Writing Across the Curriculum

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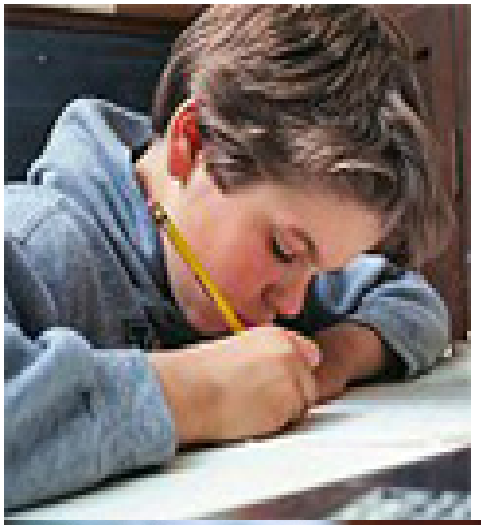
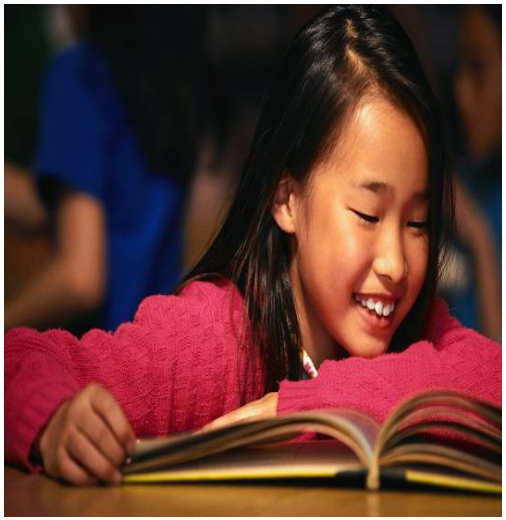
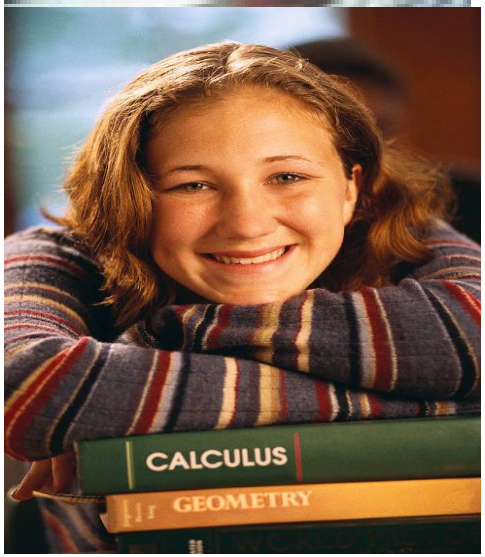
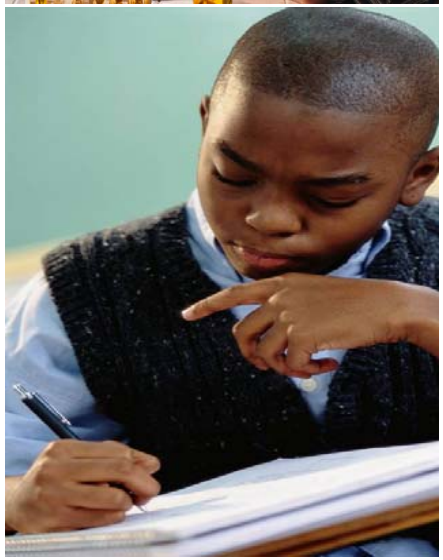








MICHIGAN Department of Education

Introduction: Writing Across the Curriculum

What is it?
Teachers across the disciplines use writing-to-learn and writing-to–demonstrate-knowledge to enhance the learning of students in all disciplines.

Basic Principles of Writing Across the Curriculum (WAC)
In response to the need of students to learn content using a variety of strategies and their need to practice writing in a variety of contexts, many teachers have adopted the strategies associated with WAC. The following principles underlie WAC:

- Writing promotes learning.
- Integration of writing and the writing process promotes student participation, a diversity of student voices, and engages students as critical thinkers while promoting their texts as important resources and thinking tools.
- Effective writing instruction integrates disciplines.
- The opportunity to write in every class develops good writers.
- Using writing as part of instruction can be used in every classroom.
- Only by practicing the thinking and writing conventions of an academic discipline will students begin to communicate effectively within that discipline.

What's In It For Teachers and Students?
Including writing in instruction has short- and long-term benefits. In the short term, students and their teachers are better able to appraise how well they grasp information and where deeper elaboration of key concepts is needed. Students are able to take small pieces of content and analyze it looking for patterns and connections. In the long run, students who use writing as a technique to learn content develop their skills as thinkers. Organization, summary, and analysis of content become easier for students, producing richer understandings. Students become more practiced at using writing to communicate their learning and thinking.

Writing is used to initiate discussion, reinforce content, and model the method of inquiry common to the field. Writing can help students discover new knowledge—to sort through previous understandings, draw connections, and uncover new ideas as they write.

Writing-to-learn activities encourage the kind of reflection on learning that improves students' metacognitive skills. The key to effectively using writing activities in every subject lies in matching the right activity to the learning situation. As you select writing strategies, ask yourself: "How well suited is this task for the objective the students are learning?" "Does this strategy fit my students' abilities and needs?" "Will this strategy complement the way my students will be assessed on content later?"

Assigned writing in all classes and courses helps students keep their writing skills sharp. Students become better readers, thinkers, and learners in a discipline by processing their ideas through writing. Writing assigned across the curriculum also helps students prepare for the day-in and day-out communicative tasks they'll face on the job, no matter what the job is. Equally important, students need to learn about how writing is used within a discipline; and utilizing many different kinds of writing assignments gives students practice with a variety of disciplinary forms and conventions.

So, why assign writing in your classes? Students will learn more content, will clarify their thinking, and will leave your classroom better prepared to face thinking and communication challenges.

Definition: Writing-To-Learn

A writing-to-learn strategy is one that teachers employ throughout and/or at the end of a lesson to engage students and develop big ideas and concepts.

Writing-to-learn fosters critical thinking, requiring analysis, application, and other higher level thinking skills. It is writing that uses impromptu, short, or informal writing tasks designed by the teacher and included throughout the lesson to help students think through key concepts and ideas. Attention is focused on ideas rather than correctness of style, grammar, or spelling. It is less structured than disciplinary writing.

This approach frequently uses journals, logs, micro-themes, responses to written or oral questions, summaries, free writing, notes, and other writing assignments that align to learning ideas and concepts.

Definition: Writing-To-Demonstrate-Knowledge

A writing-to-demonstrate-knowledge assignment is one that teachers employ when they assign reports, essays, persuasive writing, and creative or expressive writing, as well as research papers.

