Transformations

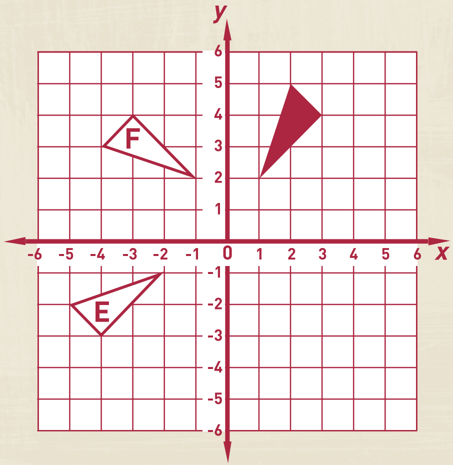
Initial assessment task

1. Draw the shaded triangle after:
   a) it has been translated -7 units horizontally and +1 units vertically. Label your answer *A*.

   b) it has been reflected in the *x* axis. Label your answer *B*.

   c) it has been rotated 90° clockwise about (0,0). Label your answer *C*.

   d) it has been reflected in the line *y* = *x*. Label your answer *D*.



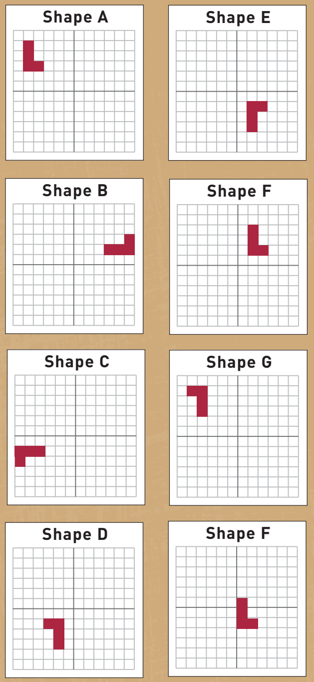
2. Describe fully the single transformation that:
   a) takes the shaded triangle onto the triangle labeled *E*.

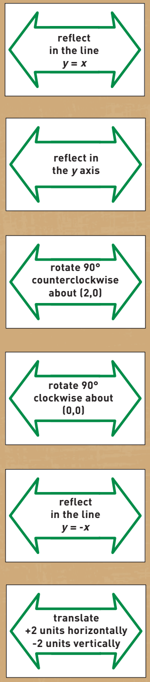
   b) takes the shaded triangle onto the triangle labeled *F*.

3. Describe a single transformation that has the same effect as rotating a shape 90°clockwise, then reflecting the result in the *x* axis.

The collaborative activity

Connect the shape cards with the word cards.





Plenary discussion

1. Show the new coordinates of the point (1,4) after it is:

   a) reflected in the x axis.

   b) reflected in the y axis.

   c) rotated 180° about (0,0).

   d) reflected in the line $y = x$.

   e) reflected in the line $y = -x$.

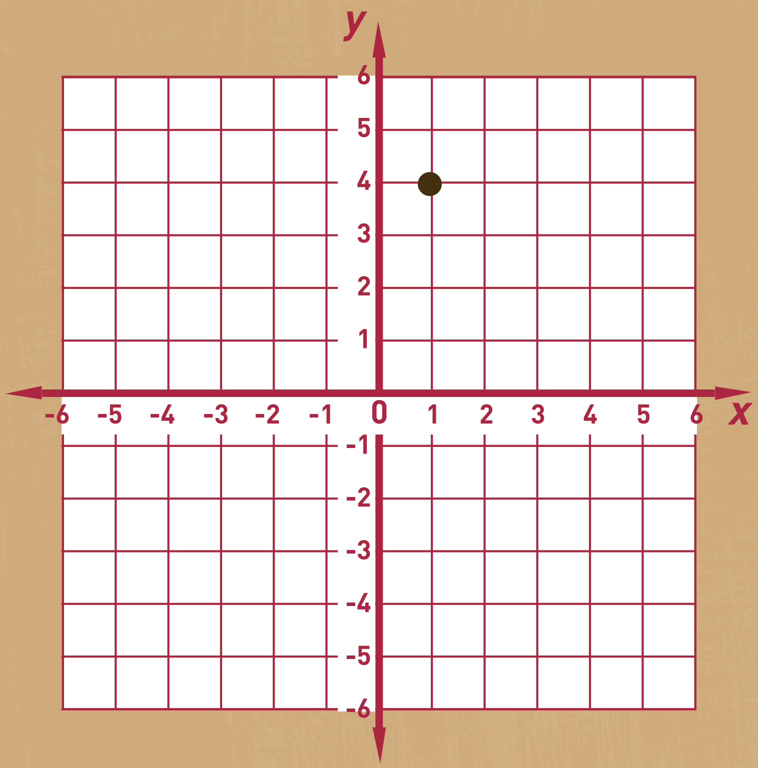
   f) rotated 90° clockwise about (0,0).

   g) rotated 90° counterclockwise about (0,0).

2. What is the single transformation that will produce the same result as:

   a) a reflection in the x axis, followed by a reflection in the y axis?

   b) a rotation 90° clockwise about (0,0), followed by a reflection in the y axis?



