions that arise in applications in terms of the context.
F.IF. 4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
Analyze functions using different representations.
F.IF. 7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases, and using technology for more complicated cases.
F.IF.7c Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
F.IF. 8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
Build a function that models a relationship between two quantities.
F.BF.I Write a function that describes a relationship between two quantities.

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## Rational Functions

A3.6.I Write the symbolic form and sketch the graph of simple rational functions.

A3.6.2 Analyze graphs of simple rational functions and understand the relationship between the zeros of the numerator and denominator and the function's intercepts, asymptotes, and domain.

## Trigonometric Functions

A3.7.I Use the unit circle to define sine and cosine; approximate values of sine and cosine; use sine and cosine to define the remaining trigonometric functions; explain why the trigonometric functions are periodic.

A3.7.2 Use the relationship between degree and radian measures to solve problems.

A3.7.3 Use the unit circle to determine the exact values of sine and cosine for integer multiples of $\pi / 6$ and $\pi / 4$.

A3.7.4 Graph the sine and cosine functions; analyze graphs by noting domain, range, period, amplitude, and location of maxima and minima.

A3.7.5 Graph transformations of basic trigonometric functions (involving changes in period, amplitude, and midline) and understand the relationship between constants in the formula and the transformed graph.

Interpret functions that arise in applications in terms of the context.
F.IF. 5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble $n$ engines in a factory, then the positive integers would be an appropriate domain for the function.*

## Analyze functions using different representations

F.IF.7d (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.*
Build a function that models a relationship between two quantities.
F.BF.I Write a function that describes a relationship between two quantities.*

Interpret functions that arise in applications in terms of the context.
F.IF. 5 Relate the domain of a function to its graph and to the quantitative relationship it describes where applicable. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble $n$ engines in a factory, then the positive integers would be an appropriate domain for the function.

## Analyze functions using different representations.

F.IF.7e Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
Build a function that models a relationship between two quantities.
F.BF.I Write a function that describes a relationship between two quantities.

## Build new functions from existing functions.

F.BF. 3 Identify the effect on the graph of replacing $f(x)$ by $f(x)+k, k f(x), f(k x)$, and $f(x+k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Extend the domain of trigonometric functions using the unit circle.
F.TF.I Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
F.TF. 2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
F.TF. 3 (+)Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi / 3, \pi / 4$ and $\pi / 6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi-\times, \pi+$ $x$, and $2 \pi-x$ in terms of their values for $x$, where $x$ is any real number.
F.TF. $4(+)$ Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
Model periodic phenomena with trigonometric functions.
F.TF. 5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

## Define trigonometric ratios and solve problems involving right triangles.

G.SRT. 6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
Find arc lengths and areas of sectors of circles.
G.C. 5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

## Mathematical Practices

I. Make sense of problems, and persevere in solving them.
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STANDARD GI: FIGURES AND THEIR

## PROPERTIES

Lines and Angles; Basic Euclidean and Coordinate
GI.I.I Solve multistep problems and construct proofs involving vertical angles, linear pairs of angles supplementary angles, complementary angles, and right angles.

GI.I. 2 Solve multistep problems and construct proofs involving corresponding angles, alternate interior angles, alternate exterior angles, and same-side (consecutive) interior angles.

GI.I. 3 Perform and justify constructions, including midpoint of a line segment and bisector of an angle, using a straightedge and compass.

GI.I. 4 Given a line and a point, construct a line through the point that is parallel to the original line using a straightedge and compass. Given a line and a point, construct a line through the point that is perpendicular to the original line. Justify the steps of the constructions.

GI.I. 5 Given a line segment in terms of its endpoints in the coordinate plane, determine its length and midpoint.

GI.I.6 Recognize Euclidean geometry as an axiom system. Know the key axioms. Understand the meaning of, and distinguish between, undefined terms, axioms, definitions, and theorems.