When writing-to-demonstrate-knowledge, students show what they have learned by synthesizing information and explaining their understanding of concepts and ideas. Students write for an audience with a specific purpose. Products may apply knowledge in new ways or use academic structures for research and/or formal writing.

Examples include essays that deal with specific questions or problems, letters, projects, and more formal assignments or papers prepared over weeks or during a course. They adhere to format and style guidelines or standards typical of professional papers, such as reports, article reviews, and research papers, and should be checked before submitted by the student for correctness of spelling, grammar, and transition word usage.

Preface: WAC In Mathematics

What is it?
Teachers in mathematics use principles of Writing Across the Curriculum such as writing-to-learn and writing- to-demonstrate-knowledge to enhance the learning of students in key areas of the curriculum.

Learning mathematics is much more complex than memorizing sets of facts and examples. In order to develop new and/or improved conceptual frameworks, students must be given the opportunity to process their ideas before, during, and after new learning takes place. This can be done orally, mentally, or in writing. This document will describe for teachers a number of writing strategies that students can use to surface their currently-held ideas and then process them in relation to new mathematical information.

In addition, it is important for all mathematicians to be able to write clearly and effectively. Not only do they have to keep complete records of their ideas and work, but they also have to be able to effectively communicate their findings to worldwide audiences. Sometimes they are also expected to write grant applications and share their findings with people outside the mathematical community. Teaching students to write well must be a part of any comprehensive mathematics program. Strategies that require students to demonstrate their knowledge of mathematics also provide opportunities to practice writing for authentic audiences.

In order for writing in mathematics to impact student learning, it must be more than just copying the notes given in class. Information must be personalized in some way. Students must be expected to include reflections and questions when they write. By making these personal connections, students will begin to develop a conceptual understanding of the mathematics they are exploring in their studies.

Many mathematics educators feel that students should already know how to write effectively when they come to their classrooms. This is not usually the case, however. Students have learned to write from their English language arts teachers, but they usually do not know how to apply these skills to mathematics. Mathematics teachers will find that they may have to explicitly teach and provide scaffolding for each of these strategies before their students will be able to implement writing in mathematics.

Teachers should implement steps from the writing process to help students use writing strategies effectively in mathematics:

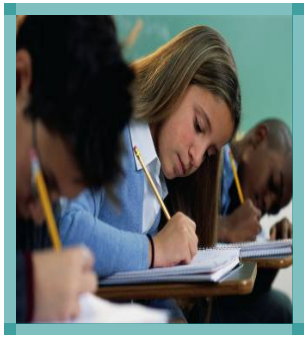
- Explain the strategy and its purpose. If appropriate, describe the real-world application and audience for the product.
- Model how to do a sample of the expected writing. While modeling, talk aloud about the thinking that goes on while preparing to write and during writing.
- Have students practice the strategy. This might be done in small groups with the teacher or with partners.
- Provide feedback on the work, encouraging students to use the feedback in their next efforts.
- Encourage students to become more independent in their practice as they build their skills.

When students have demonstrated that they have mastered the strategy, it can be used for a variety of classroom purposes. A few examples are:

- Keeping a personal record of ideas and experiences.
- Providing formative assessment data for teachers.
- Providing summative data for evaluation.
- Communicating learning with parents and other interested parties.

Each of the following strategies will include a specific example. Please note that teachers are encouraged to modify these as needed.

Table Of Contents: WAC In Mathematics



We gratefully acknowledge the Executive Board of the Michigan Council for Teachers of Mathematics for support in drafting the examples for the Mathematics Writing Across the Curriculum document. Teresa Ballard, Kevin Dykema, Jim Licht, Bob Cooper, and Betty Warren are recognized for their assistance in helping to make mathematics accessible and meaningful to Michigan students.

Writing-To-Learn: Mathematics

What is it?
A writing-to-learn strategy is one that teachers employ throughout and/or at the end of a lesson to engage students and develop big ideas and concepts.

Strategy: Cognitive Academic Language Learning Approach (CALLA)
The CALLA strategy provides support for students in learning content as well as learning to learn. Students read problems carefully and record what the problem is asking. Throughout the process, students are guided to a solution by asking them to solve, check and explain their work. Students are encouraged to write about what made the problem difficult, or strategies that helped them solve the problem (Kenney, et. al. 2005) *Mathematical Literacy: Helping Students Make Meaning in the Middle Grades.*

What does it do?
- Provides support for ESL learners in content and learning strategies.
- Helps with organization for all students.

How to implement:

Example 1: Word Problem Procedure
- Choose a partner.
- Choose a problem and write it out.
- Have one student read the problem out loud. Discuss the vocabulary and circle words you don't understand.
- Using a dictionary or partner for help, write out the definitions of the vocabulary words that you don't understand.
- Identify and write what the problem is asking you to find.
- Explore what mathematical process you should use to solve the problem.
- Consider and outline what mathematical procedures this may include.
- Solve the problem.
- Check your answer.
- Explain your answer to your partner.
- Write your explanation.
- Explain your answer to the class.
- Write a similar problem on a piece of paper.

Example 2: Mathematics Learning Strategy Checklist

There are many ways to solve problems. Check the two or three things that you did most while you worked on this problem. There are no right or wrong answers.

- ☐ **I looked for the important words to solve the problem.**
- ☐ **I read the question carefully.**
- ☐ **I remembered how I solved other problems like this one.**
- ☐ **I did the problem in my head because it was easy.**
- ☐ **I formed a picture in my head or drew a picture.**

Example 3: Math Student Self-Evaluation

- These are two important things I learned in math today/this week/this month:

  1. _____

  2. _____

- This was an easy problem for me: _____

  This was a difficult problem for me: _____

- I need more help with: _____

- This is how I feel about math today/this week/this month: (Circle your answer).

successful happy excited confused interested worried relaxed bored upset

- This is where I got help (circle words that are true):

a teacher a friend my parents the internet other (explain)

Writing-To-Learn: Mathematics

What is it?
A writing-to-learn strategy is one that teachers employ throughout and/or at the end of a lesson to engage students and develop big ideas and concepts.

Strategy: Column Notes
The double (or triple) entry journal is a two (or three) column graphic organizer. Students record important factual information from text and/or lecture in the left-hand column. The right-hand column (or middle column) is used by the student to process and record personal responses to the information. A third column can be added if needed for student response that summarizes or adds to student understanding. For more information on column notes go to: http://forpd.ucf.edu/strategies/stratCol.html

What does it do?
- Helps students recall information.
- Provides students with an opportunity to clarify information.
- Helps students make personal connections with the new information.
- Encourages students to analyze and question information presented.
- Informs teacher on extent to which students have understood the new information (formative assessment).