Back to the initial assessment task

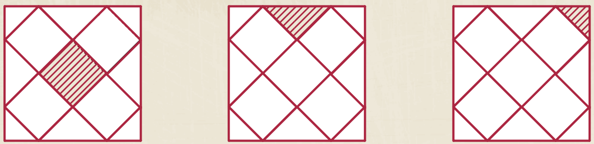
Tackle the original assessment task again, bearing in mind what you have learned during the lesson.

Generalizing Patterns: Table Tiles

Initial assessment task

Maria makes square tables, then sticks tiles to the top. She uses three types of tiles:

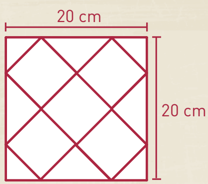
whole tiles *half tiles* *quarter tiles*



The side lengths of the square tabletops are all multiples of 10 cm.

Maria can use only quarter tiles in the corners, only half tiles along the edges of the table, and only whole tiles in the interior.

Here is one tabletop. This square table uses 5 whole tiles, 4 half tiles, and 4 quarter tiles.



How many tiles of each type will Maria need for a 40 cm x 40 cm square tabletop?

The collaborative activity

Describe a method for calculating the number of tiles of each type that Maria will need in order to tile any larger square tabletop.

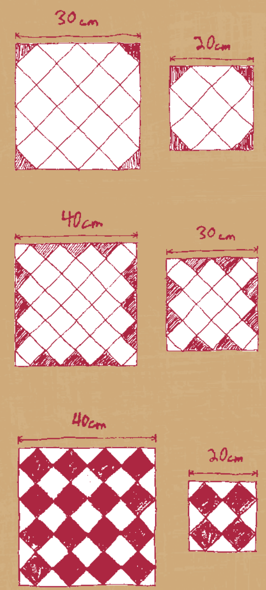
Discuss some of the patterns and generalizations. Draw some diagrams of the different possible tables.

For a 10*n* x 10*n* table, the number of quarter tiles is 4.

For a 10*n* x 10*n* table, the number of half tiles is 4(*n*-1).

Unlike the number of half tiles, the number of whole tiles increases in a way that is not linear but quadratic. This is not surprising, because the number of whole tiles increases as the area of the square increases.

Size of table	10 x 10	20 x 20	40 x 40	10n x 10n
Number of whole tiles	1	5	25	$n^2 + (n-1)^2$



Plenary discussion

1. How many quarter tiles are used for each tabletop?
2. By how many does the number of half tiles increase as each side of the square increases by 10 cm?
3. By how many does the number of whole tiles increase as each side of the square increases by 10 cm?
4. How many half tiles are needed to tile a $10n$ x $10n$ tabletop?
5. How many whole tiles are needed to tile a $10n$ x $10n$ tabletop?
6. Are the formulas suggested for calculating the number of whole tiles equivalent?
7. Someone found that the number of whole tiles needed could always be expressed as the sum of two consecutive square numbers. How do these square numbers "show up" in the tabletop drawings?

Comment on others' work

These are two possible responses. Discuss both responses in small groups before writing a commentary on the strengths and/or weaknesses of each.

Hannah's response

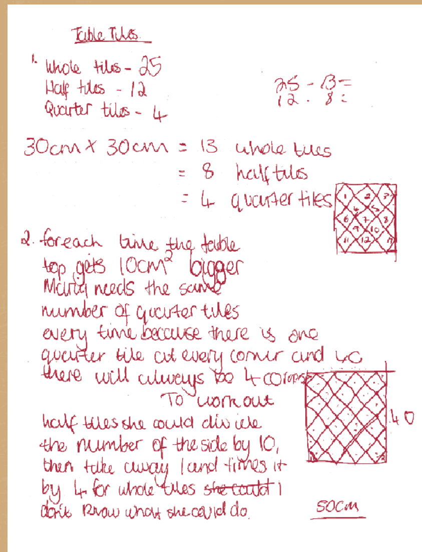


Table Tiles

1. Whole tiles - 25
Half tiles - 12
Quarter tiles - 4

25 - 13 =
12 - 8 =

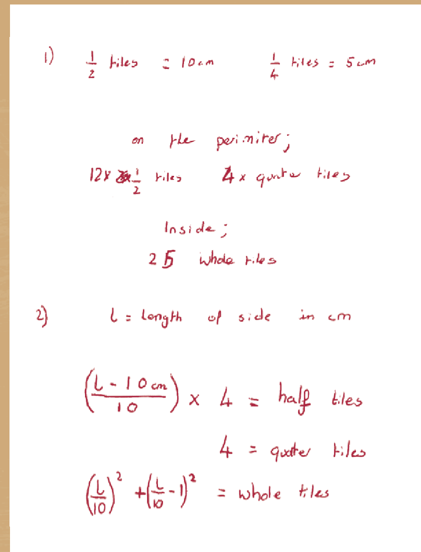
30cm x 30cm = 13 whole tiles
= 8 half tiles
= 4 quarter tiles

2. for each time the table top gets $10cm^2$ bigger Mary needs the same number of quarter tiles every time because there is one quarter tile at every corner and so there will always be 4 corners. To work out half tiles she could divide the number of the side by 10, then take away 1 and times it by 4 for whole tiles ~~she could~~ I don't know what she could do.

