



## GRADE 7 MATH: PROPORTIONAL REASONING

### UNIT OVERVIEW

This is a 3-4 week unit that focuses on identifying and using unit rates. It also develops students' understanding of proportional relationships represented in equations and graphs. Students use proportional relationships to solve multi-step ratio and percent problems.

### TASK DETAILS

**Task Name:** Proportional Reasoning

**Grade:** 7

**Subject:** Math

**Task Description:** This task consists of five extended-response questions related to proportional reasoning. The short response and extended-response questions require students to write an appropriate response.

#### **Standards Assessed:**

**7RP.1** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks  $\frac{1}{2}$  mile in each  $\frac{1}{4}$  hour, compute the unit rate as the complex fraction  $\frac{1/2}{1/4}$  miles per hour, equivalently 2 miles per hour.

**7RP.2** Recognize and represent proportional relationships between quantities.

- Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
- Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
- Represent proportional relationships by equations. For example, if total cost  $t$  is proportional to the number  $n$  of items purchased at a constant price  $p$ , the relationship between the total cost and the number of items can be expressed as  $t = pn$ .
- Explain what a point  $(x, y)$  on the graph of a proportional relationship means in terms of the situation, with special attention to the points  $(0, 0)$  and  $(1, r)$  where  $r$  is the unit rate.

**7.RP.3** Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

#### **Standards for Mathematical Practice:**

**MP.1** Make sense of problems and persevere in solving them.

**MP.2** Reason abstractly and quantitatively.

**MP.3** Construct viable arguments and critique the reasoning of others.

**MP.6** Attend to precision.



## TABLE OF CONTENTS

The task and instructional supports in the following pages are designed to help educators understand and implement tasks that are embedded in Common Core-aligned curricula. While the focus for the 2011-2012 Instructional Expectations is on engaging students in Common Core-aligned culminating tasks, it is imperative that the tasks are embedded in units of study that are also aligned to the new standards. Rather than asking teachers to introduce a task into the semester without context, this work is intended to encourage analysis of student and teacher work to understand what alignment looks like. We have learned through the 2010-2011 Common Core pilots that beginning with rigorous assessments drives significant shifts in curriculum and pedagogy. Universal Design for Learning (UDL) support is included to ensure multiple entry points for all learners, including students with disabilities and English language learners.

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Acknowledgements: The tasks were developed by the 2010-2011 NYC DOE Middle School Performance Based Assessment Pilot Design Studio Writers, in collaboration with the Institute for Learning.



# GRADE 7 MATH: PROPORTIONAL REASONING PERFORMANCE TASK

Performance-Based Assessment 1  
Proportional Reasoning – Grade 7

Student  
Name

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School

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Date

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Teacher

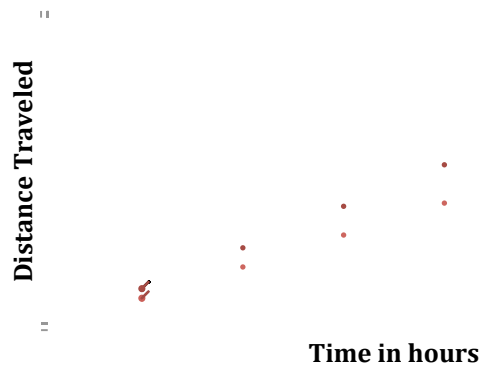
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3. Jack and Jill raced cross-country on motor bikes. Jack drove 325 miles in 5 hours; Jill took  $6\frac{1}{2}$  hours to travel the same distance as Jack.
- a. Compute the unit rates that describe Jack's average driving speed and Jill's average driving speed. Show how you made your decisions.

- b. A portion of the graph of Jack and Jill's race appears below. Identify which line segment belongs to Jack and which belongs to Jill. Explain in writing how you made your decisions.



4. Reynaldo is planning to drive from New York to San Francisco in his car. Reynaldo started to fill out the table below showing how far in miles he can travel for each gallon of gas he uses.

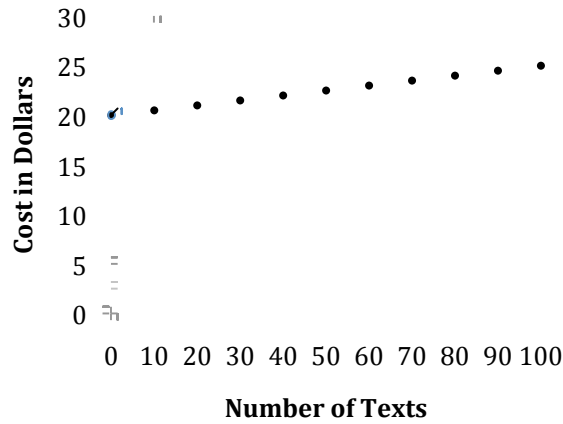
Gallons	2	4		8	10	12
Miles	56		168	224		

Use the information in Reynaldo's table to answer the questions below.

- Complete the table for Reynaldo. Assume the relationship in the table is proportional.
- Based on the table, how many miles per gallon did Reynaldo's car get? Explain in writing how you know.
- Write an equation that Reynaldo can use to find the distance ( $d$ ) he can drive on any number of gallons of gas ( $g$ ).
- When Reynaldo's tank is full, it holds 20 gallons. How far can Reynaldo drive on a full tank of gas?



5. The monthly cost of Jazmine’s cell phone plan is graphed on the grid below. Her friend Kiara selected a plan that charges \$0.25 per text, with no monthly fee, because she only uses her phone for texting.



- a. Write an equation to represent the monthly cost of Kiara’s plan for any number of texts.
  
- b. Graph the monthly cost of Kiara’s plan on the grid above.
  
- c. Using the graphs above, explain the meaning of the following coordinate pairs:
  - i. (0, 20): \_\_\_\_\_
  
  - ii. (0, 0): \_\_\_\_\_
  
  - iii. (10, 2.5): \_\_\_\_\_
  
  - iv. (100, 25): \_\_\_\_\_
  
- d. When one of the girls doubles the number of texts she sends, the cost doubles as well. Who is it? Explain in writing how you know.



# GRADE 7 MATH: PROPORTIONAL REASONING UNIVERSAL DESIGN FOR LEARNING (UDL) PRINCIPLES

## Proportional Reasoning – Math Grade 7 Common Core Learning Standards/ Universal Design for Learning

The goal of using Common Core Learning Standards (CCLS) is to provide the highest academic standards to all of our students. Universal Design for Learning (UDL) is a set of principles that provides teachers with a structure to develop their instruction to meet the needs of a diversity of learners. UDL is a research-based framework that suggests each student learns in a unique manner. A one-size-fits-all approach is not effective to meet the diverse range of learners in our schools. By creating options for how instruction is presented, how students express their ideas, and how teachers can engage students in their learning, instruction can be customized and adjusted to meet individual student needs. In this manner, we can support our students to succeed in the CCLS.

Below are some ideas of how this Common Core Task is aligned with the three principles of UDL; providing options in representation, action/expression, and engagement. As UDL calls for multiple options, the possible list is endless. Please use this as a starting point. Think about your own group of students and assess whether these are options you can use.

**REPRESENTATION:** *The “what” of learning.* How does the task present information and content in different ways? How do students gather facts and categorize what they see, hear, and read? How are they identifying letters, words, or an author's style?

*In this task, teachers can...*

- ✓ **Make explicit links between information provided in texts and any accompanying representation of that information in illustrations** by displaying time in analog and digital mode.
- ✓ **Provide text-to-speech access** by reading aloud, creating teacher-made recordings, or employing digital software.
- ✓ **Embed support for unfamiliar references within the text** by defining academic vocabulary, such as “unit rate.”

**ACTION/EXPRESSION:** *The “how” of learning.* How does the task differentiate the ways that students can express what they know? How do they plan and perform tasks? How do students organize and express their ideas?

*In this task, teachers can...*

- ✓ **Embed support for vocabulary and symbols** by providing access to online tools, such as *The Mathematics Glossary*, which uses multiple means of representation to explain concepts.
- ✓ **Provide multiple examples of novel solutions to authentic problems** by allowing students to see their peers' perspectives on proportional thinking.

**ENGAGEMENT:** *The “why” of learning.* How does the task stimulate interest and motivation for learning? How do students get engaged? How are they challenged, excited, or interested?

*In this task, teachers can...*

- ✓ **Provide alternatives in the permissible tools and scaffolds to optimize challenge** by providing calculators or designing customized mini-lessons to activate prior knowledge of the concept on proportional thinking.

Visit <http://schools.nyc.gov/Academics/CommonCoreLibrary/default.htm> to learn more information about UDL.



## GRADE 7 MATH: PROPORTIONAL REASONING BENCHMARK PAPERS WITH RUBRICS

This section contains benchmark papers that include student work samples for each of the five tasks in the Proportional Reasoning assessment. Each paper has descriptions of the traits and reasoning for the given score point, including references to the Mathematical Practices.

1. Amy and her family were traveling during their vacation. She looked at her watch at Point 1 in the diagram below, and then again at Point 2 in the diagram below. Her mom told her how far they traveled in that time, as noted below.



Point 1

Point 2

- Based on this information, what is the unit rate of the car? Explain in words what that unit rate means in the context of the problem.
- Amy's dad said that the entire trip was 1200 miles. How many hours will it take to complete the trip? Explain your reasoning in words.

NYC Grade 7 Assessment 1

Amy's Vacation  
Benchmark Papers

3 Points

The response accomplishes the prompted purpose and effectively communicates the student's mathematical understanding. The student's strategy and execution meet the content (including concepts, technique, representations, and connections), thinking processes and qualitative demands of the task. Minor omissions may exist, but do not detract from the correctness of the response.

Either verbally or symbolically, the strategy used to solve the problem is stated, as is the work used to find ratios, unit rates, and partial answers to problems. Minor arithmetic errors may be present, but no errors of reasoning appear. Complete explanations are stated, based on work shown.

Accurate reasoning processes demonstrate the Mathematical Practices, (1) Make sense of problems and persevere in solving them and (2) Reason abstractly and quantitatively (since students need to abstract information from the problem, create a mathematical representation of the problem, and correctly work with the ratio and/or proportion). Evidence of the Mathematical Practice, (3) Construct viable arguments and critique the reasoning of others, is demonstrated by complete and accurate explanations. Evidence of the Mathematical Practice, (4) Model with mathematics, is demonstrated by representing the problem with ratios or proportions. Evidence of the Mathematical Practice, (6) Attend to precision, can include proper use of ratio notation and proper labeling of quantities. Evidence of the Mathematical Practice, (7) Look for and make use of structure, may be demonstrated by student recognition, e.g., that the multiples used in a table need not be of equal intervals (see table below) or that part b may be solved with a properly formed proportion, where the variable is equal to the numbers of hours a 1200-mile trip will take.

The reasoning used to solve the parts of the problem may include:

- a. Indicating the 80-mile trip took two hours; scaling down the 80 mile : 2 hour rate in tabular or fraction form to 40/1 or 40 miles per hour.
- b. Using a proportion or proportional reasoning (e.g., 2 hours is twice 1 hour, so I can half 80 miles (or divide 80 miles by 2) to find the unit rate.
- c. Scaling up the 80 mile: 2 hours or 40 mile : 1 hour rate in tabular or fraction form to 1200 miles : 30 hours.
- d. Using a proportion or proportional reasoning (e.g., 1200 miles is 40 miles times 30, so I can multiply 1 hour by 30.)

miles	40	400	1200
hours	1	10	30

a.  $\frac{80 \text{ mi}}{2 \text{ hr}} = \frac{2}{2} = \frac{40 \text{ mi}}{1 \text{ hr}}$

The unit of the car is 40 miles per hour

b.  $\frac{1 \text{ hr}}{1} = \frac{40 \text{ mi}}{1200 \text{ mi}}$

$\frac{1200}{40} = \frac{40x}{40}$

$x = 30$  hours

It will take them 30 hours to complete the trip. I know because I used a proportion and got 30 hours per 1200 miles.

check my work ✓

$\frac{1 \text{ hr}}{30 \text{ hr}} \Rightarrow \frac{40 \text{ mi}}{1200 \text{ mi}}$

1200  $\div$  40 = 30

NYC Grade 7 Assessment 1

Amy's Vacation  
Benchmark Papers

2 Points

The response demonstrates adequate evidence of the learning and strategic tools necessary to complete the prompted purpose. It may contain overlooked issues, misleading assumptions, and/or errors in execution. Evidence in the response demonstrates that the student can revise the work to accomplish the task with the help of written feedback or dialogue.