How to implement:
- Teacher models use of the organizer by describing how to identify important information and modeling how to take notes.
- Teacher explains the purpose of the right side of the organizer and models how to add a personal response.
- Teacher lectures for 10 minutes while students take notes on the left side of the organizer.
- Students write and/or sketch reactions to their notes on the right side of the organizer.
- Students share reactions with a partner and then repeat the steps.
- At the end of the lesson, students write a summary paragraph about what they have learned and compare their summaries with a partner.

Example:

Notes	Personal Connection	More examples/ summary
Fractions, Percents and Decimals **½ = .5= 50%** **3/3 = 1.00 = 100%**	**One-half of a candy bar is the same as 50% of the candy bar. I could divide it equally between 2 people.**	**On a number line, 50% is the same place as ½ and .50**

Writing-To-Learn: Mathematics

What is it?
A writing-to-learn strategy is one that teachers employ throughout and/or at the end of a lesson to engage students and develop big ideas and concepts.

Strategy: Compare and Contrast
Students collect information about two or more mathematical concepts or examples. The key attributes are recorded on a two-column graphic organizer/chart to clarify similarities and differences.

What Does It Do?
- Encourages students to examine systems being compared analytically.
- Helps students organize/classify the information in a personal manner.
- Provides structure for remembering characteristics for mathematical examples.

How to Implement:
- Students collect information about the concept being studied.
- Students carefully analyze information.
- Students record key attributes.
- Students summarize similarities and differences (first recording similarities and then recording differences) in a chart or short explanation.

Example 1: Comparison of Rhombus and Square
Compare and contrast the attributes of a rhombus with those of a square.

Rhombus	Square
Similarities: Quadrilateral Polygon 2 pairs of parallel sides all sides equal	**Similarities:** Quadrilateral Polygon 2 pairs of parallel sides all sides equal
Differences:	**Differences:** four 90^0 angles

Example 2: Comparing Problems and Answers
Compare and contrast finding 1 – 4/5 with 13 – 4/5 .
These are similar problems because if I find 1 – 4/5 = 1/5, I will know that 13 – 4/5 is 12 more, so the answer equals 12 1/5. They are different problems because they have a different answer.

Example 3: These are / These are not

These are equations	These are not equations
N = 3	X > 2
3x + 5 = 7	4 < 7
3 + 4 – 1 = 5 – 2 + 3	5 - 3
$y = x^2$	½ + ¾
2A = lw	
L = ½ w	

(Examples 1,2,3 are adapted from, Thompson, Kersaint, Richards, Hundsader, & Rubenstein, *Mathematical Literacy)*

Writing-To-Learn: Mathematics

What is it?

A writing-to-learn strategy is one that teachers employ throughout and/or at the end of a lesson to engage students and develop big ideas and concepts.

Strategy: Concept Definition Map

A visual representation in which sub-concepts, vocabulary terms and examples are related to a main topic.

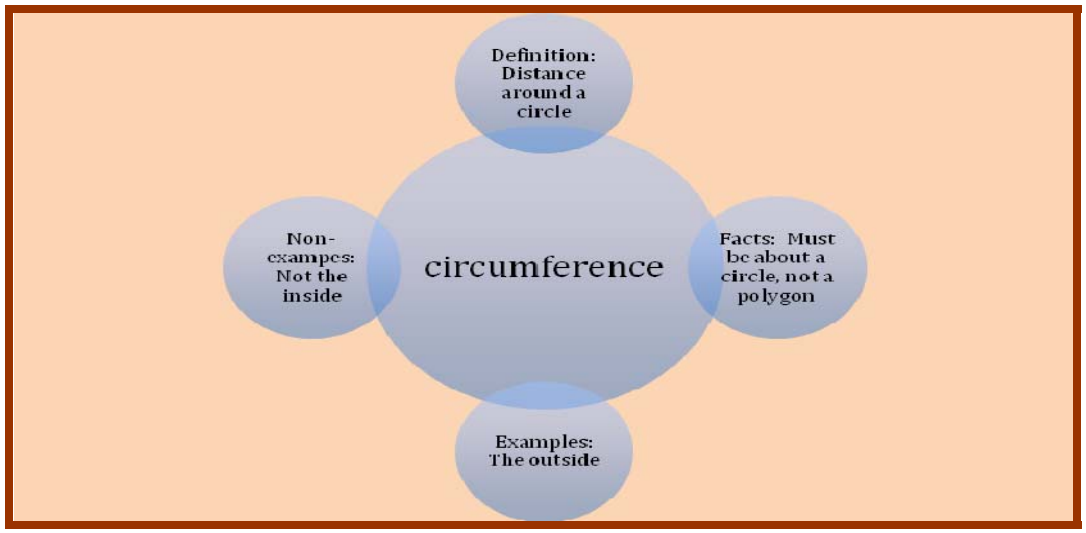
What does it do?

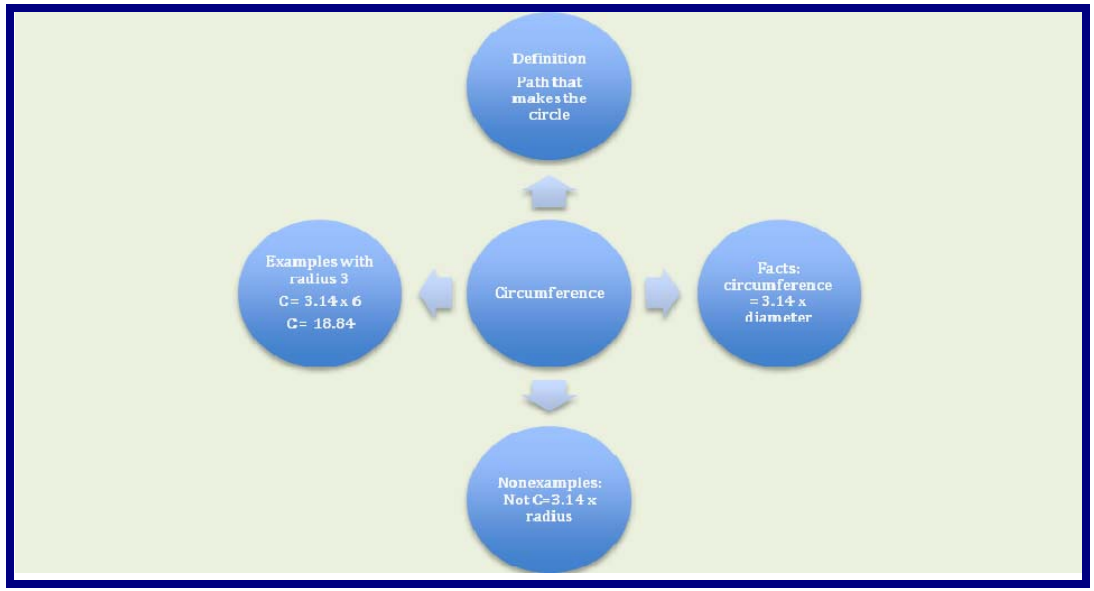
- Helps students make connections between ideas.
- Provides opportunities for review.
- Provides a tool for reflection on changes in understanding.
- Provides an opportunity to access prior knowledge.