CCSS Cluster Statements and Standards

## Experiment with transformations in the plane.

G.CO.I Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

## Prove geometric theorems.

G.CO.9 Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

## Make geometric constructions.

G.CO. 12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.
G.CO.I 3 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Use coordinates to prove simple geometric theorems algebraically.
G.GPE. 4 Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and contains the point $(0,2)$.
G.GPE. 6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
G.GPE. 7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

STANDARD GI: FIGURES AND THEIR

## PROPERTIES

## Triangles and Their Properties

GI.2.I Prove that the angle sum of a triangle is $180^{\circ}$ and that an exterior angle of a triangle is the sum of the two remote interior angles.

GI.2.2 Construct and justify arguments and solve multistep problems involving angle measure, side length, perimeter, and area of all types of triangles.

GI.2.3 Know a proof of the Pythagorean Theorem, and use the Pythagorean Theorem and its converse to solve multistep problems.

GI.2.5 Solve multistep problems and construct proofs about the properties of medians, altitudes, perpendicular bisectors to the sides of a triangle, and the angle bisectors of a triangle. Using a straightedge and compass, construct these lines.

## Triangles and Trigonometry

GI.3.I: Define the sine, cosine, and tangent of acute angles in a right triangle as ratios of sides. Solve problems about angles, side lengths, or areas using trigonometric ratios in right triangles.

GI.3.2 Know and use the Law of Sines and the Law of Cosines and use them to solve problems. Find the area of a triangle with sides $a$ and $b$ and included angle $\square$ using the formula Area $=(1 / 2)$ (ab) $\sin \square$.

## Prove geometric theorems.

G.CO. 10 Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180 degrees; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

## Define trigonometric ratios and solve problems involving right triangles.

G.SRT. 8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Define trigonometric ratios and solve problems involving right triangles.
G.SRT. 6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
G.SRT. 7 Explain and use the relationship between the sine and cosine of complementary angles.
G.SRT. 8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

## Apply trigonometry to general triangles.

G.SRT. $9(+)$ Derive the formula $A=(1 / 2) a b$ $\sin (C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
G.SRT.IO (+) Prove the Laws of Sines and Cosines and use them to solve problems.
G.SRT.II (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

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STANDARD GI: FIGURES AND THEIR

## PROPERTIES

## Quadrilaterals and Their Properties

GI.4.I Solve multistep problems and construct proofs involving angle measure, side length, diagonal length, perimeter, and area of squares, rectangles, parallelograms, kites, and trapezoids.

GI.4.2 Solve multistep problems and construct proofs involving quadrilaterals using Euclidean methods or coordinate geometry.

GI.4.3 Describe and justify hierarchical relationships among quadrilaterals.

GI.4.4 Prove theorems about the interior and exterior angle sums of a quadrilateral.

## Other Polygons and Their Properties

GI.5.I Know and use subdivision or circumscription methods to find areas of polygons.

## Circles and Their Properties

GI.6.I Solve multistep problems involving circumference and area of circles.

G I.6.2 Circles and Their Properties: Solve problems and justify arguments about chords and lines tangent to circles.

GI.6.3 Circles and Their Properties: Solve problems and justify arguments about central angles, inscribed angles, and triangles in circles.

G I.6.4 Circles and Their Properties: Know and use properties of arcs and sectors and find lengths of arcs and areas of sectors.

## Prove geometric theorems.

G.CO.II Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.
Use coordinates to prove simple geometric theorems algebraically.
G.GPE. 4 For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point $(0,2)$.
G.GPE. 5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
G.GPE. 7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

## Explain volume formulas and use them to solve

 problems.G.GMD.I Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.

## Understand and apply theorems about circles.

G.C.I Prove that all circles are similar.
G.C. 2 Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
G.C. 3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

STANDARD GI: FIGURES AND THEIR
PROPERTIES
Circles and Properties (continued)

## Conic Sections and Their Properties

G.I.7.I Find an equation of a circle given its center and radius; given the equation of a circle, find its center and radius.

GI.7.2 Identify and distinguish among geometric representations of parabolas, circles, ellipses, and hyperbolas; describe their symmetries, and explain how they are related to cones.
GI.7.3 Graph ellipses and hyperbolas with axes parallel to the $x$ - and $y$-axes, given equations.

CCSS Cluster Statements and Standards
G.C. $4(+)$ Understand and apply theorems about circles. Construct a tangent line from a point outside a given circle to the circle.
Find arc lengths and areas of sectors of circles.
G.C. 5 Derive, using similarity, the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Explain volume formulas and use them to solve problems.
G.GMD.I Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.