Either verbally or symbolically, the strategy used to solve the problem is stated, as is the work used to find ratios, unit rates, and partial answers to problems. Reasoning may contain incomplete, ambiguous or misrepresentative ideas. Partial explanations are stated, based on work shown. Accurate reasoning processes demonstrate the Mathematical Practices, (1) Make sense of problems and persevere in solving them and (2) Reason abstractly and quantitatively (since students need to abstract information from the problem, create a mathematical representation of the problem, and correctly work with the ratio and/or proportion). Evidence of the Mathematical Practice, (3) Construct viable arguments and critique the reasoning of others, is demonstrated by complete and accurate explanations. Evidence of the Mathematical Practice, (4) Model with mathematics, is demonstrated by representing the problem with ratios or proportions. Evidence of the Mathematical Practice, (6) Attend to precision, can include proper use of ratio notation and proper labeling of quantities. Evidence of the Mathematical Practice, (7) Look for and make use of structure, may be demonstrated by student recognition, e.g., that the multiples used in a table need not be of equal intervals (see table below) or that part b may be solved with a properly formed proportion, where the variable is equal to the numbers of hours a 1200-mile trip will take.

The reasoning used to solve the parts of the problem may include:

- a. Indicating the 80-mile trip took some number other than 2 hours; or the distance traveled for the two-hour trip was some number other than 80 miles; scaling down in tabular or fraction form to a unit rate consistent with the number chosen.
- b. Correctly using a proportion or proportional reasoning to find the unit rate, but choosing a distance other than 80 miles or a time other than 2 hours.
- c. Scaling up the 80 mile : 2 hour or 40 mile : 1 hour rate in tabular or fraction form, but failing to reach or failing to stop at 1200 miles.
- d. Using a proportion or proportional reasoning (e.g., 1200 miles is 80 miles times 15), but failing to multiply 2 hours by 15.

miles	40	80	120	160
hours	1	2	3	4

traveled 2 hours

A. unit rate is  $\frac{40 \text{ mi}}{1 \text{ hour}}$  the unit rate needs to have a denominator of one. So the 2 was going to be the denominator so I divided  $2 \overline{)80}$  to get half,

B. Every 2 hour is 80 mi  
It will take them 15 hours to get to their destination because

$$\begin{array}{r} 15 \\ 80 \overline{)1200} \\ \underline{-80} \phantom{0} \\ 400 \\ \underline{-400} \\ 0 \end{array}$$

NYC Grade 7 Assessment 1

Amy's Vacation

Benchmark Papers

1 Point

The response demonstrates some evidence of mathematical knowledge that is appropriate to the intent of the prompted purpose. An effort was made to accomplish the task, but with little success. Evidence in the response demonstrates that, with instruction, the student can revise the work to accomplish the task.

Some evidence of reasoning is demonstrated either verbally or symbolically, but may be based on misleading assumptions, and/or contain errors in execution. Some work is used to find ratios, or unit rates; or partial answers to portions of the task are evident. Explanations are incorrect, incomplete or not based on work shown. Accurate reasoning processes demonstrate the Mathematical Practices, (1) Make sense of problems and persevere in solving them and (2) Reason abstractly and quantitatively (since students need to abstract information from the problem, create a mathematical representation of the problem, and correctly work with the ratio and/or proportion). Evidence of the Mathematical Practice, (3) Construct viable arguments and critique the reasoning of others, is demonstrated by complete and accurate explanations. Evidence of the Mathematical Practice, (4) Model with mathematics, is demonstrated by representing the problem with ratios or proportions. Evidence of the Mathematical Practice, (6) Attend to precision, can include proper use of ratio notation and proper labeling of quantities. Evidence of the Mathematical Practice, (7) Look for and make use of structure, may be demonstrated by student recognition, e.g., that the multiples used in a table need not be of equal intervals (see table below) or that part b may be solved with a properly formed proportion, where the variable is equal to the numbers of hours a 1200-mile trip will take.

The reasoning used to solve the parts of the problem may include:

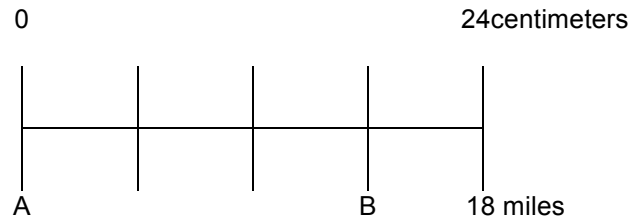
- Failing to find a unit rate.
- Choosing numbers other than 80 miles and 2 hours to scale down; but scaling down in tabular or fraction form to a unit rate consistent with both incorrect numbers chosen; or correctly using a proportion or proportional reasoning to find the unit rate, but choosing a distance other than 80 miles and a time other than 2 hours.
- Using a proportion or proportional reasoning, but choosing a distance other than 80 miles and a time other than 2 hours.

Handwritten student work showing five multiplication problems:

$$\begin{array}{r} 80 \\ \times 10 \\ \hline 800 \end{array}$$
$$\begin{array}{r} 80 \\ \times 12 \\ \hline 960 \end{array}$$
$$\begin{array}{r} 80 \\ \times 13 \\ \hline 1040 \end{array}$$
$$\begin{array}{r} 80 \\ \times 14 \\ \hline 1120 \end{array}$$
$$\begin{array}{r} 80 \\ \times 15 \\ \hline 1200 \end{array}$$



2. On a map of the United States, 24 centimeters represents 18 miles, and the 24 centimeter segment is divided into four equal pieces, as shown in the picture below.



- How many centimeters represent one mile?
- How long is the line segment between A and B in centimeters? Use mathematical reasoning to justify your response.
- If A and B represent two cities, what is the actual distance between the two cities? Use mathematical reasoning to justify your response.

NYC Grade 7 Assessment 1

Map Task

Benchmark Papers

3 Points

The response accomplishes the prompted purpose and effectively communicates the student's mathematical understanding. The student's strategy and execution meet the content (including concepts, technique, representations, and connections), thinking processes and qualitative demands of the task. Minor omissions may exist, but do not detract from the correctness of the response.

Either verbally or symbolically, the strategy used to solve each part of the problem is stated, as is the work used to find ratios, proportions, and partial answers to problems. Minor arithmetic errors may be present, but no errors of reasoning appear.

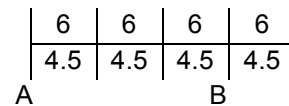
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The reasoning used to solve the parts of the problem may include:

- Dividing the line segment into equal-sized pieces and reasoning from the picture.
- Forming the ratio 24 cm : 18 mi. and scaling down to a denominator of 1 or dividing 24 by 18; possibly drawing a picture first and using 6/4.5; possibly using similar reasoning in part c.
- Noting that the ratio of line segment AB to the 24 cm segment is  $\frac{3}{4}$ , and finding  $\frac{3}{4}(24)$ ; using the proportion  $\frac{3}{4} = \frac{x}{24}$  then scaling 4 up to 24 and 3 up to 18 with tables or multiplication; or solving the proportion as an equation; possibly using similar reasoning in part c.

0

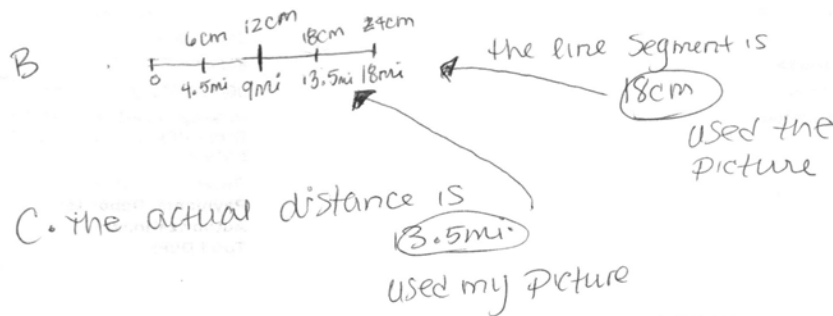
24centimeters



18 miles

A.  $\frac{24\text{cm}}{18\text{mi}} = 1.\bar{3}\text{cm}$  represents 1 mile

$$\begin{array}{r} 1.3333\dots \\ 18 \overline{) 24.00} \\ \underline{18} \phantom{00} \\ 60 \phantom{0} \\ \underline{54} \phantom{0} \\ 60 \phantom{0} \\ \underline{60} \phantom{0} \\ 0 \phantom{0} \end{array}$$



NYC Grade 7 Assessment 1

Map Task

Benchmark Papers

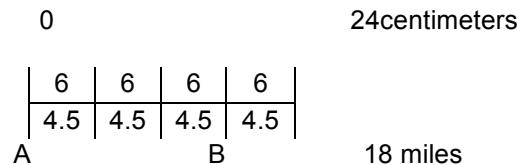
2 Points

The response demonstrates adequate evidence of the learning and strategic tools necessary to complete the prompted purpose. It may contain overlooked issues, misleading assumptions, and/or errors in execution. Evidence in the response demonstrates that the student can revise the work to accomplish the task with the help of written feedback or dialogue.

Either verbally or symbolically, the strategy used to solve each part of the problem is stated, as is the work used to find ratios, proportions, and partial answers to problems. Minor arithmetic errors may be present. Reasoning may contain incomplete, ambiguous or misrepresentative ideas. Accurate reasoning processes demonstrate the Mathematical Practices, (1) Make sense of problems and persevere in solving them and (2) Reason abstractly and quantitatively (since students need to abstract information from the problem, create a mathematical representation of the problem, and correctly work with unit rate, ratios and/or proportion). Evidence of the Mathematical Practice, (3) Construct viable arguments and critique the reasoning of others, is demonstrated by complete and accurate explanations. Evidence of the Mathematical Practice, (4) Model with mathematics, is demonstrated by representing the problem as pictorially, or with ratios and/or proportions. Evidence of the Mathematical Practice, (6) Attend to precision, can include proper use of ratio notation, proportion, and proper labeling of pictures and quantities. Evidence of the Mathematical Practice, (7) Look for and make use of structure, may be demonstrated by student recognition that parts b and c of this problem may successfully be solved with a proportion. Evidence of the Mathematical Practice, (8) Look for and express regularity in repeated reasoning, may be demonstrated by student use of the same reasoning process (repeated calculations) in both parts b and c.

The reasoning used to solve the parts of the problem may include:

- Dividing the line segment into equal-sized pieces and reasoning from the picture, but indicating, e.g., that the 6 cm. segment represents one mile.
- Forming the ratio 18 cm : 24 mi. or 18 mi. : 24 cm and scaling down to a denominator of 1 or dividing 18 by 24; possibly drawing a picture first and using 4.5/6; possibly using similar reasoning in part c.
- Incorrectly forming the ratio of line segment AB to the 24 cm segment, but then correctly finding that fraction of 24, or correctly forming the ratio of line segment AB to the 24 cm segment, but then finding that fraction of 18; possibly using scaling or proportions to do so; possibly using similar reasoning in part c.
- Correctly attempting only two parts of the problem.



a.  $\frac{24 \text{ mi}}{18 \text{ cm}} = \frac{18}{1 \text{ mi}} = 11.3 \text{ cm}$   
 1.3 cm represent 1 mile

b. The line segment between A and B is 4.5 cm

c.  $\frac{11.3 \text{ cm}}{40.5 \text{ cm}} = \frac{1 \text{ mi}}{X}$   
 $\frac{11.3}{40.5} = \frac{1}{X}$   
 $X = 3.46$   
 The actual distance between the two cities is 3.46 miles

1 Point

The response demonstrates some evidence of mathematical knowledge that is appropriate to the intent of the prompted purpose. An effort was made to accomplish the task, but with little success. Evidence in the response demonstrates that, with instruction, the student can revise the work to accomplish the task.

Some evidence of reasoning is demonstrated either verbally or symbolically, is often based on misleading assumptions, and/or contains errors in execution. Some work is used to find ratios or proportion or partial answers to portions of the task are evident. Accurate reasoning processes demonstrate the Mathematical Practices, (1) Make sense of problems and persevere in solving them and (2) Reason abstractly and quantitatively (since students need to abstract information from the problem, create a mathematical representation of the problem, and correctly work with unit rate, ratios and/or proportion). Evidence of the Mathematical Practice, (3) Construct viable arguments and critique the reasoning of others, is demonstrated by complete and accurate explanations. Evidence of the Mathematical Practice, (4) Model with mathematics, is demonstrated by representing the problem as pictorially, or with ratios and/or proportions. Evidence of the Mathematical Practice, (6) Attend to precision, can include proper use of ratio notation, proportion, and proper labeling of pictures and quantities. Evidence of the Mathematical Practice, (7) Look for and make use of structure, may be demonstrated by student recognition that parts b and c of this problem may successfully be solved with a proportion. Evidence of the Mathematical Practice, (8) Look for and express regularity in repeated reasoning, may be demonstrated by student use of the same reasoning process (repeated calculations) in both parts b and c.