How to implement:

- Teacher models how to create a concept map using everyday examples.
- Class creates a concept map together on a topic previously studied.
- Teacher provides a list of terms related to the upcoming topic of study and any scaffolding needed for using this strategy.
- Students independently create a concept map that includes the terms provided, additional terms of their choosing, and connecting phrases.
- Students revisit their map periodically during the unit, adding and changing pieces as needed.
- Students write a reflective paragraph on how their thinking has grown or changed during the unit.

Examples for " Circumference" (next page):


Definition: Distance around a circle
Non-exampes: Not the inside
circumference
Facts: Must be about a circle, not a polygon
Examples: The outside

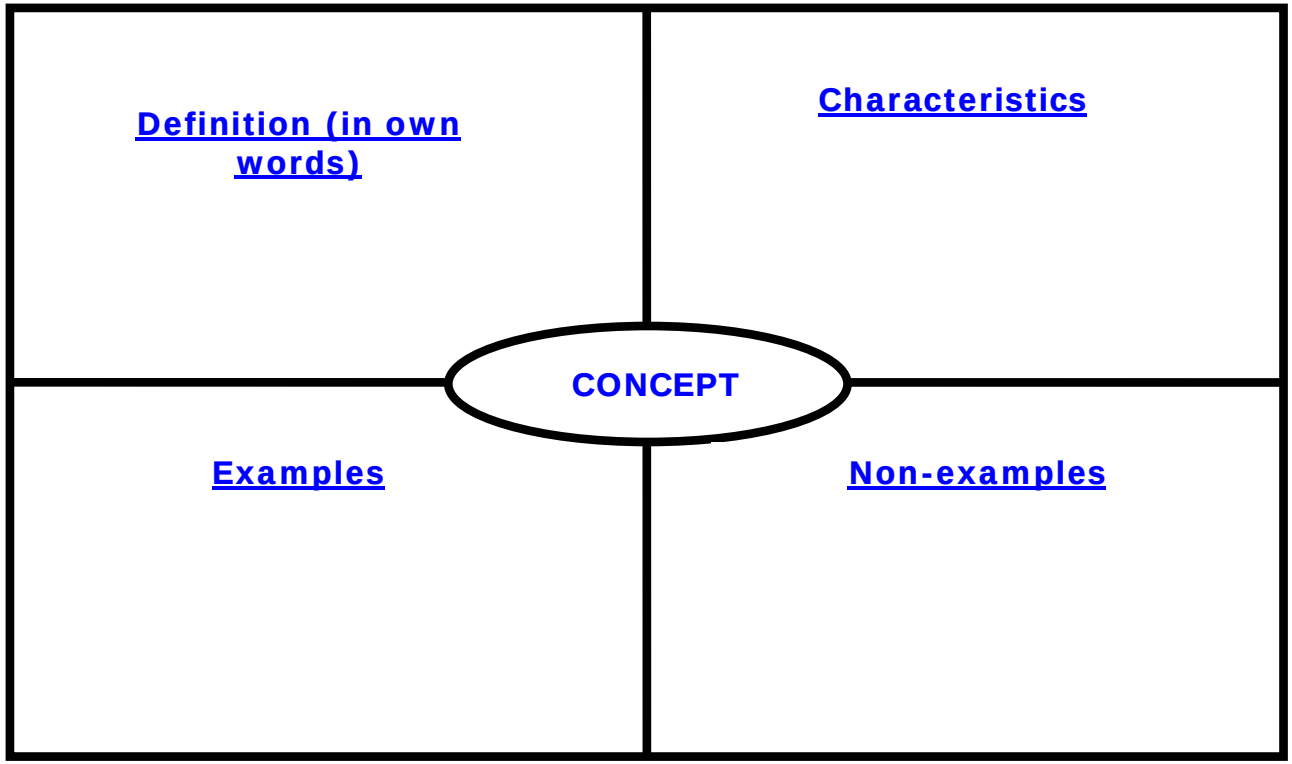

Definition
Path that makes the circle
Examples with radius 3
C= 3.14 x 6
C= 18.84
Circumference
Facts: circumference = 3.14 x diameter
Nonexamples: Not C=3.14 x radius

Writing-To-Learn: Mathematics

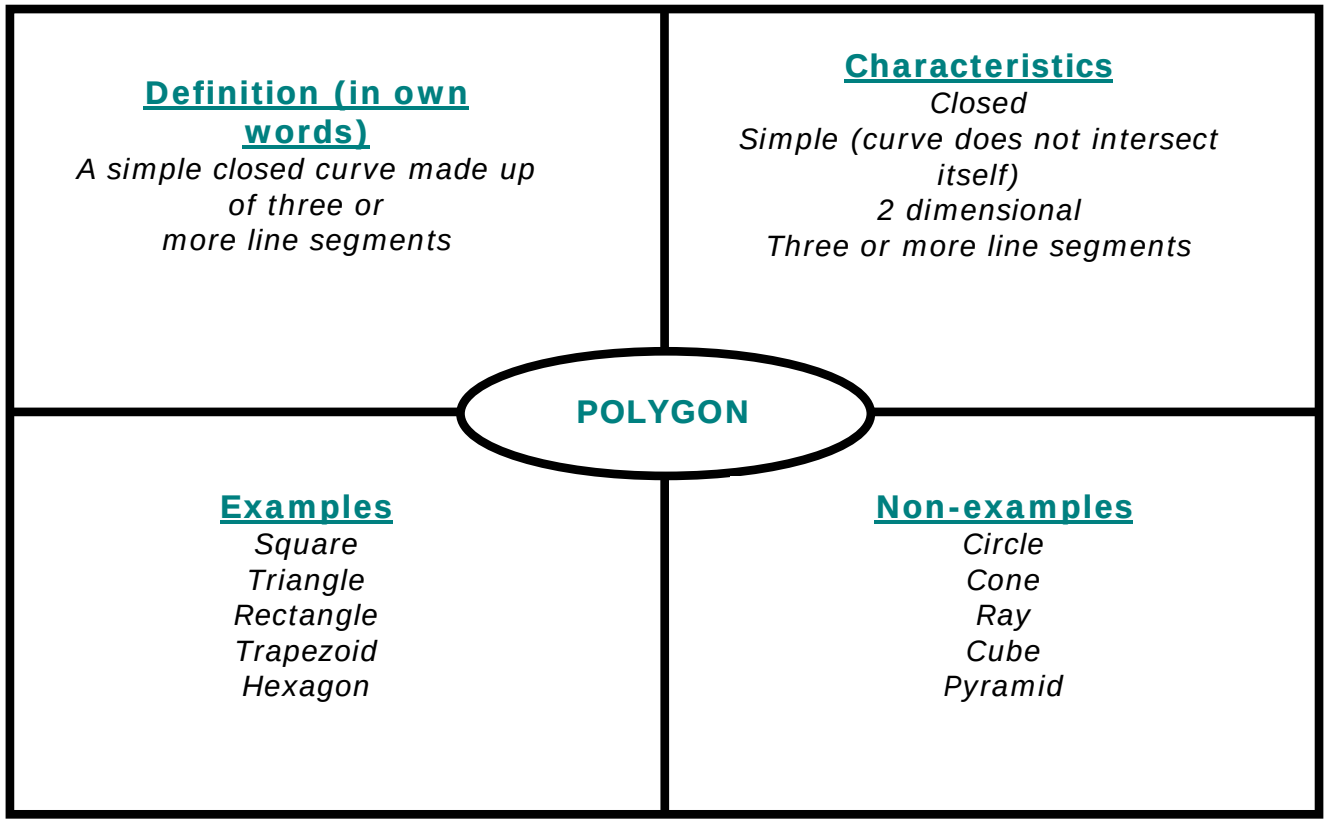
What is it?
A writing-to-learn strategy is one that teachers employ throughout and/or at the end of a lesson to engage students and develop big ideas and concepts.