40

50cm

Thomas's response



1) $\frac{1}{2}$ tiles = 10cm $\frac{1}{4}$ tiles = 5cm

on the perimiter;

12 x $\frac{1}{2}$ tiles 4 x quarter tiles

Inside;

25 whole tiles

2) l = length of side in cm

$$\left(\frac{l - 10\text{cm}}{10}\right) \times 4 = \text{half tiles}$$

$$4 = \text{quarter tiles}$$

$$\left(\frac{l}{10}\right)^2 + \left(\frac{l}{10} - 1\right)^2 = \text{whole tiles}$$

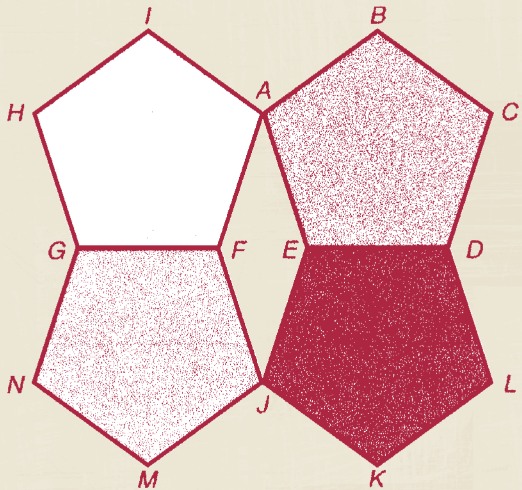
Back to the initial assessment task

Tackle the original assessment task again, bearing in mind what you have learned during the lesson.

Solving Pentagon Problems

Initial assessment task

1. Find the measure of angle *AEJ*. Please show all your work.
2. Find the measure of angle *EJF*. Please show all your work.
3. Find the measure of angle *KJM*. Please show all your work.

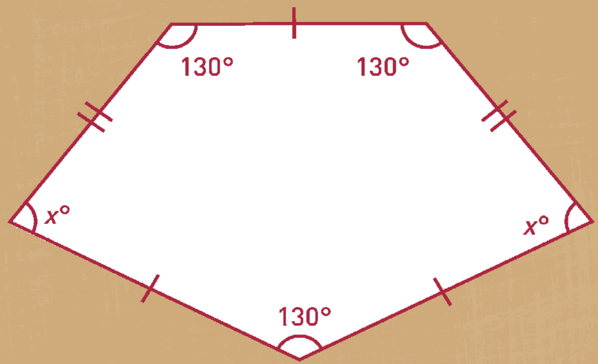


The collaborative activity

This pentagon has three sides of equal length at the top and two sides of equal length at the bottom.

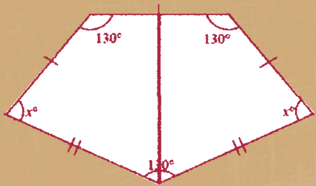
Three of the angles have a measure of 130°.

Figure out the measure of the angles marked *x*, and explain your reasoning.

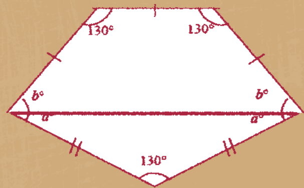


Consider these four different starting points that four different students used to arrive at their solutions.

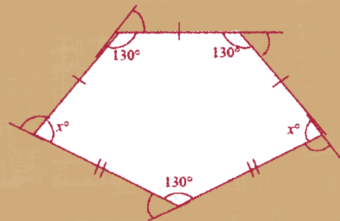
Annabel drew a vertical line down the middle of the pentagon and calculated the measure of the angles marked *x* in one of the quadrilaterals she had made.



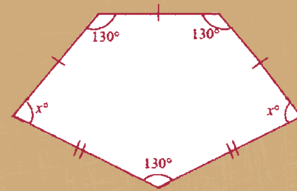
Brian drew a horizontal line that divided the pentagon into a trapezoid and a triangle. The angles marked *x* had also been cut into two parts, so he labeled the parts *a* and *b*.



Carlos used the exterior angles of the pentagon to figure out the measure of the angles marked *x*.



Diane used the sum of the interior angles to calculate the measure of the angles marked *x*.



Plenary discussion

Discuss each response in small groups before writing a commentary on the strengths and/or weaknesses of each.

Annabel's response

$(5-2)360 \cdot \frac{1080}{5} : 216$

Brian's response

$30 + \frac{1}{2}130 + x = 360$

$195 + x = 360$

$x = 165$

Carlos's response

$180 - 130 = 50$

$\frac{50}{2} = 25$

$a = 25°$

$b = 155°$

$180 - 25 = 155$

Diane's response

$180 \cdot 4 = 720$

$130 + 130 + 130 = 390 -$

$330 \div 2$

165

Back to the initial assessment task

Tackle the original assessment task again, bearing in mind what you have learned during the lesson.