The reasoning used to solve the parts of the problem may include:

- a. Dividing the line segment into equal-sized pieces and reasoning from the picture, but indicating, e.g., that the 6 cm. or the 4.5 mi. segment represents answers to several parts of the problem.
- b. Forming some appropriate ratios, but failing to scale appropriately.
- c. Correctly attempting only one part of the problem.

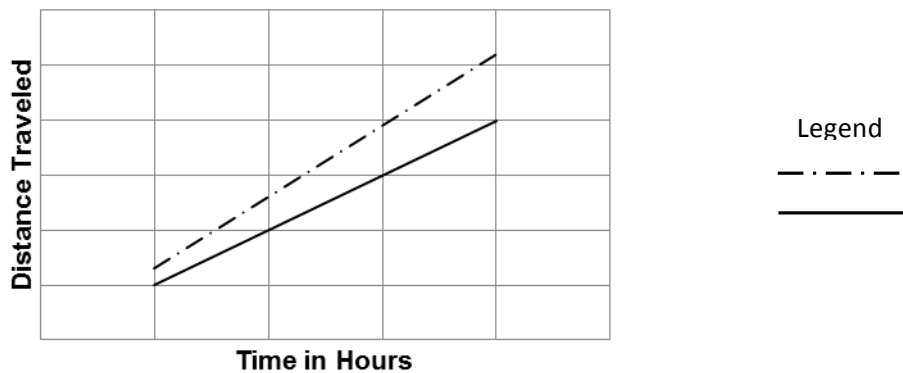
a. ~~1m = 6cm~~  $24\text{cm} = 18\text{miles}$   
 $6\text{cm} = 1\text{mile}$

b.  $4\frac{1}{2}\text{cm}$        $1.3 = \frac{4}{3} \times 18$

c. ~~18cm~~  $12\text{cm}$  ~~miles~~  $4.5$   
 $12\text{m}$        $18 \div 4 = 4.5$

The diagram at the bottom shows a line segment with three equal parts. The top part is labeled '18m', the middle part is labeled '6m', and the bottom part is labeled '12m'. A large checkmark is drawn to the right of the diagram.

3. Jack and Jill raced cross-country on motor bikes. Jack drove 325 miles in 5 hours; Jill took  $6\frac{1}{2}$  hours to travel the same distance as Jack.
- Compute the unit rates that describe Jack's average driving speed and Jill's average driving speed. Show how you made your decisions.
  - A portion of the graph of Jack and Jill's race appears below. Identify which line segment belongs to Jack and which belongs to Jill. Explain in words how you decided which line segment belongs to Jack and which belongs to Jill.



NYC Grade 7 Assessment 1  
 Jack and Jill's Race Task  
 Benchmark Papers

3 Points

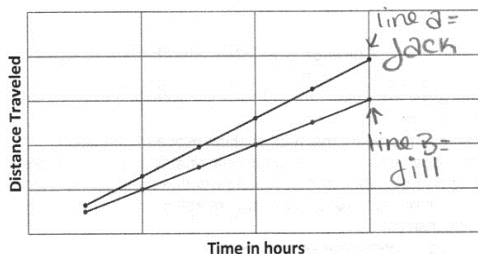
The response accomplishes the prompted purpose and effectively communicates the student's mathematical understanding. The student's strategy and execution meet the content (including concepts, technique, representations, and connections), thinking processes and qualitative demands of the task. Minor omissions may exist, but do not detract from the correctness of the response.

Either verbally or symbolically, the strategy used to solve the problem is stated, as is the work used to find ratios and unit rates. Minor arithmetic errors may be present, but no errors of reasoning appear. Complete explanations are stated, based on work shown.

Accurate reasoning processes demonstrate the Mathematical Practices, (1) Make sense of problems and persevere in solving them and (2) Reason abstractly and quantitatively (since students need to abstract information from the problem, create a mathematical representation of the problem, and correctly work with unit rate/constant of proportionality.) Evidence of the Mathematical Practice, (3) Construct viable arguments and critique the reasoning of others, is demonstrated by complete and accurate explanations. Evidence of the Mathematical Practice, (4) Model with mathematics, is demonstrated by representing the problem with ratios and/or proportions. Evidence of the Mathematical Practice, (6) Attend to precision, can include proper use of ratio notation and proper labeling of quantities.

The reasoning used to solve part b of the problem may include:

- Recognizing that the line rising more quickly must represent the faster biker.
- Noting that, for any one or all points on the graph, those appearing on the upper line show a larger distance traveled in the same amount of time as those points on the lower line; may or may not reference that the x-coordinate represents amount of time traveled while the y-coordinate represents distance traveled.
- Building a table of values for each biker and matching the table to the graph, possibly by choosing scales for the axes.



d.  $\frac{325m}{5hrs} = \frac{65mils}{1hr}$        $\frac{325m}{6hrs} = \frac{x=50}{1}$   
 $325 \div 5 = 65$        $325 \div 6.5 = 50$   
 Jack rode 65 miles Per hour.      Jill rode 50 miles Per hour.

I think that line A shows Jack's Progress because it is moving upward at a faster Pace than line B. This is Because Jack rode faster than Jill

NYC Grade 7 Assessment 1  
 Jack and Jill's Race Task  
 Benchmark Papers

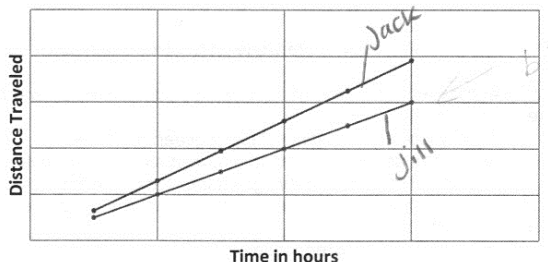
2 Points

The response demonstrates adequate evidence of the learning and strategic tools necessary to complete the prompted purpose. It may contain overlooked issues, misleading assumptions, and/or errors in execution. Evidence in the response demonstrates that the student can revise the work to accomplish the task with the help of written feedback or dialogue.

Either verbally or symbolically, the strategy used to solve the problem is stated, as is the work used to find ratios and unit rates. Reasoning may contain incomplete, ambiguous or misrepresentative ideas. Partial explanations are stated, based on work shown. Accurate reasoning processes demonstrate the Mathematical Practices, (1) Make sense of problems and persevere in solving them and (2) Reason abstractly and quantitatively (since students need to abstract information from the problem, create a mathematical representation of the problem, and correctly work with unit rate/constant of proportionality). Evidence of the Mathematical Practice, (3) Construct viable arguments and critique the reasoning of others, is demonstrated by complete and accurate explanations. Evidence of the Mathematical Practice, (4) Model with mathematics, is demonstrated by representing the problem with ratios and/or proportions. Evidence of the Mathematical Practice, (6) Attend to precision, can include proper use of ratio notation and proper labeling of quantities.

The reasoning used to solve part b of the problem may include:

- Recognizing that the top line must represent the faster biker, but failing to note that the top line is rising more quickly.
- Generally noting that the points appearing on the upper line show a larger distance traveled without noting that the larger distance is occurring in the same amount of time.
- Building a table of values for each biker and matching the table to the graph, possibly by choosing scales for the axes; scales may be inappropriately chosen or tables of values may be arbitrary.



$$a.i \frac{325 \text{ mi}}{5 \text{ hr}} \div \frac{5}{5} = \frac{65}{1 \text{ hr}}$$

Jack's average speed is 65 mi/hr

$$ii. \frac{325 \text{ mi}}{6.5 \text{ hr}} \div \frac{6.5}{6.5} = \frac{50}{1 \text{ hr}}$$

Jill's average driving speed is 50 mi/hr

b.ii I made my decision because Jack drives at a higher rate per hour

NYC Grade 7 Assessment 1  
 Jack and Jill's Race Task  
 Benchmark Papers

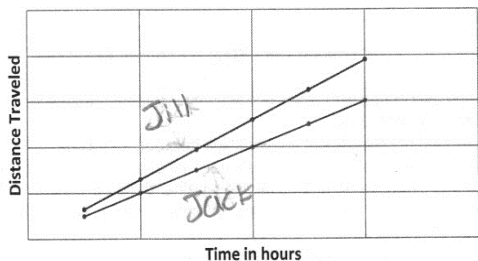
1 Point

The response demonstrates some evidence of mathematical knowledge that is appropriate to the intent of the prompted purpose. An effort was made to accomplish the task, but with little success. Evidence in the response demonstrates that, with instruction, the student can revise the work to accomplish the task.

Some evidence of reasoning is demonstrated either verbally or symbolically, is often based on misleading assumptions, and/or contains errors in execution. Some work is used to find ratios and unit rates or partial answers to portions of the task are evident. Explanations are incorrect, incomplete or not based on work shown. Accurate reasoning processes demonstrate the Mathematical Practices, (1) Make sense of problems and persevere in solving them and (2) Reason abstractly and quantitatively (since students need to abstract information from the problem, create a mathematical representation of the problem, and correctly work with unit rate/constant of proportionality). Evidence of the Mathematical Practice, (3) Construct viable arguments and critique the reasoning of others, is demonstrated by complete and accurate explanations. Evidence of the Mathematical Practice, (4) Model with mathematics, is demonstrated by representing the problem with ratios and/or proportions. Evidence of the Mathematical Practice, (6) Attend to precision, can include proper use of ratio notation and proper labeling of quantities.

The reasoning used to solve the parts of the problem may include:

- a. Assuming proportionality rather than trying to find unit rate.
- b. Reference to the length of the segments rather than distance and time.
- c. Assuming the upper line suggests more time rather than a larger distance when compared to time.



a  $\frac{5 \text{ hours}}{325 \text{ miles}} = \frac{6\frac{1}{2} \text{ hours}}{x}$

325

$\frac{5x}{5} = 325 \cdot 6\frac{1}{2} = 2112.5$   
 $x = 422.5$

b = I think the line at the bottom is Jack and the top is Jill because it took Jill  $6\frac{1}{2}$  hours just to go as far as Jack while it took Jack 5 hours to go 325 miles

$$\begin{array}{r} 42.25 \\ 5 \overline{) 2112.5} \\ \underline{10} \phantom{00} \\ 12 \phantom{00} \\ \underline{10} \phantom{00} \\ 25 \phantom{00} \\ \underline{25} \phantom{00} \\ 00 \end{array}$$



4. Reynaldo is planning to drive from New York to San Francisco in his car. Reynaldo started to fill out the table below showing how far in miles he can travel for each gallon of gas he uses.

Gallons	2	4		8	10	12
Miles	56		168	224		

Use the information in Reynaldo's table to answer the questions below.

- Complete the table for Reynaldo. Assume the relationship in the table is proportional.
- Based on the table, how many miles per gallon did Reynaldo's car get? Explain your reasoning in words.
- Write an equation that Reynaldo can use to find the distance ( $d$ ) he can drive on any number of gallons of gas ( $g$ ).
- When Reynaldo's tank is full, it holds 20 gallons. How far can Reynaldo drive on a full tank of gas?

NYC Grade 7 Assessment 1  
 Reynaldo's Drive Task  
 Benchmark Papers

3 Points

The response accomplishes the prompted purpose and effectively communicates the student's mathematical understanding. The student's strategy and execution meet the content (including concepts, technique, representations, and connections), thinking processes and qualitative demands of the task. Minor omissions may exist, but do not detract from the correctness of the response.

Either verbally or symbolically, the strategy used to solve the problem is stated, as is the work used to find ratio, unit rates, and equation. Minor arithmetic errors may be present, but no errors of reasoning appear. Complete explanations are stated, based on work shown.

Accurate reasoning processes demonstrate the Mathematical Practices (1) Make sense of problems and persevere in solving them, and (2) Reason abstractly and quantitatively (since students need to abstract information from the problem, create a mathematical representation of the problem, and correctly work both with unit rate/constant of proportionality). Evidence of the Mathematical Practice, (3) Construct viable arguments and critique the reasoning of others, is demonstrated by complete and accurate explanations. Evidence of the Mathematical Practice, (4) Model with mathematics, is demonstrated by the linear equation shown in part c. Evidence of the Mathematical Practice, (6) Attend to precision, can include proper use of ratio and/or proportion notation, proper symbolism and proper labeling.

The reasoning used to solve the parts of the problem may include:

- Indicating that the unit rate can be multiplied by the number of gallons to find the distance, and using that process.
- Using a proportion or proportional reasoning (e.g., 2 times 10 equals 20, so I can multiply 56 by 10 to find distance).
- Extending the table of values.