Strategy: Frayer Model of Concept Development
Students use a variety of methods and modes (oral, visual, written) to develop a personal, in-depth understanding of key mathematical concepts and terms. The Frayer Model (Frayer, Federick, & Klausmeier, 1969) is a strategy designed to help students understand concepts. This strategy uses a graphic organizer to help students understand a concept using definitions, characteristics, examples and non-examples to clarify meaning.

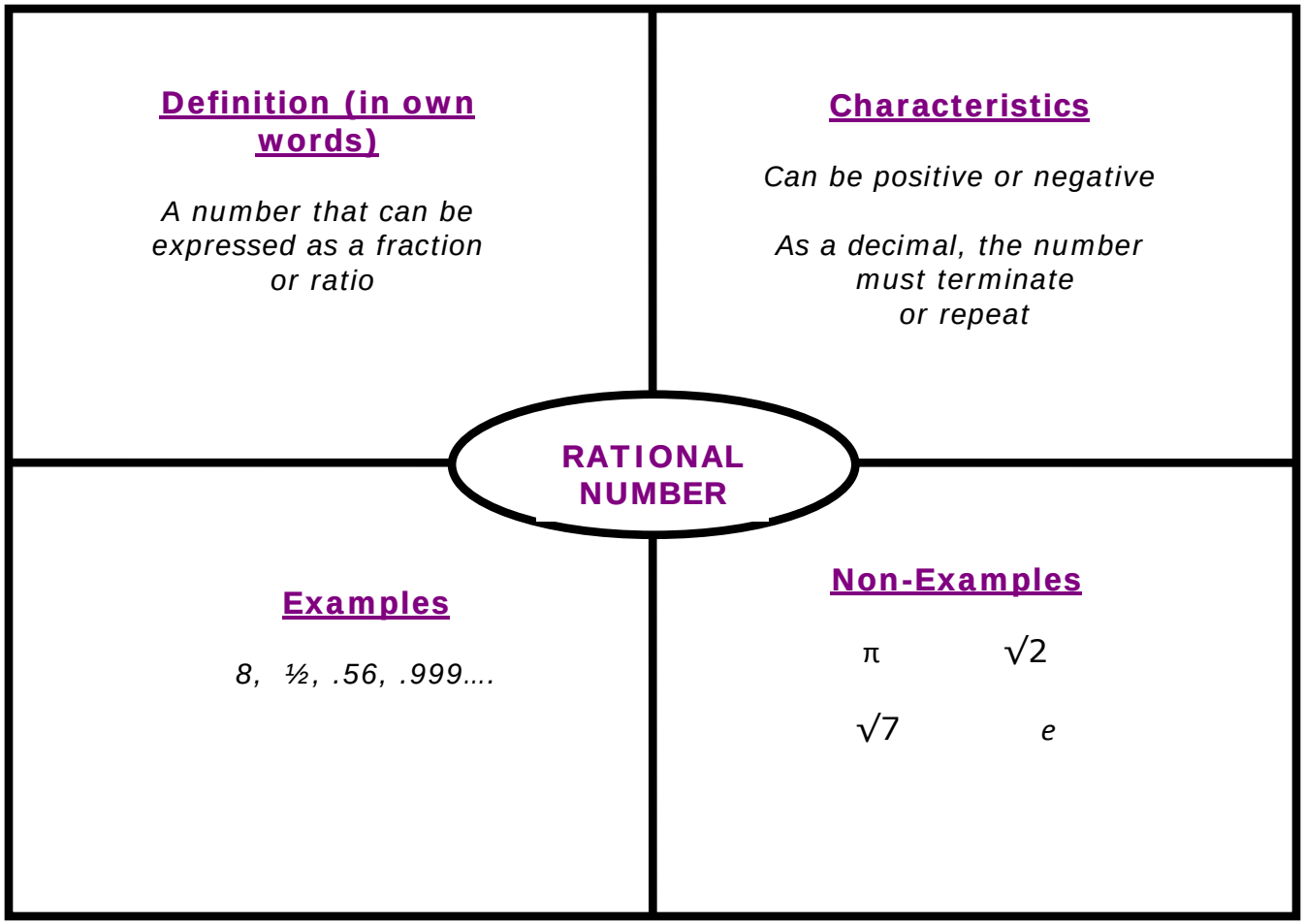
Template:



Example 1: Concept Of Polygon



Example 2: Concept Of Rational Number:



Writing-To-Learn: Mathematics

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Strategy: GIST (Generating Interactions Between Schemata and Text)
GIST is a step-by-step process for summarizing text material. It is particularly valuable with students who have difficulty putting what they have read into their own words. Students restate main ideas from mathematics text, omitting specific examples and/or evidence used to support or illustrate the concept(s) (Cunningham, 1982).

What does it do?
- Improves student comprehension of mathematics text.
- Helps students process new information and make connections with own ideas/experiences.
- Provides a structure for identifying and remembering key ideas.
- Helps students surface questions about what they have read.

How to implement:
Assign a mathematics passage for students. Teacher models the following steps and then scaffolds practice as needed until students are able to use them independently.
- Read the text (or a portion of for longer pieces) and write down the important ideas.
- Using the important ideas, write a summary of no more than 15 words.
- Continue portion by portion for longer pieces until assigned text is completed.
- Write a short summary of the text selection in own words.