Translate between the geometric description and the equation for a conic section.
G.GPE.I Derive the equation of a circle, given center and radius, using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
G.GPE. 3 (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

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STANDARD GI: FIGURES AND THEIR PROPERTIES

## Three- Dimensional Figures

GI.8.I Solve multistep problems involving surface area and volume of pyramids, prisms, cones, cylinders, hemispheres, and spheres.

## STANDARD G2: RELATIONSHIPS

 BETWEEN FIGURES
## Relationships Between Area and Volume Formulas

G2.I. 3 Know and use the relationship between the volumes of pyramids and prisms (of equal base and height), and cones and cylinders (of equal base and height).

## Relationships Between Two-dimensional and Threedimensional Representations

G2.2.I Identify or sketch a possible threedimensional figure, given two-dimensional views. Create a two-dimensional representation of a three-dimensional figure.

G2.2.2 Relationships Between Two-dimensional and Three-dimensional Representations: Identify or sketch cross sections of three-dimensional figures. Identify or sketch solids formed by revolving two-dimensional figures around lines.

## Explain volume formulas and use them to solve problems.

G.GMD.I Give an informal argument for the formulas for the circumference of a circle, area of a circle, and the volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.
G.GMD. 3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

Apply geometric concepts in modeling situations.
G.MG. 2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).

## CCSS Cluster Statements and Standards

Explain volume formulas and use them to solve problems.
G.GMD.I Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.
G.GMD. 3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

Visualize relationships between two-dimensional and three-dimensional objects.
G.GMD. 4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

STANDARD G2: RELATIONSHIPS
BETWEEN FIGURES

## Congruence and Similarity

G2.3.I Prove that triangles are congruent using the SSS, SAS, ASA, and AAS criteria, and that right triangles, are congruent using the hypotenuse-leg criterion.

G2.3.2 Use theorems about congruent triangles to prove additional theorems and solve problems, with and without use of coordinates.

G2.3.3 Prove that triangles are similar by using SSS , SAS, and AA conditions for similarity.

G2.3.4 Use theorems about similar triangles to solve problems with and without use of coordinates

G2.3.5 Know and apply the theorem stating that the effect of a scale factor of $k$ relating one twodimensional figure to another or one threedimensional figure to another, on the length, area, and volume of the figures, is to multiply each by k , $k^{2}$, and $k^{3}$, respectively.

CCSS Cluster Statements and Standards

## Understand congruence in terms of rigid motions.

G.CO. 7 Use the definition of congruence, in terms of rigid motions, to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
G.CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

## Understand similarity in terms of similarity transformations.

G.SRT. 2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles, and the proportionality of all corresponding pairs of sides.

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STANDARD G2: RELATIONSHIPS
BETWEEN FIGURES

CCSS Cluster Statements and Standards
G.SRT. 3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Prove theorems involving similarity.
G.SRT. 4 Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.
G.SRT. 5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Use coordinates to prove simple geometric theorems algebraically.
G.GPE. 4 Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin, containing the point ( 0 , 2).
G.GPE. 7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

STANDARD G3: TRANFORMATION OF FIGURES INTHE PLANE

Distance-preserving Transformations: Isometries
G3.I.I: Define reflection, rotation, translation, and glide reflection and find the image of a figure under a given isometry.

G3.I. 2 Isometries: Given two figures that are images of each other under an isometry, find the isometry and describe it completely.

G3.1.3 Find the image of a figure under the composition of two or more isometries and determine whether the resulting figure is a reflection, rotation, translation, or glide reflection image of the original figure.

## CCSS Cluster Statements and Standards

## Experiment with transformations in the plane.

G.CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
G.CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

STANDARD G3: TRANFORMATION OF
FIGURES INTHE PLANE

## CCSS Cluster Statements and Standards

G.CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
Understand congruence in terms of rigid motions.
G.CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

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STANDARD G3: TRANFORMATION OF FIGURES INTHE PLANE

## Shape-preserving Transformations: Dilations and Isometries

G3.2.I Know the definition of dilation and find the image of a figure under a given dilation.
G3.2.2 Given two figures that are images of each other under some dilation, identify the center and magnitude of the dilation.