Gallons	2	4	6	8	10	12
Miles	56	112	168	224	280	336

28 because 2 gallons equal 56 miles  
 so unit rate of it is  $\frac{1}{28}$ .

$$D = 28 \times g$$

I double  $\frac{70}{280}$  I double it which is  $\frac{70}{560}$ .

NYC Grade 7 Assessment 1  
 Reynaldo's Drive Task  
 Benchmark Papers

2 Points

The response demonstrates adequate evidence of the learning and strategic tools necessary to complete the prompted purpose. It may contain overlooked issues, misleading assumptions, and/or errors in execution. Evidence in the response demonstrates that the student can revise the work to accomplish the task with the help of written feedback or dialogue.

Either verbally or symbolically, the strategy used to solve the problem is stated, as is the work used to find ratio, unit rates, proportionality and equation. Reasoning may contain incomplete, ambiguous or misrepresentative ideas. Partial explanations are stated, based on work shown. Accurate reasoning processes demonstrate the Mathematical Practices (1) Make sense of problems and persevere in solving them, and (2) Reason abstractly and quantitatively (since students need to abstract information from the problem, create a mathematical representation of the problem, and correctly work both with unit rate/constant of proportionality). Evidence of the Mathematical Practice, (3) Construct viable arguments and critique the reasoning of others, is demonstrated by complete and accurate explanations. Evidence of the Mathematical Practice, (4) Model with mathematics, is demonstrated by the linear equation shown in part c. Evidence of the Mathematical Practice, (6) Attend to precision, can include proper use of ratio and/or proportion notation, proper symbolism and proper labeling.

The reasoning used to solve the parts of the problem may include:

- Indicating that the unit rate can be multiplied by the number of gallons to find the distance, and using that process, but using an incorrect unit rate or number of gallons.
- Using a proportion or proportional reasoning incorrectly (e.g., 20 gallons is 2 times 10, so I can multiply 10 by 10 to find distance).
- Extending the table of values, but failing to recognize the need to stop at 20 gallons.

Gallons	2	4	6	8	10	12
Miles	56	112	168	224	280	336

b.  $1. \frac{56}{2} = \frac{28 \text{ mi}}{1 \text{ gal}}$   
 Reynaldo used 28 miles per gallon

c. a constant rate of 28 miles per gallon  
 $d = 28g$

d.  $\frac{1 \text{ gal}}{20 \text{ gal}} = \frac{28 \text{ mi}}{x}$  Reynaldo can drive 560 miles on a full tank  
 $\frac{x}{20} = \frac{560}{1}$   
 $x = 560$

NYC Grade 7 Assessment 1  
Reynaldo's Drive Task  
Benchmark Papers

1 Point

The response demonstrates some evidence of mathematical knowledge that is appropriate to the intent of the prompted purpose. An effort was made to accomplish the task, but with little success. Evidence in the response demonstrates that, with instruction, the student can revise the work to accomplish the task.

Some evidence of reasoning is demonstrated either verbally or symbolically, is often based on misleading assumptions, and/or contains errors in execution. Some work is used to find ratio, unit rates, proportionality and equation or partial answers to portions of the task are evident. Explanations are incorrect, incomplete or not based on work shown. Accurate reasoning processes demonstrate the Mathematical Practices (1) Make sense of problems and persevere in solving them, and (2) Reason abstractly and quantitatively (since students need to abstract information from the problem, create a mathematical representation of the problem, and correctly work both with unit rate/constant of proportionality). Evidence of the Mathematical Practice, (3) Construct viable arguments and critique the reasoning of others, is demonstrated by complete and accurate explanations. Evidence of the Mathematical Practice, (4) Model with mathematics, is demonstrated by the linear equation shown in part c. Evidence of the Mathematical Practice, (6) Attend to precision, can include proper use of ratio and/or proportion notation, proper symbolism and proper labeling.

The reasoning used to solve the parts of the problem may include:

- No attempt at mathematical reasoning to respond to part b.
- Some attempt to scale, but failure to maintain the ratio, typically by reverting to addition.
- Failure to attempt at least two parts of the problem.

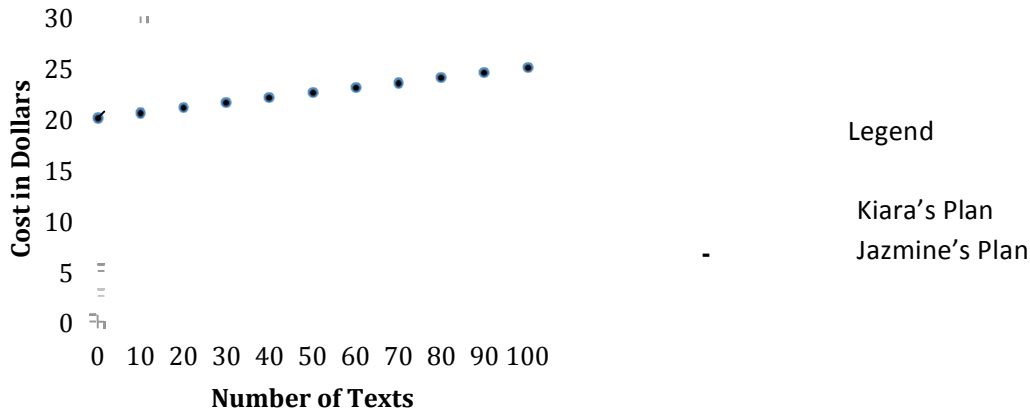
a)

gallons	2	4	6	8	10	12
Miles	56	112	168	224	280	336

b) He used the number 56.

c) Keep adding 2 for the gallons and keep adding 56 for the miles

5. The monthly cost of Jazmine's cell phone plan is graphed on the grid below.



Her friend Kiara selected a plan that charges 25¢ per text, with no monthly fee, because she only uses her phone for texting.

- a. Write an equation to represent the monthly cost of Kiara's plan for any number of texts.
- b. Graph the monthly cost of Kiara's plan on the grid above.
- c. Using the graphs above, explain the meaning of the following coordinate pairs:
  - i. (0, 20): \_\_\_\_\_
  - ii. (0, 0): \_\_\_\_\_
  - iii. (10, 2.5): \_\_\_\_\_
  - iv. (100, 25): \_\_\_\_\_
- d. When one of the girls doubles the number of texts she sends, the cost doubles as well. Who is it? Explain your reasoning in words.

NYC Grade 7 Assessment 1  
Jazmine's Cell Phone Plan Task  
Benchmark Papers

3 Points

The response accomplishes the prompted purpose and effectively communicates the student's mathematical understanding. The student's strategy and execution meet the content (including concepts, technique, representations, and connections), thinking processes and qualitative demands of the task. Minor omissions may exist, but do not detract from the correctness of the response.

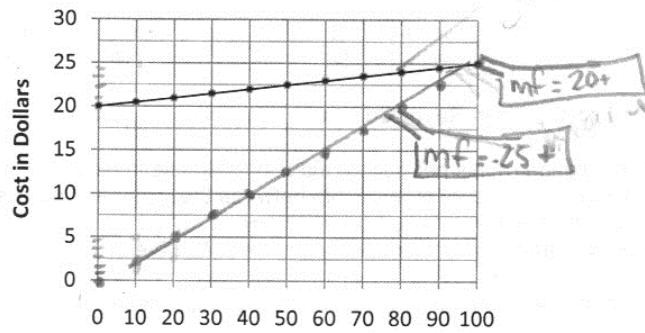
Either verbally or symbolically, the strategy used to solve the problem is stated, as is the work used to find equation and graph. Appropriately identify to which girl each coordinate pair is associated and refer to the x-coordinate as number of texts and the y-coordinate as cost in dollars. Minor arithmetic errors may be present, but no errors of reasoning appear. Complete explanations are stated, based on work shown.

Accurate reasoning processes demonstrate the Mathematical Practices, (1) Make sense of problems and persevere in solving them, and (2) Reason abstractly and quantitatively (since students need to abstract information from the problem, create a mathematical representation of the problem, and correctly work with the ratios, proportions or proportional reasoning, and/or equations). Evidence of the Mathematical Practice, (3) Construct viable arguments and critique the reasoning of others, is demonstrated by complete and accurate explanations. Evidence of the Mathematical Practice, (4) Model with mathematics, is demonstrated by representing Kiara's plan as an equation, correctly representing her information graphically, and using proportions, tables and/or multiplication statements to determine the answer to part c. Evidence of the Mathematical Practice, (6) Attend to precision, can include proper use of ratio and or proportion notation and proper labeling of quantities. Evidence of the Mathematical Practices, (7) Look for and make use of structure, and (8) Look for and express regularity in repeated reasoning, may be demonstrated by student recognition from accurate responses to parts iii. and iv. -- that Kiara's plan is the only one that describes a proportional relationship OR from comparison of the two linear equations, and noting that Kiara's plan is the only one where  $b = 0$ , the indicator of a proportional relationship.

The reasoning used to solve the parts of the problem may include:

- a. For part a, indicating that the unit rate 25¢ can be multiplied by the number of texts to find the cost.
- b. For part d, correctly identifying Kiara as the girl whose cost doubles. Possible arguments for this result include:
  - i. using a proportion or proportional reasoning if appropriate (e.g., for Kiara, 10 texts cost \$2.50 while 20 cost \$5. So, Kiara's cost doubles. For Jazmine, 10 texts cost \$20 while 20 cost a little more than \$20, not double \$20, or \$40.
  - ii. using parts iii. and iv. as evidence, if correctly identified in part c.
  - iii. building a table of values for each girl and observing the results.
  - iv. noting only Kiara's line passes through (0, 0); citing as evidence that a line representing a proportional relationship must pass through (0, 0) and that, if a line doesn't pass through (0, 0), the relationship is not proportional. Explaining why that signifies a proportional relationship.
  - v. finding the equation for Jazmine's plan and noting as evidence that, in the equation describing Jazmine's plan,  $b \neq 0$  while in the equation describing Kiara's plan,  $b = 0$ ; explaining why that signifies a proportional relationship.

NYC Grade 7 Assessment 1  
 Jazmine's Cell Phone Plan Task  
 Benchmark Papers



a. Monthly fee = .25 +

- i. (0, 20): 20 dollars per 0 text
- ii. (0, 0): 0 dollars per 0 text
- iii. (10, 2.5): 2.5 dollars per 10 texts
- iv. (100, 25): 25 dollars per 100 texts

texts, t	0	10	20	30	40	50	60	70	80	90	100
Costs, c	20	20.5	21	21.5	22	22.5	23	23.5	24	24.5	25

d. Kiara is the girl because she is charged at a constant rate of .25 texts per hour starting at 0,0.

NYC Grade 7 Assessment 1  
Jazmine's Cell Phone Plan Task  
Benchmark Papers

2 Points

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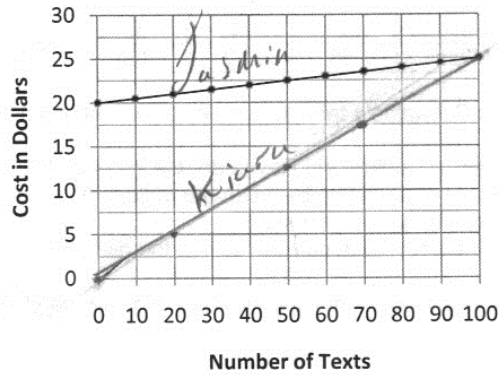
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The reasoning used to solve the parts of the problem may include:

- a. Possibly indicating that the unit rate 25¢ can be added repeatedly to find the cost.
- b. For part c, reversing the meaning of the x- and y-variables in the explanation.
- c. For part d, correctly identifying Kiara as the girl whose cost doubles, but failing to provide an adequate explanation as to why. Possible arguments for this result include:
  - i. Inaccurately or inadequately using a proportion or proportional reasoning (e.g., Kiara's cost doubles because she pays 25¢ per text.)
  - ii. Attempting to use parts iii. and iv. from part c as evidence, but failing to correctly identify information there.
  - iii. building a table of values for each girl and observing the results, but failing to cite evidence from the table in the explanation.
  - iv. noting only Kiara's line passes through (0, 0); citing as evidence that a line representing a proportional relationship must pass through (0, 0) and that, if a line doesn't pass through (0, 0), the relationship is not proportional. Failing to explain why that signifies a proportional relationship.
  - v. finding the equation for Jazmine's plan and noting as evidence that, in the equation describing Jazmine's plan,  $b \neq 0$  while in the equation describing Kiara's plan,  $b = 0$ ; failing to explain why that signifies a proportional relationship.
- d. Failing to correctly answer at least three of the four parts of the problem.