Grade 3-5 Example:

Text Passage

Numbers like 46,839 have 5 digits. Each digit in the number has a different place value. The first digit is in the ten thousands place. The number 46,839 has 4 ten thousands. The second digit is the thousands place. In the example, there are 6 thousands. The third digit is the hundreds place, so the example has 8 hundreds. The next digit is in the tens place, so the example has 3 tens. Finally the last digit is in the ones place. The number has 9 ones.

Important Ideas:

- 5 digits
- Place Value
- ten thousands place
- thousands place
- hundreds place
- tens place
- ones place

Summary:

The number 46839 has 4 ten thousands, 6 thousands, 8 hundreds, 3 tens, 9 ones.

Grade 6-8 Example:

Text Passage

Ratios express how one number is related to another. It may be written as a/b, a:b, or as a phrase, *a to b*. For example the ratio of 1:8 is read as 1 to 8, and means that the second number is eight times as large as the first. A proportion is a statement of equivalency for two or more proportions. Given the proportion of a:b = 3:8 and asked to find b if a = 12, follow these steps:

1. **First substitute 12 in the proportion for a, 12:b = 3:8; or** $\frac{12}{b} = \frac{3}{8}$
2. **Then use cross products, 3 x b = 12 x 8.**
3. **Solve the equation 3b = 96, b = 32.**
4. **Therefore, if the ratio of a to b is 3:8 and a = 12, then b = 32.**

Important Ideas:

- How numbers relate
- Can be written different ways
- a:b is 3:8 and a=12
- The proportion is 12:b = 3:8
- 3 x b = 12 x 8 which is 3b =96
- 96 ÷ 3 = 32
- b = 32

Summary:

Ratios express how numbers relate and if the ratio is 3:8 and a = 12, then b = 32.

Writing-To-Learn: Mathematics

What is it?
A writing-to-learn strategy is one that teachers employ throughout and/or at the end of a lesson to engage students and develop big ideas and concepts.

Strategy: Journaling
A journal, or math notebook, is a record of student learning experiences over a period of time. This notebook can be used exclusively for discoveries, or it can be used to include reflections from reading or class discussions. It is more than a collection of observations, facts learned, and procedures conducted. The notebook also documents student reflections, questions, predictions, and conclusions.

What does it do?
- Provides permanence and stability for student investigation results, reflections, and questions.
- Provides a resource for reviewing information learned during mathematics lessons.
- Provides feedback about students' investigatory skills: measuring, recording data, graphing, etc. (i.e. both self and formative assessment).

How to implement:
- Teacher explains purpose and provides specific format for mathematics notebook. (Audience is student, but format is still important.)
- Students use notebook prior to learning new concept to record their thinking (including questions and predictions).
- During learning of new concept, students record procedure uses and observations. Students should not be expected to copy information from a handout of directions or from the text-these can be referenced.
- After the lesson is completed, students should record conclusions, reflections, and any new understandings they gained as a result of the lesson. In addition, they record questions they still have or any new questions stimulated by the lesson.
- During class discussions, students record new information and reflect on their learning.
- Teacher provides feedback on entries and encourages student to continue to process ideas.

Variation: Learning Log
Students write regularly in their notebook. This can be done the first five minutes of class to review or raise questions, in the middle of a lesson to process ideas, or during the last five minutes of class to summarize the day's learning.

***Sample prompts*:**
- Which part of your homework assignment was most difficult?
- What are two questions you would like answered during today's (tomorrow's) lesson?
- Summarize what you learned about this topic today/yesterday.
- What is one thing that is still puzzling you?

Writing-To-Learn: Mathematics

What is it?
A writing-to-learn strategy is one that teachers employ throughout and/or at the end of a lesson to engage students and develop big ideas and concepts.

Strategy: Marginal Notes

Marginal notes are short written statements in which students record their interactions with the text in the margins while they are reading.

What does it do?

- Activates students' background knowledge.
- Helps students monitor their comprehension of text.
- Stimulates questioning and analysis of text.
- Helps students become aware of connection between text and their thinking (meta-cognition).

How to implement:

- Teacher explains purpose for reading and taking marginal notes.
- Teacher uses "think-aloud" while modeling how to record responses to text.
- Students read the text segment (or math problems), make notes, and highlight or underline key points.
- Students record short notes in margins of text as they re-read (either directly or on sticky notes).
- Students discuss notes with partners and/or class.

Example 1:

Factor This Quadratic:	Student Response:
$X^2 + 6x + 8$	*I have to think about how to solve this quadratic.*
$X^2 + 4x + 2x + 8$	*I don't know where the 4x and 2x came from.*
x(x+4) + 2(x+4)	*It looks like x was factored from the first 2 terms and 2 was factored from the last 2 terms.*
(x + 4) (x + 2)	*I don't know how an addition problem became a multiplication problem.*

If this is discussed with a partner and/or read by the teacher, suggestions specific to this student's confusion may be addressed and corrected immediately.

Writing-To-Learn: Mathematics

What is it?

A writing-to-learn strategy is one that teachers employ throughout and/or at the end of a lesson to engage students and develop big ideas and concepts.

Strategy: Quick Write

Quick writing asks students to independently record everything that they can think of in 3-5 minutes. The prompt students are given can be open ended or specific, depending on the teacher's purpose. This strategy can be used before, during, or after learning. It should not be a high risk, "graded" activity.

What does it do?

- Surfaces student thinking, making it visible and transparent.
- Provides information for teacher on lingering misconceptions, i.e. formative assessment.
- Helps students monitor their own learning.
- Raises questions for future lessons.
- Provides direction for follow-up instruction, i.e. formative assessment.

How to implement:

- Teacher explains the purpose of the activity and that their products will be read but not graded.
- Teacher explains and models how he/she would respond to a given prompt.
- Students are told that they should write what first comes to mind on the topic and that they should not consult their notes or classmates.
- Teacher then assigns a topic, which can be open ended or very specific.

Example 1: Explanation

Provide students with a visual prompt and ask them to respond with a description or explanation. The prompt can be a quote, demonstration, photograph, or drawing.

- Students are provided with a linear equation and they are asked to write about what the graph and the table would look like.
- Students are provided with an addition problem involving two digit numbers with regrouping and they are asked to describe how to find the answer.
- Students are given a model of a 3-D object (pyramid, prism, cone) and asked to write all the words that can be used to describe the object.

Example 2: Exit Slip

- At the end of a lesson, students summarize in one sentence what they think was the main point of the lesson. Select 3-5 cards randomly and read aloud or post at the beginning of the next class. Discuss how well the selected statements reflected the lesson.
- Formative Assessment: At the beginning of the lesson, explain the Quick Write Prompt- it can be general or specific: *"Today we are going to be learning about what characteristics define different types of quadrilaterals. At the end of class, I will ask you to list four facts from today."*

At the end of the lesson, repeat the prompt and use student responses for planning. Separate cards into groups. If most students respond appropriately, meet individually with those who are still having problems. If the responses are varied, set up differentiated instruction for the next lesson: enrichment, review, and small group tutoring.