## CCSS Cluster Statements and Standards

## Experiment with transformations in the plane.

G.CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

## Understand similarity in terms of similarity transformations.

G.SRT.I Verify experimentally the properties of dilations given by a center and a scale factor:
a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
G.SRT. 2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain, using similarity transformations, the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
G.SRT. 3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

## STRAND 4: STATISTICS AND PROBABILITY

STANDARD SI UNIVARIATE DATA EXAMINING DISTRIBUTIONS

## Producing and Interpreting Plots

SI.I.IConstruct and interpret dot plots, histograms, relative frequency histograms, bar graphs, basic control charts, and box plots with appropriate labels and scales; determine which kinds of plots are appropriate for different types of data; compare data sets and interpret differences based on graphs and summary statistics.

SI.I. 2 Given a distribution of a variable in a data set, describe its shape, including symmetry or skewedness, and state how the shape is related to measures of center (mean and median) and measures of variation (range and standard deviation), with particular attention to the effects of outliers on these measures.

## Measures of Center and Variation

SI.2.I Calculate and interpret measures of center including: mean, median, and mode; explain uses, advantages and disadvantages of each measure given a particular set of data and its context.

SI.2.2 Estimate the position of the mean, median, and mode in both symmetrical and skewed distributions, and from a frequency distribution or histogram.

SI.2.3 Compute and interpret measures of variation, including percentiles, quartiles, interquartile range, variance, and standard deviation.

CCSS Cluster Statements and Standards

Reason quantitatively and use units to solve problems.
N.Q.I Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

Summarize, represent, and interpret data on a single count or measurement variable.
S.ID.I Represent data with plots on the real number line (dot plots, histograms, and box plots).
S.ID. 2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
S.ID. 3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
Make inferences and justify conclusions from sample surveys, experiments, and observational studies.
S.IC. 6 Evaluate reports based on data.

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## STANDARD SI UNIVARIATE DATA -

EXAMINING DISTRIBUTIONS

## The Normal Distribution

SI.3.I Explain the concept of distribution and the relationship between summary statistics for a data set and parameters of a distribution.

SI.3.2 Describe characteristics of the normal distribution, including its shape and the relationships among its mean, median, and mode.

SI.3.3 Know and use the fact that about 68\%, 95\%, and $99.7 \%$ of the data lie within one, two, and three standard deviations of the mean, respectively, in a normal distribution.

SI.3.4 Calculate z-scores, use z-scores to recognize outliers, and use $z$-scores to make informed decisions.

## CCSS Cluster Statements and Standards

Summarize, represent, and interpret data on a single count or measurement variable.
S.ID.I Represent data with plots on the real number line (dot plots, histograms, and box plots).
S.ID. 2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
S.ID. 4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.
S.IC. 6 Evaluate reports based on data.

Calculate expected values and use them to solve problems.
S.MD.I (+) Calculate expected values and use them to solve problems. Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.

Use probability to evaluate outcomes of decisions.
S.MD. 7 (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

STANDARD $\mathbf{S 2}$ BIVARIATE DATA EXAMINING RELATIONSHIPS

## Scatter plots and Correlation

S2.I.I Construct a scatter plot for a bivariate data set with appropriate labels and scales.

S2.I. 2 Given a scatter plot, identify patterns, clusters, and outliers. Recognize no correlation, weak correlation, and strong correlation.

S2.I.3 Estimate and interpret Pearson's correlation coefficient for a scatter plot of a bivariate data set. Recognize that correlation measures the strength of linear association.

## CCSS Cluster Statements and Standards

Summarize, represent, and interpret data on two categorical and quantitative variables.
S.ID. 6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

Interpret linear models.
S.ID. 8 Compute (using technology) and interpret the correlation coefficient of a linear fit.
S.ID. 9 Distinguish between correlation and causation.

STANDARD $\mathbf{S 2}$ BIVARIATE DATA -
EXAMINING RELATIONSHIPS
S2.I. 4 Differentiate between correlation and causation. Know that a strong correlation does not imply a cause-and-effect relationship. Recognize the role of lurking variables in correlation.