NYC Grade 7 Assessment 1  
 Jazmine's Cell Phone Plan Task  
 Benchmark Papers



Kiara pay = ~~\$0.25~~ 25 x text

- i. (0, 20): on Jazmine graph she is sending no text messages
- ii. (0, 0): on Jazmine no text no bill.
- iii. (10, 2.5): ten texts on Jazmine
- iv. (100, 25): 100 on Jazmine graph.

A.

Month	0	1	2	3	4
Bill	0	25	50	75	100

D. Kiara because she pay \$0.25 for every text so if she double the number of texts the bill doubles too.

NYC Grade 7 Assessment 1  
 Jazmine's Cell Phone Plan Task  
 Benchmark Papers

1 Point

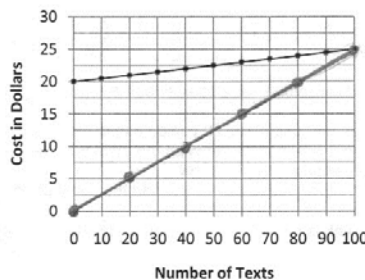
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The reasoning used to solve the parts of the problem may include:

- Possibly indicating that the unit rate 25¢ can be divided by the number of texts to find the cost.
- For part c, reversing the meaning of the x- and y-variables in the explanation.
- For part d, incorrectly identifying Jazmine as the girl whose cost doubles, but failing to provide an adequate explanation as to why.
- Failing to correctly answer at least two of the four parts of the problem.



(a)  $C = 0.25 \times T$   $C = \text{cost}$ ,  $T = \text{text}$

- (0, 20): A Monthly Plan For Jazmine's Plan
- (0, 0): 0 number of text with No charge
- (10, 2.5): The Number of text with Kiara
- (100, 25): The amount of text from Jazmine and Kiara.

(b)

Dollar	25¢	50¢	75¢	100¢
texts	100	200	300	400

- (c) No because the value of one text +  
 (d) is not the same as a dollar.

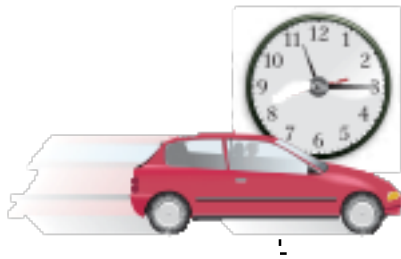


## GRADE 7 MATH: PROPORTIONAL REASONING ANNOTATED STUDENT WORK

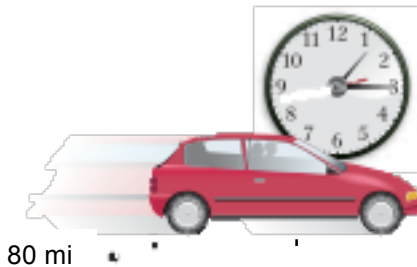
This section contains annotated student work at a range of score points for the Proportional Reasoning Assessment.

### Assessment 1: Question 1

1. Amy and her family were traveling during their vacation. She looked at her watch at Point 1 in the diagram below, and then again at Point 2 in the diagram below. Her mom told her how far they traveled in that time, as noted below.



Point 1



Point 2

- Based on this information, what is the unit rate of the car? Explain in words what that unit rate means in the context of the problem.
- Amy's dad said that the entire trip was 1200 miles. How many hours will it take to complete the trip? Explain your reasoning in words.

#### CCSS (Content) Addressed by this Task

- 7.RP.2b Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams and verbal descriptions.
- 7.RP.3 Use proportional relationships to solve multistep ratio and percent problems.

#### CCSS for Mathematical Practice Addressed by the Task:

- Make sense of problems and persevere in solving them
- Reason abstractly and quantitatively
- Construct viable arguments and critique the reasoning of others
- Model with mathematics
- Attend to precision
- Look for, make use of structure

**Annotation of Student Work With a Score of 3**

a.  $\frac{80 \text{ mi}}{2 \text{ hr}} = \frac{2}{2} = \frac{40 \text{ mi}}{1 \text{ hr}}$   
 The unit of the car is  
 40 miles per hour

b.  $\frac{1 \text{ hr}}{x} = \frac{40 \text{ mi}}{1200 \text{ mi}}$  It will take  
 them 30 hours  
 to complete  
 the trip.  
 $\frac{1200}{40} = \frac{40x}{40}$  I know because  
 I used a proportion  
 and got 30 hours  
 per 1200 miles.  
 $x = 30$  hours

check my work ✓

$\frac{1 \text{ hr}}{30 \text{ hr}} \rightleftharpoons \frac{40 \text{ mi}}{1200 \text{ mi}}$

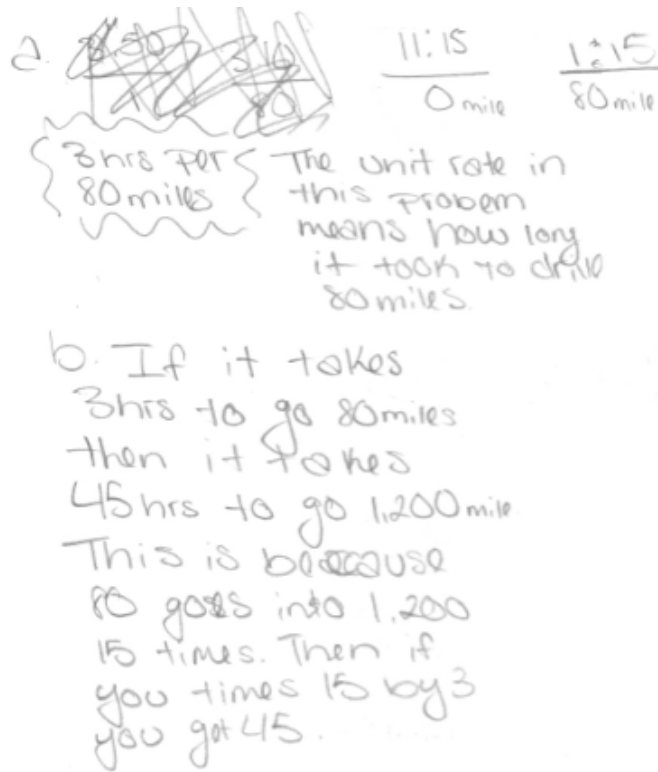
1200  $\div$  40 = 30

**Content Standards:** The student received a score of 3 because s/he uses information given in the task to determine and correctly write a ratio of distance traveled to time elapsed (7.RP.2b). Although the ratio is then scaled down to arrive at a unit rate, the student then imprecisely states, “The unit of the car is 40 miles per hour.” In part b, the student sets up a proportion:  $\frac{\text{unit time}}{\text{time}} = \frac{\text{distance1}}{\text{distance2}}$  and then solves it algorithmically to correctly state that the 1200-mile trip will take 30 hours to complete (7.RP.3).

**Mathematical Practices:** Practice 1 is exhibited in the student’s work because both parts of the question are answered and then checked. The student reasons both abstractly and quantitatively (Practice 2) by using contextual information to determine the elapsed time, and then creating a ratio using that time and the distance shown in the diagram. The student scales the ratio down to a unit rate and then contextualizes that rate to 40 miles per hour. However, Practice 3 is not clearly demonstrated; because the student writes, “the unit of the car is 40 miles per hour,” it is not evident whether there is a misunderstanding about unit rates or whether the student has simply left out part of the explanation. Proportional relationships are correctly represented and labeled (Practices 6 and 7), allowing the student to create a mathematical model (Practice 4) and then to calculate an accurate answer for the second part of the question.

**Next Instructional Steps:** Ask the student what this sentence means, “The unit of the car is 40 miles per hour.” Ask the student to describe similarities and differences between a rate and a unit rate. Ask the student to give examples of a unit and then examples of a unit rate.

**Annotation of Student Work With a Score of 2**



a. ~~Diagram~~ 11:15 1:15  
0 mile 80 mile

3 hrs for 80 miles

The unit rate in this problem means how long it took to drive 80 miles.

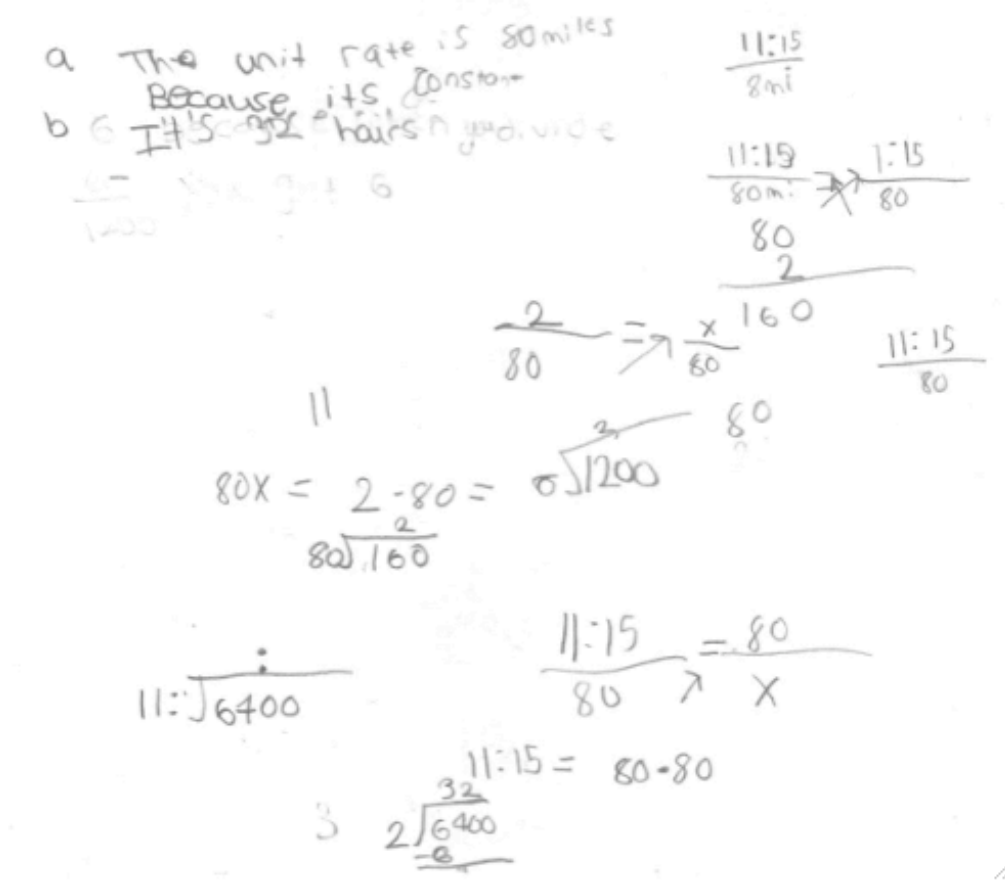
b. If it takes 3 hrs to go 80 miles then it takes 45 hrs to go 1,200 miles. This is because 80 goes into 1,200 15 times. Then if you times 15 by 3 you get 45.

**Content Standards:** The student received a score of 2 because, while s/he correctly reasons through the question in part b) (7.RP.3), a misunderstanding of unit rate in part a) prevents the student from successfully calculating the time needed to travel 1200 miles. Instead of correctly calculating the elapsed time as 2 hours, the student incorrectly calculates the interval from 11:15 to 1:15 and arrives at a total elapsed time of 3 hours. The statement, “The unit rate in this problem means how long it took to drive 80 miles” shows the student has a misunderstanding of the concept of unit rate (7.RP.2b).

**Mathematical Practices:** The student makes sense of the problem and exhibits perseverance in solving it (Practice 1). S/he demonstrates Practice 4 by using information in the diagram to create a mathematical model of the problem. The student uses contextual information in the diagram to analyze the situation and then to represent it numerically (Practice 2). Despite initially miscalculating the elapsed time, the student abstracts from given information and provides a sound written argument (Practice 3) to reach a solution to part b), although it is incorrect because of the error in part a). S/he creates a sound argument for how to use the multiplicative relationship between the distances given in the problem in order to scale the amount of elapsed time shown in the diagram to answer the question about how many hours the trip will take (MP 3). The student’s representation of the time in digital format rather than as a quantity hinders the development of a correct structure (Practice 6 and Practice 7).

**Next Instructional Steps:** Since the student’s explanation of unit rate in the problem is incorrect, ask the student to explain similarities and differences between a rate and a unit rate and to give an example of each in this situation. A possible sequence of questions for the student might be: “If one car traveled 50 miles in 1 hour and another car traveled 150 miles in 3 hours, which car was traveling at a faster rate? What is the unit rate for each car? How do you know? What is the difference between a rate and a unit rate?” Additionally, ask the student how s/he determined 3 hours, since it is likely this is a simple calculation error.