Writing-To-Learn: Mathematics

What is it?
A writing-to-learn strategy is one that teachers employ throughout and/or at the end of a lesson to engage students and develop big ideas and concepts.

Strategy: Response Journal
Students write comments and questions about what they have read (or solve a problem alone) before engaging in small-group discussions. Following a whole-group discussion, the teacher may assign a related writing assignment (Kenney, et al 2005; Literacy Strategies for Improving Mathematics Instruction).

What does it do?
- Gives ALL students the opportunity and responsibility to interact with what they have read (or solved).
- Students write about *what is* being done and *why* it works.
- The process of writing about a mathematical problem will, itself often lead to a solution.
- The range of individual differences (of students) will be revealed.
- A variety of alternative methods provides students an opportunity to compare and contrast connections.

How to implement:

GUIDELINES

- **The problem must be appropriate for students.**
- **Students must know how to use blocks, diagrams, or grids to work out their solutions before writing about them.**
- **Students must have confidence in their ability to respond to the problem.**
- **Students must feel comfortable sharing their answers.**
- **The problem must be discussed with the whole class with all strategies reported.**

Example:
Four students could not go on a field trip. They had paid $32 in total for their tickets. How much money was returned to each student?

Directions to student:

1. *Read the problem.*
2. *Write down one possible strategy to solve the problem. Use diagrams or pictures when possible.*
3. *Write down any questions you have about the problem.*

Possible Student Response #1

Student #1: $5, $2, $1 = $8
Student #2: $5, $2, $1 = $8
Student #3: $5, $2, $1 = $8
Student #4: $5, $2, $1 =$8

*I know that each student could not get $10 because 4 * 10 = $40, so I returned $5 to each student for a total of $20. I had $12 left. I gave each student $2. I had $4 left so I gave each student $1.*

Possible Student Response #2

I knew that $32 had to be shared equally among four students, so I gave each student $1 until all my bills were gone. I gave out $4 at a time, 8 times. Each student received $8.

By the time students have finished studying a problem like this, they have looked at it in depth from their own perspective and the perspectives of others. It allows students to untangle what is in their minds first, get it on paper, and then share their thinking with others.

Writing-To-Learn: Mathematics

What is it?
A writing-to-learn strategy is one that teachers employ throughout and/or at the end of a lesson to engage students and develop big ideas and concepts.

Strategy: Symbols, Meanings, Writing

Symbols, Meanings, and Writing offers a chance for students to provide their reasoning about each step they employ when they solve a mathematics problem. It enables students to capture explicitly the reasoning and thinking that occurs in proceeding from one step to another (Thompson, et. al.: 2008, *Mathematical Literacy: Helping Students Make Meaning in the Middle Grades).*

What does it do?

- Offers a chance for students to provide their reasoning about each step they employ as they solve a mathematics problem.
- Enables students to capture explicitly the reasoning and thinking that occurs in proceeding from one step to another.
- Student's thinking becomes visible.
- Students can compare their thinking to peers or teacher.
- Allows teachers to learn what understandings or misconceptions a student may have about a problem.

How to implement:

- Fold a piece of paper vertically.
- In the left column, write the symbolic notation for the steps taken.
- In the right column, write the reasoning and thinking that justified the transitions between steps.
- The right column includes more than a simple reason; it involves the student talking to himself about why certain actions are taken.

Example:

Problem: **A stove costs $428. You make a $50 down payment and monthly payments of $42. How many months will it take to finish paying for the stove?**

SYMBOLS	MEANINGS
The stove costs $428; I paid $50 and will pay $42 per month. Let x = the number of months I pay	*I made sure I understood the story and what the numbers represented. I labeled the variable.*
428 = 50 + 42x	*I know the cost (428) has to be the same as the down payment plus the cost per month times the number of months. 42 dollars/month times some months will equal dollars. There are dollars on both sides of the equation.*
428 = 50 +42x -50 = -50	*I know I paid the down payment so I can subtract that from both sides.*
378 = 42x	*I subtracted the $50 from each side of the equation. The equation is still equal.*
$\frac{378}{42} = \frac{42x}{42}$	*I will pay $378 at $42 per month. To find the number of months, I divide by 42.*
9 = x	*I will pay for 9 months.*
Check: Is 42(9) + 50 = 428	*I can check by substituting the 9 months.*
9x42 =378 and 378 + 50=428	*The 9 month works.*

Writing-To-Learn: Mathematics

What is it?

A writing-to-learn strategy is one that teachers employ throughout and/or at the end of a lesson to engage students and develop big ideas and concepts.

Strategy: Test Corrections

The goal for this strategy is to help students make sense of the mathematics. Minimal understandings of some concepts make it difficult for students to understand concepts studied later. As students compare and contrast what they did on the problem to what they should have done, they address their misunderstandings. This opportunity helps students to make sense of the mathematics so they can move on (Thompson, et. al., 2008, *Mathematical Literacy: Helping Students Make Meaning in the Middle Grades*).

What does it do:

- Students address their understanding (or misunderstanding) of mathematical concepts.
- Students have an opportunity to make sense of mathematical concepts they did not previously understand correctly so they have the foundations to move forward.
- Some teachers enable students to improve their exam grade for *Test Corrections* done appropriately.

How to implement:

The following steps are required for the problems or questions that students answered incorrectly:

1. Write about what you did incorrectly. What was your misunderstanding?
2. Write about what you should have done and why.
3. Redo the problem.

Writing-To-Learn: Mathematics

What is it?
A writing-to-learn strategy is one that teachers employ throughout and/or at the end of a lesson to engage students and develop big ideas and concepts.

Strategy: Visualizing and Recording Mental Images

This strategy encourages students to use images to describe patterns. Students have their own unique way of making sense of a variety of patterns.

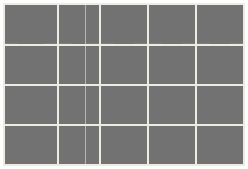
What does it do?

- Engages multiple dimensions of a student's thinking process.
- Accesses student's personal schema for the concept.
- Encourages students to synthesize information.
- Surfaces student questions and lingering misunderstanding (formative assessment).

How to implement:

- Students are directed to lay pencils down and just look at a pattern displayed on the overhead. They have 10 seconds (depending on the pattern and age of students).
- The image is covered up; students have a limited time to draw what they remember.
- This is repeated one more time with students adjusting their original sketch.

Example:



Russell, S. & Econonomopoulos, K. (2007). *Quick Images"* from Investigations in Data, Space and Number.