## Linear Regression

S2.2.I For bivariate data that appear to form a linear pattern, find the least squares regression line by estimating visually and by calculating the equation of the regression line. Interpret the slope of the equation for a regression line.
S2.2.2 Use the equation of the least squares regression line to make appropriate predictions.

CCSS Cluster Statements and Standards

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.
S.IC. 6 Evaluate reports based on data.

Construct and compare linear, quadratic, and exponential models and solve problems.
F.LE. 2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two inputoutput pairs (include reading these from a table).
Interpret expressions for functions in terms of the situation they model
F.LE. 5 Interpret the parameters in a linear or exponential function in terms of a context.
Make inferences and justify conclusions from sample surveys, experiments, and observational studies.
S.IC. 6 Evaluate reports based on data.

Summarize, represent, and interpret data on two categorical and quantitative variables.
S.ID. 6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related:
a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.
b. Informally assess the fit of a function by plotting and analyzing residuals.
c. Fit a linear function for a scatter plot that suggests a linear association.
Interpret linear models.
S.ID. 7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

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STANDARD S3 SAMPLES, SURVEYS, EXPERIMENTS

## Data Collection and Analysis

S3.I.I Know the meanings of a sample from a population and a census of a population, and distinguish between sample statistics and population parameters.

S3.I.2 Identify possible sources of bias in data collection, sampling methods and simple experiments; describe how such bias can be reduced and controlled by random sampling; explain the impact of such bias on conclusions made from analysis of the data; know the effect of replication on the precision of estimates.

S3.I. 3 Distinguish between an observational study and an experimental study and identify, in context, the conclusions that can be drawn from each.

## STANDARD $\mathbf{S 4}$ PROBABILITY MODELS AND

 PROBABILITY CALCULATIONS
## Probability

S4.I.I Understand and construct sample spaces in simple situations.

S4.I. 2 Define mutually exclusive events, independent events, dependent events, compound events, complementary events and conditional probabilities; use the definitions to compute probabilities.

## CCSS Cluster Statements and Standards

Understand and evaluate random processes underlying statistical experiments.
S.IC.I Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
Make inferences and justify conclusions from sample surveys, experiments, and observational studies.
S.IC. 3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
S.IC. 6 Evaluate reports based on data.

## CCSS Cluster Statements and Standards

Understand and evaluate random processes underlying statistical experiments.
S.IC. 2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls head side up with probability 0.5 . Would a result of 5 tails in a row cause you to question the model?

Understand independence and conditional probability and use them to interpret data.
S.CP.I Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
S.CP. 2 Understand that two events $A$ and $B$ are independent if the probability of A and B occurring together is the product of their probabilities; use this characterization to determine if they are independent.
S.CP. 3 Understand the conditional probability of $A$ given $B$ as $P(A$ and $B) / P(B)$, and interpret independence of $A$ and $B$ as saying that the conditional probability of $A$ given $B$ is the same as the probability of $A$, and the conditional probability of $B$ given $A$ is the same as the probability of $B$.

STANDARD S4 PROBABILITY MODELS AND PROBABILITY CALCULATIONS

Probability (continued)

CCSS Cluster Statements and Standards
S.CP. 4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.
S.CP. 5 Recognize and explain the concepts of conditional probability and independence in everyday language and situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.

Calculate expected values and use them to solve problems.
S.MD. 3 (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.
Use probability to evaluate outcomes of decisions
S.MD.5b (+) Evaluate and compare strategies on the basis of expected values. For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.
S.MD. 6 (+) Use probability to evaluate outcomes of decisions. Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
S.MD. 7 (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

## Mathematical Practices

I. Make sense of problems, and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments, and critique the reasoning of others.
4. Model with mathematics.
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STANDARD $\mathbf{S 4}$ PROBABILITY MODELS AND PROBABILITY CALCULATIONS

## Application and Representation

S4.2.I Compute probabilities of events using tree diagrams, formulas for combinations and permutations, Venn diagrams, or other counting techniques.

S4.2.2 Apply probability concepts to practical situations in such settings as finance, health, ecology, or epidemiology, to make informed decisions.