**Annotation of Student Work With a Score of 1**



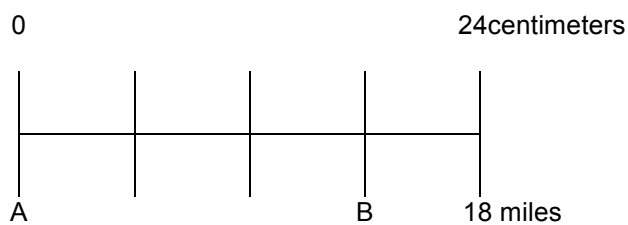
**Content Standards:** The student received a score of 1 because, while the work demonstrates some evidence of mathematical knowledge appropriate to the task at hand, s/he fails to find a unit rate (7.RP.2b). In fact, the student exhibits a misunderstanding of the concept of a unit rate, stating, “The unit rate is 80 miles because it’s [sic] constant.” Although s/he writes and correctly interprets the time as given by the two clocks in the diagram, the ratio comparison is time to distance rather than distance to time. Because the equations shown in the work do not truly represent proportions, the student is not able to solve the problem as requested (7.RP.3).

**Mathematical Practices:** The student shows perseverance in that s/he writes down several attempts to make sense of the quantities in the proportional relationship (Practice 1). S/he also attempts to use Practice 2 to reason quantitatively; however, the work does not flow in a way that helps the student develop a solid mathematical model (Practice 4), nor does it help the student to create a viable argument that might help to create a solid model (Practice 3). While there is some evidence of an understanding of the multiplicative structure in the proportional relationship (given the arrows in the ratio  $11:15/80 \text{ mi} = 1:15/80$ ) the lack of precision in attending to both units and quantities prevents the student from moving forward toward a solution (Practice 6).

**Next Instructional Steps:** This student would benefit by being grouped with other students who are able to articulate and describe how to determine a unit rate and what it means. S/he should be challenged to repeat back what other capable students are saying and asked to restate what was said in his or her own words. Another step might be to ask the student what quantities are to be compared in this situation. Ask her/him to examine some proportions that have been arranged correctly and describe what is being compared in these proportions.

## Assessment 1: Question 2

2. On a map of the United States, 24 centimeters represents 18 miles, and the 24-centimeter segment is divided into four equal pieces, as shown in the picture below.



- How many centimeters represent one mile?
- How long is the line segment between A and B in centimeters? Use mathematical reasoning to justify your response.
- If A and B represent two cities, what is the actual distance between the two cities? Use mathematical reasoning to justify your response.

### CCSS (Content) Addressed by this Task

- 7.RP.1 Compute unit rates associated with ratios of fractions.
- 7.RP.2 Recognize and represent proportional relationships between quantities.
- 7.RP.3 Use proportional relationships to solve multistep ratio and percent problems.

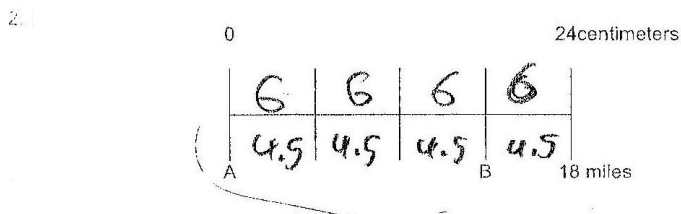
### CCSS for Mathematical Practice Addressed by the Task:

- Make sense of problems and persevere in solving them
- Reason abstractly and quantitatively
- Construct viable arguments and critique the reasoning of others
- Model with mathematics
- Attend to precision
- Look for and make use of structure
- Look for and express regularity in repeated reasoning



NYC Grade 7 Assessment 1  
 Map Task  
 Annotated Student Work

Annotation of Student Work With a Score of 3



- a.  $24 \text{ cm} : 18 \text{ mi.}$   
 $1.\overline{33}$  centimeter represents 3/4 mile
- b.  $3 \times 6$  18 centimeters
- c. 18 kilometers instead  
 $3 \times 4.5$  13.5 miles

**Content Standards:** The student received a score of 3 because s/he writes down the correct unit rate associated with the map scale (7.RP.1), and then uses the information that the map segment is divided into 4 equal parts to correctly partition the segments in the drawing. The 24:18 that the student writes shows that s/he recognizes the proportional relationship inherent in that representation (7.RP.2). S/he provides evidence that s/he uses the concept of proportionality to answer part c) by writing  $3 \times 4.5$  before providing the correct distance (7.RP.3).

**Mathematical Practices:** The student makes sense of the context and s/he answers all parts associated with the task (Practice 1). S/he correctly interprets the given information in the task and relates it to the graph in the correct quantitative amounts (Practice 2). By partitioning the map segment, the student uses mathematical tools (Practice 4), and s/he provides further justification of his/her thinking by writing  $3 \times 4.5$ . With one exception (writing kilometers instead of centimeters) the student uses correct units in his/her work, and s/he recognizes that part c) requires an answer in miles (Practice 6). The student makes use of the underlying mathematical structure (Practice 7) in the problem by providing evidence of a linkage between the identified segment lengths on the drawing and the written representation of that as  $3 \times 4.5$  and uses that again in part c) (Practice 8).

**Next Instructional Steps:** Ask the student to explain how s/he calculated the unit rate for part a). Challenge the student to explain the thinking represented by his/her sketch in a mathematical expression or equation.

Annotation of Student Work With a Score of 2

2.

$\frac{24}{18} = \frac{x}{1}$

$\frac{18}{24} = \frac{x}{24}$

$$\begin{array}{r} 24 \overline{) 180} \\ \underline{54} \phantom{0} \\ 54 \phantom{0} \\ \underline{54} \phantom{0} \\ 0 \phantom{0} \end{array}$$

$$\begin{array}{r} 0.133 \\ 24 \overline{) 1800} \\ \underline{72} \phantom{00} \\ 108 \phantom{0} \\ \underline{72} \phantom{0} \\ 360 \\ \underline{360} \\ 0 \end{array}$$

24 centimeters

18 miles

A B

a.

$$\begin{array}{r} 0.06 \\ 24 \overline{) 180} \\ \underline{144} \\ 360 \\ \underline{360} \\ 0 \end{array}$$

$$\begin{array}{r} 18x = 24 \\ x = 1.33 \end{array}$$

1.33 centimeters represents one mile.

$$\begin{array}{r} 24 \\ \times 5 \\ \hline 120 \\ 24 \\ \hline 120 \\ \times 6 \\ \hline 144 \\ 24 \\ \hline 168 \end{array}$$

$$\begin{array}{r} 1 \\ 2 \\ 3 \\ 4 \\ \hline \end{array}$$

The line segment between A and B is 18 cm long because each segment is 6 centimeters.

The actual distance between the two cities are 18 centimeters.

$$\begin{array}{r} 4 \overline{) 124} \\ \underline{8} \phantom{0} \\ 44 \\ \underline{40} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

NYC Grade 7 Assessment 1

Map Task

Annotated Student Work

**Content Standards:** The student received a score of 2 because s/he recognizes that there is a proportional relationship between the centimeters and miles on the map (7.RP.2) and s/he correctly computes the unit rate for the map scale (7.RP.1). S/he uses contextual information that the full segment is divided into equal sections to correctly determine the length of the map segment between A and B. However, the student does not show that s/he is able to work simultaneously with the two units (centimeters and miles) represented on the map and so is unable to represent the proportional relationship for miles mathematically. This prevents him/her from correctly solving part c) (7.RP.3).

**Mathematical Practices:** The student makes sense of the problem and presents answers to all questions, thereby exhibiting Practice 1. However, s/he does not present his/her work in a way that easily permits a reader to follow his/her thinking (Practice 3). S/he does use the drawing as a tool to help in reasoning through part b) of the problem (Practice 4). S/he demonstrates the ability to reason quantitatively as she connects contextual information from the task with the map representation, but his/her work shows that s/he has difficulty reasoning abstractly (Practice 2). The student correctly displays the units required for the unit rate (Practice 6). Although the student does use repeated reasoning in his/her work, s/he does not clearly demonstrate that s/he recognizes the underlying structure in the proportional relationship embedded within the problem (Practice 7 and Practice 8).

**Next Instructional Steps:** Ask the student to explain why s/he chose to divide 24 by 18. Ask him/her what the corresponding distance would be for a different map length, given the same scale. Ask him/her how s/he could use the approach from part b) to help solve part c). Encourage him/her to consider making a separate work space for 'scratch' work so that the 'presentation' work is easier to follow.

NYC Grade 7 Assessment 1  
Map Task  
Annotated Student Work

Annotation of Student Work With a Score of 1

a.  $\frac{24 \text{ cm}}{18 \text{ m}} = \frac{x \text{ cm}}{1 \text{ m}}$   
 $18x = 24.1$   
 $18x = 24$  | Answer: 1.3 cm |

b.  $4 \frac{1}{2} \text{ cm}$

c. ~~3.375 mile~~ 3.375 mile

**Content Standards:** The student received a score of 1 because, although s/he correctly represents the proportional relationship shown on the map (7.RP.2), s/he does not identify the resulting answer as a unit rate (7.RP.1). The student does not show evidence that she recognizes the proportionality as carrying through to parts b) and c) (7.RP.2). S/he does not use the proportional relationship to correctly answer either part b) or part c) (7.RP.3).

**Mathematical Practices:** The student makes initial sense of the problem and attempts to answer all associated parts (Practice 1). Although s/he demonstrates some ability to reason quantitatively in part a), s/he does not show evidence that s/he clearly relates this to the other parts of the problem (Practice 2). The student work does not demonstrate that s/he is able to justify his/her thinking (Practice 3). The student creates and sets out a tool s/he can use to model the relationship between miles and centimeters, but then fails to use it to help solve the rest of the problem (Practice 4). Although the student does correctly align the units in his/her proportional relationship in part a), his/her subsequent work does not use correct units (Practice 6). S/he does not provide evidence that she recognizes the underlying structure present in all parts of the problem (Practice 7), nor does s/he exhibit recognition of the repeated reasoning that connects all parts of the problem (Practice 8).

**Next Instructional Steps:** Ask the student to explain his/her understanding of a unit rate, and then ask him/her to explain how that understanding relates to the work in part a). Ask the student to explain how s/he would partition the segment in the drawing, and how s/he understands the phrase 'the line segment between A and B,' because his/her answer suggests that s/he sees that as referring to one of the subsegments.

### Assessment 1: Question 3

3. Jack and Jill raced cross-country on motor bikes. Jack drove 325 miles in 5 hours; Jill took  $6\frac{1}{2}$  hours to travel the same distance as Jack.
- Compute the unit rates that describe Jack's average driving speed and Jill's average driving speed. Show how you made your decisions.
  - A portion of the graph of Jack and Jill's race appears below. Identify which line segment belongs to Jack and which belongs to Jill. Explain in words how you decided which line segment belongs to Jack and which belongs to Jill.



#### CCSS (Content) Addressed by this Task

- 7.RP.1 Compute unit rates associated with rates of fractions.
- 7.RP.2 Recognize and represent proportional relationships between quantities.
- 7.RP.2b Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions.

#### CCSS for Mathematical Practice Addressed by the Task:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
6. Attend to precision

**Annotation of Student Work With a Score of 3**

a.  $\frac{325 \text{ m}}{5 \text{ hrs}}$     $\frac{65 \text{ miles}}{1 \text{ hr}}$     $\frac{325 \text{ m}}{6.5 \text{ hrs}} = \frac{x = 50}{1}$

$325 \div 5 = 65$     ~~$325 \div 6.5 = 50$~~

Jack rode 65 miles  
Per hour.

Jill rode 50 miles  
Per hour.

I think that line A shows Jack's progress because it is moving upward at a faster pace than line B. This is because Jack rode faster than Jill.

**Content Standards:** The student received a score of 3 because s/he finds both Jack's and Jill's average driving speeds or unit rates by dividing the total number of miles by the total hours (7.RP.1), sets up correct ratios and proportions and accurately determines miles per hour (7.RP.2). In addition, the student determines the unit rate for Jack's and Jill's trip from the verbal description in the problem and in part b), the student associates the larger unit rate with the line having a steeper slope. (7.RP.2b).

**Mathematical Practices:** The student shows evidence of Practice 1 by making sense of Jack and Jill's trips, by developing a pathway for solving the problem and by arriving at the solution. S/he demonstrates Practice 2 by abstracting information from the context of the problem, dealing with it numerically and writing a ratio and a proportion, and then correctly referring back to the context of the problem and accurately associating the steeper line with the faster speed. The student constructs an argument (Practice 3) when s/he accurately claims that Jack has gone faster but s/he does not deliberately defend his/her claim by referring to the information in part a). A more convincing argument might sound like "I think that line A shows Jack's progress because the vertical change (distance traveled) in line A is greater than what is shown in line B." The student models with mathematics (Practice 4) by writing a ratio comparison of distance to time for both Jack's and Jill's average speeds, by setting up a proportion and determining miles per hour, and by accurately making connections between the ratios, the context and the information on the graph. Finally, the student accurately sets up a ratio and proportional relationship with quantities labeled for both Jack's and Jill's trips (Practice 6).