Writing-To-Learn: Mathematics

What is it?
A writing-to-learn strategy is one that teachers employ throughout and/or at the end of a lesson to engage students and develop big ideas and concepts.

Strategy: Vocabulary Development
Students use a variety of methods and modes (oral, visual, written) to develop a personal, in-depth understanding of key mathematical terms/concepts. Vocabulary development must be intentional, transparent (modeling), useable, personal, and a priority for instruction.

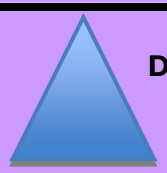
What Does It Do?
- Moves vocabulary development from rote memorization to conceptualization.
- Helps students remember critical concepts.
- Builds connections to new learning.
- Builds background knowledge needed for reading comprehension.

How to Implement
- Teacher provides description, explanation, and/or example.
- Students restate concept/term in their own words.
- Revisit periodically using a variety of engaging activities.
- Monitor student understanding (self and formative assessment).
- Provide additional explanation and modeling, as needed (Marzano & Pickering, 2005).

Example 1: Using Word Walls
Word walls are an organized collection of words displayed in large letters on a wall. To be effective, teachers must use the word walls. Word walls help students remember words because of the frequency of interaction with the words and their meaning and most importantly, these interactions must be fun (Fisher & Frey, 2008, Word Wise and Content Rich).

Example 2: Bingo
Students create cards with the words. Teachers read the definitions aloud. Students play to simulate rules of the game, Bingo. Play continues until someone gets a "Bingo".

Example 3: Word Cards
Students collect the cards and practice with each other, parents, and teachers.

Triangle (the word)	**A polygon with exactly three sides(student's words for what the word means)**
 **Diagram or illustration**	**Circle, rectangle, square, (What the word *ISN'T)***

Example 4*:* Structured Word Sort
Teacher provides students with a list of vocabulary words (before and during the unit) which students cut out and place on a chart similar to the chart below. This can be used as pre-assessment and then formative assessments throughout the unit.

I learned this in elementary	***I've heard this word***	***I have no clue***
Sum Multiplication Subtraction Division Dividend Divisor	Algebra Equation Expression Solution	Function Linear equation Quadratic equation

Writing-To-Demonstrate-Knowledge: Mathematics

What is it?

When writing-to-demonstrate-knowledge students show what they have learned by synthesizing information and explaining or applying their understanding of concepts and ideas.

Strategy: Argumentation

In argumentation, a claim is developed and proven using logical reasoning and examples. In mathematics this can be as simple as explaining why 2 + 3 = 3 + 2 or more complex involving geometric proofs involving congruent figures or algebraic proofs such as that the square root of 2 is irrational.

What does it do?

- Deepens student's understanding of the content.
- Promotes critical thinking, analytical skills, and logical reasoning.
- Prepares students for ACT and postsecondary mathematics experiences.
- Provides evidence of what a student knows and can apply about a specific area of content.

How to implement:

- Teacher encourages student to 'prove' by using examples.
- Teacher models more abstract proof using real world situations.
- Students practice 'proof' from a personal point of view.
- Teacher models a variety of types of 'proof'.
- Students practice by creating and critiquing each other's work.

Examples:

Elementary

2 x 3 = 6 because 2 x 3 is the same as 2 added together 3 times: 2 + 2 + 2 = 6

2 x 4 = 8 because 2 x 3 = 6 so 2 x 4 is 2 more, = 8

Middle and High School

Use of geometric proofs, as well as algebraic proofs, can demonstrate depth of knowledge of the topic.

Writing-To-Demonstrate-Knowledge: Mathematics

What is it?

When writing-to-demonstrate-knowledge students show what they have learned by synthesizing information and explaining or applying their understanding of concepts and ideas.

Strategy: Structured Writing Guide

A structured writing guide provides structure for students as they convey their learning in an organized manner. The writing guide includes step-by-step formatting for each paragraph as well as suggested sentence starters. This structure supports students as they learn to write about what they are learning (Kenney, et. al., 2005).

What does it do?

- Supports students in writing an informative short essay.
- Provides a structure with hints for what to write.

How to implement:

- Determine the structure.
- Provide modeling and examples of expectations.
- Practice each part with students.
- Show student work and work with students to 'grade' these samples.
- Use peer editing for improvement strategies. This also supports student learning of the concept.

Example:

SAMPLE STRUCTURED WRITING GUIDE

Paragraph One: Problem Statement

Write answers to these questions:

- **What is the problem about?**
- **What am I supposed to find?**

Paragraph Two: Work Write-Up

Explain step-by-step and in detail everything you did to arrive at each of your answers. Think of this as a recipe for someone to follow. Complete the following sentences:

- *First I ……*
- *Then I….*
- *Next I….*
- *After that I….*
- *Finally I …*

Paragraph Three: Answer

Prove that your answer is correct by referring to the math that you did. Do NOT write that you checked it on the calculator, you did it twice, or that your friend told you it looked OK. Complete the following sentences:

- *My answer is…*
- *My answer makes sense because…*

Source: Jubinville,L. (2002). *Guide For Writing Responses In Mathematics.* Unpublished Manuscript. Reprinted with permission.

Writing-To-Demonstrate-Knowledge: Mathematics

What is it?
A writing-to-learn strategy is one that teachers employ throughout and/or at the end of a lesson to engage students and develop big ideas and concepts.

Strategy: Write Your Own Problem

Students write their own word problems based on the mathematics they are studying. Students choose a situation from their own experiences and write a problem for peers to solve. The problem must have complete information and clear, specific instructions. Students learn that writing must convey meaning as they revise or modify their work (Thompson, et. al.: 2008, *Mathematical Literacy: Helping Students Make Meaning in the Middle Grades*).

What does it do?

- Helps students understand the mathematical concepts and important principals.
- Helps students understand the structure of problems.
- Often they choose situations from their own experience, thus see how mathematics applies to their own lives.
- This application may give students more confidence to read and solve word problems.
- Demands clear, specific, and complete instructions.
- As students share with their peers, any difficulties in their problems become apparent.
- Revisions become necessary.

How to implement:

- Students are instructed to write a word problem demonstrating concepts they are working with, modeling other problems or as problems to given equations or situations.
- Students share with a peer: each student solves (or attempts to solve) their peer's problem.
- Issues are identified and shared with the author of the problem.
- Revisions made and reviewed.
- Students practice using a rubric to assess other problems.

Example Problem and Response:

Problem:

Create a question about a real situation that can be answered by solving the equation: $4x + 8 = 5x + 2$. Be sure to tell what x represents.

Student Response:

Ping has $8.00 already in her money jar and she plans to save $4.00 more each week. Scott has $2.00 but he says he will save $5.00 a week. How long before Scott has more money than Ping? X is the number of weeks.