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Make inferences and justify conclusions from sample surveys, experiments, and observational studies
S.IC. 4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
S.IC. 5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.*

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(continued)

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## CONTENT THAT IS DIFFERENT

## Content moving out of high school

Number Systems and Number Sense
LI.I. 4 Describe the reasons for the different effects of multiplication by, or exponentiation of, a positive number by a number less than 0 , a number between 0 and I, and a number greater than I.
LI.I. 5 Justify numerical relationships

## Representations and Relationships

## LI.2.2 Interpret representations that reflect

absolute value relationships in such contexts as error tolerance.

## CONTENT THAT IS DIFFERENT

## Content moving out of high school

## Calculation Using Real and Complex Numbers

L2.I.I Explain the meaning and uses of weighted averages.

## Language and Laws of Logic

L3.2.4 Write the converse, inverse, and contrapositive of an "if..., then..." statement. Use the fact, in mathematical and everyday settings, that the contrapositive is logically equivalent to the original, while the inverse and converse are not.

## Proof

L3.3.I Know the basic structure for the proof of an "if..., then..." statement (assuming the hypothesis and ending with the conclusion) and that proving the contrapositive is equivalent.

L3.3.2 Construct proofs by contradiction. Use counterexamples, when appropriate, to disprove a statement.

L3.3.3 Explain the difference between a necessary and a sufficient condition within the statement of a theorem. Determine the correct conclusions based on interpreting a theorem in which necessary or sufficient conditions in the theorem or hypothesis are satisfied.

## No alignment

No alignment

No alignment

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## Michigan HS Content Expectations

## Power Functions

A3.4.2 Power Functions: Express direct and inverse relationships as functions and recognize their characteristics.

A 3.4.3 Power Functions: Analyze the graphs of power functions, noting reflectional or rotational symmetry.

## Triangles and Their Properties

GI.2.4 Prove and use the relationships among the side lengths and the angles of $30^{\circ}-60^{\circ}-90^{\circ}$ triangles and $45^{\circ}-45^{\circ}-90^{\circ}$ triangles.

## Triangles and Trigonometry

GI.3.3 Determine the exact values of sine, cosine, and tangent for $0^{\circ}, 30^{\circ}, 45^{\circ}, 60^{\circ}$ and their integer multiples, and apply in various contexts.

## Other Polygons and Their Properties

G1.5.2 Know, justify, and use formulas for the perimeter and area of a regular $n$-gon, and formulas to find interior and exterior angles of a regular n -gon and their sums.

## Three- Dimensional Figures

GI.8.2 Identify symmetries of pyramids, prisms, cones, cylinders, hemispheres, and spheres.

## Relationships Between Area and Volume Formulas

G2.I.I: Know and demonstrate the relationships between the area formula of a triangle, the area formula of a parallelogram, and the area formula of a trapezoid.

## G2.I. 2 Know and demonstrate the relationships

 between the area formulas of various quadrilaterals.
## No alignment

## No alignment

## No alignment

No alignment

## CCSS Common Core State Standards

## Michigan HS Content Expectations

Partial Alignment to AI.I. 7
(HSCE Recommended I I/07)

Partial Alignment to A2.2.4
(HSCE Recommended II/07)

## Prove and apply trigonometric identities

F.TF. 8 Prove the Pythagorean identity $\sin ^{2}(\square)+\cos ^{2}$ $(\square)=I$ and use it to find $\sin \square, \cos \square$, or $\tan \square$, given $\sin \square, \cos \square$, or $\tan \square$, and the quadrant of the angle.
Build new functions from existing functions
F.BF. 4 Find inverse functions.
a. Solve an equation of the form $f(x)=c$ for a simple function $f$ that has an inverse and write an expression for the inverse. For example, $f(x)=2\left(x^{3}\right)$ or $f(x)=(x+1) /(x-1)$ for $x \neq 1(x$ not equal to 1$)$.

No alignment

Partial Alignment to GI.7.4
(HSCE Recommended I I/07)
G.MG. 3 Apply geometric concepts in modeling situations. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).
G.GPE. 2 Translate between the geometric description and the equation for a conic section. Derive the equation of a parabola given a focus and directrix.

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Education

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