**Next Instructional Steps:** Ask the student to explain how s/he knows his/her claim is accurate. Ask the student if s/he can make a connection between the unit rates and the graphical representations on the grid.

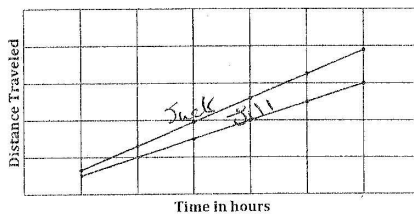
NYC Grade 7 Assessment 1  
 Jack and Jill  
 Annotated Student Work

Annotation of Student Work With a Score of 2

$$\frac{5}{325} \quad \frac{6.5}{325}$$

Jack =  $\frac{1}{65}$  Jill  $\frac{1}{50}$

Jack 65 miles per hour       $325 \div 5 = 65$   
 Jill 50 miles every hour       $325 \div 6.5 = 50$



Jack made it faster in less time so  
 he is on top Jill still took more  
 time. Jill went less of a distance  
 each hour

**Content Standards:** The student received a score of 2 because the student determines both Jack's and Jill's average driving speed (unit rate) by dividing the total number of miles by the total hours to find the miles per hour ( $325 \div 5 = 65$  and  $325 \div 6.5 = 50$ ) (7.RP.1). While the student sets up ratios that permit him/her to solve the problem and correctly determines both Jack's and Jill's average speeds in miles per hour, the ratios the student writes compare time to distance rather than distance to time. The student's conclusion does not follow directly from the ratios s/he writes because, e.g.,  $1/50$  does not equal 50 miles per hour, nor is it true that 'Jack =  $1/50$ .' (7.RP.2). In part b), the student makes both accurate and inaccurate statements. The use of both accurate and inaccurate statements provides conflicting information about what the student understands regarding constant of proportionality (7.RP.2b).

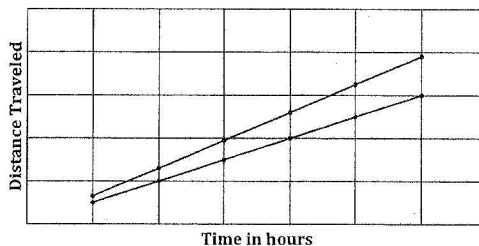
**Mathematical Practices:** The student makes sense of Jack's and Jill's trips. The student has a pathway for solving the problem and arrives at a correct solution to part a), but delivers an ambiguous response to part b) (Practice 1). The student abstracts information from the problem, deals with it numerically, and competently works with unit rate because his/her paper shows a unit rate. However, the student does not reason as well from a graphical representation. While the graph is labeled correctly, the explanation contains inaccuracies, which call into question exactly what the student understands (Practice 2). The student fails to support his/her arguments (Practice 3) because s/he makes both accurate and inaccurate claims. The student builds a mathematical model (Practice 4) when s/he writes a ratio and determines the unit rate. The student attempts to apply what s/he knows through numerical calculation of the average speeds in part a) to the graphical model of the situation in part b); however, the student is not explicit in identifying important qualities of the graph. The student's response lacks precision (Practice 6) because although s/he accurately divides to determine and then to label the average speed for both Jack's and Jill's trips, s/he inaccurately states that Jill =  $1/50$  and Jack =  $1/65$ , without labeling the quantities.

**Next Instructional Steps:** Ask the student to clarify what units are associated with his/her ratios of  $1/50$  and  $1/65$ ; also ask which numbers are the divisors and dividends in the ratios. Ask the student what evidence s/he sees in the graph to support his/her statement that 'Jill still took more time.' Ask the student how s/he knows that Jack 'made it faster [sic] in less time.'

Annotation of Student Work With a Score of 1

$$\begin{array}{r} \text{Jack } 6.65 \\ 5 \overline{)325} \end{array}$$

$$\begin{array}{r} \text{Jill } 54.1 \\ 6 \overline{)325} \end{array}$$



Jace race appears longer  
then jill because his unit  
rate was higher then jill

**Content Standards:** The student received a score of 1 because, although the student does not write ratios, s/he does select the appropriate operation of division to calculate the unit rate. There is an error in the second division problem, since the student uses 6 instead of 6.5 as the divisor, thus missing the target of this content standard (7.RP.1). The student fails to demonstrate that s/he recognizes a proportional relationship (7.RP.2). The student uses the verbal description in the problem to write the two division problems in part a), one of which is appropriate to calculate the unit rate. In part b) s/he refers to unit rate; however, his/her answer does not indicate that s/he associates his/her response to part a) with the unit rate referred to in part b), nor how s/he associates these quantities with the graph (7.RP.2b).

**Mathematical Practices:** The student shows some evidence of Practice 1 when s/he divides the total number of miles by the hours. Although s/he ignores the decimal in Jill's number of hours, s/he uses the correct operation to calculate the miles per hour. The student shows incomplete reasoning (Practice 2) when s/he works with the correct distance and time for Jack's trip but does not do the same for Jill's trip. The student also makes an incorrect claim when s/he states that 'Jace race appears longer'. As a result, the student fails to construct a viable argument (Practice 3). Although the student uses division appropriately in this situation, his/her response does not indicate that s/he knows how to interpret it in the context of the problem, or how it might be useful in answering the question about interpreting the graph in part b) (Practice 4). The student fails to show precision (Practice 6) when s/he inaccurately reads and selects the numbers in the problem. S/he uses 6 instead of 6.5 as the divisor in part a). What the student understands about the meaning of the quotients in the context of the problem is unknown because s/he uses no units. S/he does not label his/her graph, and s/he suggests that Jack's race is longer rather than faster than Jill's.

**Next Instructional Steps:** Ask the student to explain his/her thinking in omitting the decimal portion of Jill's speed when dividing in part a). Ask him/her to explain the units associated with the division operation in part a) and how s/he knows s/he has calculated a unit rate. Ask the student to explain the meaning of the statement 'Jace race appears longer then [sic] Jill.'



### Assessment 1: Question 4

4. Reynaldo is planning to drive from New York to San Francisco in his car. Reynaldo started to fill out the table below showing how far in miles he can travel for each gallon of gas he uses.

Gallons	2	4		8	10	12
Miles	56		168	224		

Use the information in Reynaldo's table to answer the questions below.

- Complete the table for Reynaldo. Assume the relationship in the table is proportional.
- Based on the table, how many miles per gallon did Reynaldo's car get? Explain your reasoning in words.
- Write an equation that Reynaldo can use to find the distance ( $d$ ) he can drive on any number of gallons of gas ( $g$ ).
- When Reynaldo's tank is full, it holds 20 gallons. How far can Reynaldo drive on a full tank of gas?

#### CCSS (Content) Addressed by this Task

- 7.RP.2 Recognize and represent proportional relationships between quantities.
- 7.RP.2b Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams and verbal descriptions.
- 7.RP.2c Represent proportional relationships by equations.
- 7.RP.3 Use proportional relationships to solve multistep ratio and percent problems.

#### CCSS for Mathematical Practice Addressed by the Task:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
6. Attend to precision

**Annotation of Student Work With a Score of 3**

Gallons	2	4	6	8	10	12
Miles	56	112	168	224	280	336

b.  $1. \frac{56}{2g} = \frac{2}{2} = \frac{28 \text{ mi}}{1 \text{ gal}}$   
 Reynaldo used 28 miles per gallon

c. a constant rate of 28 miles per gallon  
 $d = 28g$

d.  $\frac{1 \text{ gal}}{20 \text{ gal}} = \frac{28 \text{ mi}}{x}$  Reynaldo can drive 560 miles on a full tank  
 $\frac{x}{20} = \frac{560}{1}$   
 $x = 560$

**Content Standards:** This paper received a score of 3 because the student demonstrates proficiency with all 4 standards associated with this task. The student represents the proportional relationship between miles traveled and gallons of gas used by correctly completing the table (7.RP.2). Using information from the table, s/he then calculates the unit rate (7.RP.2b). By creating the equation  $d = 28g$ , the student meets 7.RP.2c. Using the given information that the tank holds 20 gallons when full, s/he uses the equation from part c) to correctly calculate that the car can travel 560 miles (7.RP.3).

**Mathematical Practices:** The student makes sense of the problem and works through all associated parts (Practice 1). The student abstracts information both from the problem and the table, and creates an accurate representation of the unit rate with the statement that  $\frac{56}{2} \div \frac{2}{2} = \frac{28 \text{ mi}}{1 \text{ gal}}$ , then successfully interprets his/her results as 28 miles per gallon (Practice 2).

Additionally, s/he displays the ability to model with mathematics (Practice 4) by scaling down the quotient  $56/2$  to a unit rate, and by writing both a correct proportion and equation. Although the student correctly solves the problem and the work is presented in an organized manner, s/he can further develop an explanation for the line of reasoning used (Practice 3). The procedural steps flow clearly; this exhibits precision in thinking and the student is attentive to the units required (Practice 6).

**Next Instructional Steps:** Ask the student to explain what the variables in the equation mean, particularly the variable  $g$ . Give the student a model of writing in which a student has made a claim and provides a coherent and concise explanation of his/her solution path. Challenge the student to revise his/her response to add clarity and depth, and then to write a learning reflection on the process.

**Annotation of Student Work With a Score of 2**

(A)

Gallons	2	4	6	8	10	12	14	16	18	20
Miles	56	112	168	224	280	336	392	448	506	562

(B) Reynaldo uses 2 gallons 56 miles  $\rightarrow$  Rate = 2 gallons = 56 miles

(C) 28 miles per gallon (MPG)

(D) IF Reynaldo's tank is full with 20 Gallons that means that Reynaldo can drive 562 Miles.

**Content Standards:** This paper received a score of 2 because, as stated in the rubric, the student response shows evidence that demonstrates that s/he can revise the work to accomplish all parts of the task with the help of written feedback or dialogue. The student recognizes the proportional relationship between gallons of gas and miles traveled (7.RP.2), but s/he makes a calculation error in the table when going from 16 to 18 gallons. This is then carried over to the 20-gallon calculation. The student recognizes that Reynaldo's car travels 56 miles on 2 gallons of gas, and this information is converted to a unit rate in part b), thus meeting standard 7.RP.2b. The student fails to create an equation to describe the proportional relationship shown in the table (7.RP.2c). However, s/he does determine the number of miles that Reynaldo can travel on a full tank of gas (7.RP.3).

**Mathematical Practices:** The student perseveres to reach a solution to the final question in the task (Practice 1). S/he exhibits the ability to reason quantitatively, as demonstrated by completion of the table, but does not display a methodology to show how s/he then calculates the unit rate (Practice 2). The student provides partial explanations of the work, but does not clearly demonstrate his/her line of thinking (Practice 3). The student uses an additive approach to complete the table, but does not demonstrate how to model that thinking (Practice 4). The student is careful to indicate appropriate units, but does not correct the calculation error in the table (Practice 6).

**Next Instructional Steps:** Ask the student if the additive relationship s/he used to extend the table can be converted to a multiplicative one. Ask the student if s/he can think of a way to create an equation to describe the relationship being shown in the table.

**Annotation of Student Work With a Score of 1**

a.

gallons	2	4	6	8	10	12
Miles	56	112	168	224	280	336

b) He used the number 56.

c) Keep adding 2 for the gallons and keep adding 56 for the miles

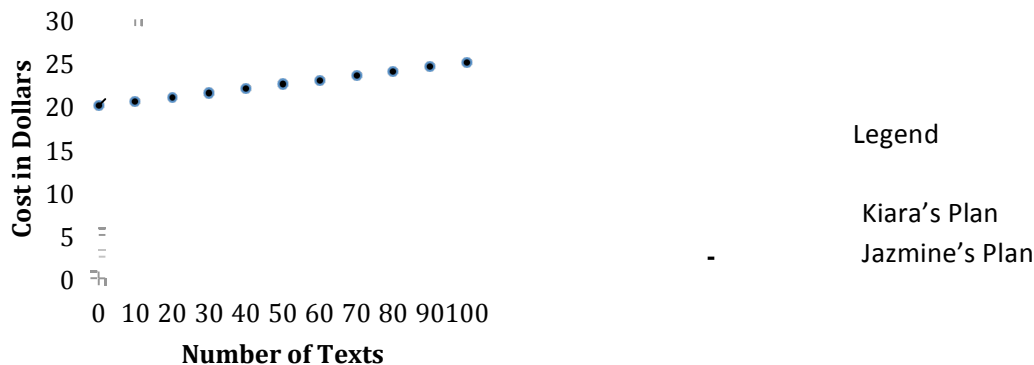
**Content Standards:** This paper received a score of 1 because, although the student correctly completes the table by filling in missing gallons and miles traveled (7.RP.2), s/he does not find a unit rate (7.RP.2b). The student recognizes that it is necessary to add 56 to obtain values for subsequent columns in the table, but does not develop an equation that can be used to calculate distances Reynaldo traveled (7.RP.2c). The student identifies a strategy that can be used to solve the question in part d), but fails to carry the strategy through to a solution (7.RP.3).

**Mathematical Practices:** The student does attempt to make sense of the problem and works through the parts of the problem. The student reasons quantitatively, but s/he does not display a methodology that allows for a correct calculation of the number of miles driven if the number of gallons increases by an amount other than 2 (Practice 2). Since the student does not determine a unit rate appropriate to the given situation, s/he does not demonstrate a clear understanding of the relationships inherent in the problem (Practice 4). The student presents an argument for his/her thinking, but does not use it to obtain a solution (Practice 3), nor does s/he precisely explain how the strategy outlined in part d) can be used to reach a solution (Practice 6).

**Next Instructional Steps:** Ask the student if s/he thinks Reynaldo's car gets good gas mileage and then why or why not. Ask the student what the word "unit" means to him/her, and, if s/he responds with "one", then ask what s/he now thinks the unit rate might mean. Ask the student to look more closely at the table and see if s/he can think of a way to shift from an additive approach to a multiplicative structure.

### Assessment 1: Question 5

5. The monthly cost of Jazmine's cell phone plan is graphed on the grid below.



Her friend Kiara selected a plan that charges 25¢ per text, with no monthly fee, because she only uses her phone for texting.

- Write an equation to represent the monthly cost of Kiara's plan for any number of texts.
- Graph the monthly cost of Kiara's plan on the grid above.
- Using the graphs above, explain the meaning of the following coordinate pairs:
  - (0, 20): \_\_\_\_\_
  - (0, 0): \_\_\_\_\_
  - (10, 2.5): \_\_\_\_\_
  - (100, 25): \_\_\_\_\_
- When one of the girls doubles the number of texts she sends, the cost doubles as well. Who is it? Explain your reasoning in words.

#### CCSS (Content) Addressed by this Task

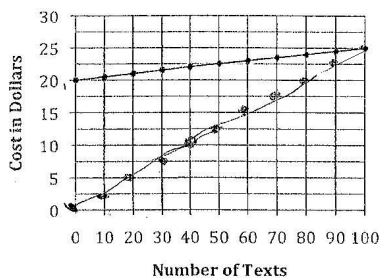
- 7.RP.2 Recognize and represent proportional relationships between quantities.
- 7.RP.2b Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams and verbal descriptions.
- 7.RP.2c Represent proportional relationships by equations.
- 7.RP.2d Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation.
- 7.RP.3 Use proportional relationships to solve multistep ratio and percent problems.

#### CCSS for Mathematical Practice Addressed by the Task:

- Make sense of problems and persevere in solving them
- Reason abstractly and quantitatively
- Construct viable arguments and critique the reasoning of others
- Model with mathematics
- Attend to precision
- Look for and make use of structure
- Look for and express regularity in repeated reasoning

NYC Grade 7 Assessment 1  
 Jazmine's Cell Phone Plan  
 Annotated Student Work

Annotation of Student Work With a Score of 3



- a.  $C = 0.25 \times T$
- c. Using the graphs above, explain the meaning of the following coordinate pairs:
- $(0, 20)$ : It's cost if she send 200 text
  - $(0, 0)$ : It's cost 200 text 200 dollar
  - $(10, 2.5)$ : It's cost 10 text <sup>cost</sup> 2.50¢
  - $(100, 25)$ : It's cost 100 text is 25 dollars
- d. Kiara when she double the text she doubled the price for example when you double 250 to 10 you get 500\$ to 20 text +

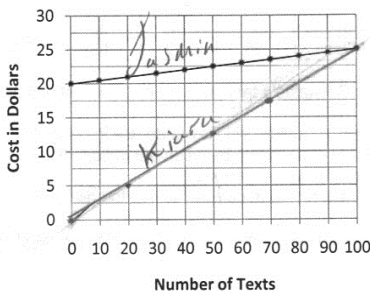
**Content Standards:** The student received a score of 3 because s/he recognizes that the unit rate can be multiplied by the number of texts to determine the total cost of Kiara's plan (7.RP.2). S/he correctly graphs the monthly cost of Kiara's plan on the grid (7.RP.2c). Although the student does not explicitly identify the constant of proportionality, s/he makes use of it, with correct units, in his/her equation (7.RP.2b). The student successfully explains the meaning of various points that relate to the proportional relationship described by Kiara's plan (7.RP.2d). S/he also successfully uses the proportional relationship represented in this problem to solve multiple parts of this problem—graphing, writing an equation, describing the meaning of various points representative of the relationship (7.RP.3).

**Mathematical Practices:** The student makes sense of the problem and exhibits perseverance by working through the multiple parts of this task (Practice 1). The student demonstrates Practice 2 by reasoning abstractly—in describing the meaning of the points—and quantitatively—in creating the equation. S/he successfully uses tools, e.g. graph, equation, to model with mathematics (Practice 4). With the exception of part c iii), the student attends to issues of precision by successfully representing 25¢ in decimal form and by converting cents to dollars (Practice 6). S/he presents a partial argument for why Kiara's plan is the one that doubles in that s/he writes 'when she double the text she doubled the price for example when you double 250 to 10 you get 500\$ to 20 text.' However, the student fails to complete the argument, since s/he doesn't compare and contrast the two plans (Practice 3), nor does s/he clearly identify Kiara's plan as describing a proportional relationship (Practices 7 and 8).

**Next Instructional Steps:** Ask the student to explain why the two graphs have different intercepts and ask how those differences are reflected in the equation in part a). Ask the student to explain why Kiara's plan is the only one that doubles. Clarify U.S. conventions regarding placement of currency symbols when writing monetary values.

NYC Grade 7 Assessment 1  
Jazmine's Cell Phone Plan  
Annotated Student Work

Annotation of Student Work With a Score of 2



a.

Month	0	1	2	3	4
Bill	0	25¢	50¢	75¢	100¢

- c. Using the scenario above, explain the meaning of the following coordinate pairs:
- i. (0, 20): on Jazmine graph she is sending no text messages
  - ii. (0, 0): on Jazmine no text no bill.
  - iii. (10, 2.5): ten texts on Jazmine
  - iv. (100, 25): 100 on Jazmine graph.

of Kiara because she pay \$0.25 for every text so if she double the number of texts the bill doubles too.

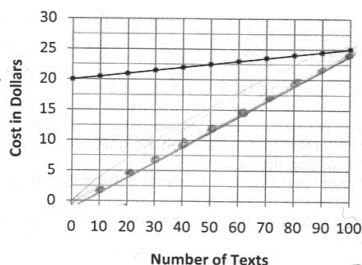
**Content Standards:** The student received a score of 2 because his/her work demonstrates adequate evidence of the learning and tools necessary to complete the requested task even though s/he fails to correctly answer the minimum 3 of the 4 parts of the problem. Although the student identifies all the points referenced in part c) as belonging to Jazmine's graph, the explanation of their contextual meaning is correct (7.RP.2d). S/he recognizes the proportionality inherent in Kiara's plan when s/he states that Kiara pays '\$0.25 for every text so if she double [sic] the number of texts the bill doubles too' (7.RP.2). The student creates a table to display the proportional relationship represented by Kiara's plan, but labels the texts as 'months'. However, the student fails to identify the constant of proportionality (7.RP.2b), and, as a result, does not write an equation as required in part a) (7.RP.2c). His/her table indicates that the unit rate of 25 cents can be added to find the total cost, but it is not clear whether s/he recognizes the pattern as being a proportional relationship (7.RP.3).

**Mathematical Practices:** The student explanations make sense of the problem parts and s/he answers all parts of the problem (Practice 1). The student reasons abstractly by correctly explaining the meaning of certain points on the graph and quantitatively interprets information on the table s/he creates (Practice 2). The student exhibits evidence of Practice 4 by creating a graph and a table for Kiara's plan, both of which are mathematical tools. The student attends to precision (Practice 6) by labeling both graphs on the grid and correctly using currency symbols. S/he displays some evidence of Practice 3 in that s/he explains why Kiara's cost doubles as the number of texts doubles, but fails to compare and contrast that plan against Jazmine's. Practices 7 and 8 are not evident, since s/he does not use language or display work that clearly shows an understanding of the proportional relationship described by Kiara's plan.

**Next Instructional Steps:** Ask the student to explain the meaning of unit rate and also what s/he understands a proportional relationship to be, and then to identify these terms within the context of this problem. Ask the student if s/he can create another table that displays a multiplicative, rather than additive, relationship between the number of texts sent and the total bill. Clarify U.S. conventions regarding placement of currency symbols when writing monetary values.

NYC Grade 7 Assessment 1  
 Jazmine's Cell Phone Plan  
 Annotated Student Work

Annotation of Student Work With a Score of 1



$.25 \times 10 = 2.50$

- a.  
 c. Using the scenario above, explain the meaning of the following coordinate pairs:

- i. (0, 20): \$5.00
- ii. (0, 0): \$0.00
- iii. (10, 2.5): \_\_\_\_\_
- iv. (100, 25): \$25.00

It is Kiara because she only uses her cell phone for texting so if she starts to double the number of texts she's doubling the money.

Texts	0	10	20	30	40	50	60	70	80	90	100
Money	0	2.50	5.00	7.50	10.00	12.50	15.00	17.50	20.00	22.50	25.00

**Content Standards:** The student received a score of 1 because, while s/he does demonstrate a beginning ability to make sense of the contextual information and to recognize its underlying mathematical structure, and s/he is able to correctly graph relevant points on the grid, s/he fails to answer any of the parts of the problem regarding proportionality (7.RP.3). Although s/he writes ' $.25 \times 10 = 2.50$ ,' s/he does not provide evidence that s/he is able to abstract that information and to produce an equation to describe Kiara's plan (7.RP.2c). The student does not explain the meaning of any of the coordinate points given in part c) (7.RP.2d). The student represents the proportional relationship between Kiara's number of texts and her cost in the table, but does not demonstrate that s/he recognizes it as proportional (7.RP.2), since s/he fails to identify a unit rate (7.RP.2b).

**Mathematical Practices:** The student makes an attempt to make sense of the problem and to answer all associated parts (Practice 1). Although s/he does reason quantitatively in that s/he is able to create an accurate table and to write ' $.25 \times 10 = 2.50$ ,' s/he does not display evidence of abstract reasoning regarding proportionality (Practice 2), and the explanation presented in part d), while true, is not justified mathematically (Practice 3). The student does use a mathematical tool to help him/her make sense of the problem when s/he creates a table and s/he uses this table to graph Kiara's plan (Practice 4). His/her work does not show a clear understanding of underlying mathematical structure (Practice 7), since s/he does not exhibit the ability to shift perspective when working with coordinate points—the student can graph points from a table but cannot interpret the meaning of points identified on the grid. The work displays some underlying precision in that the decimal point is correctly interpreted and used in the table and equation, but units are not explicitly noted (Practice 6). The student's table



NYC Grade 7 Assessment 1  
Jazmine's Cell Phone Plan  
Annotated Student Work

shows evidence of the ability to recognize a regular pattern but s/he does not show that s/he can generalize from the pattern (Practice 8).

**Next Instructional Steps:** Since the student is able to correctly graph points from his/her table, point to one of the points s/he graphed and ask him/her to explain its meaning without looking back to the table. Then see if s/he can explain the meaning of one of the points in part c). Challenge the student to extend his/her level of abstract thinking by asking how s/he might generalize the equation written under the grid.



## GRADE 7 MATH: PROPORTIONAL REASONING INSTRUCTIONAL SUPPORTS

These instructional supports include three arcs of related lessons: a sequence of high-level instructional tasks that address the set of targeted Common Core State Standards for Mathematical Content and Common Core State Standards for Mathematical Practice assessed in the first performance-based assessment related to Ratios and Proportional Relationships. The tasks are designed to support student learning in preparation for Assessment #1. Each of the high-level instructional tasks are accompanied by a lesson guide.

The lesson guides (which begin on page 72) provide teachers with the mathematical goals of the lesson, as well as possible student solution paths, errors and misconceptions, and talk moves for engaging students in rigorous teaching and learning. Teachers may choose to use them to support their planning and instruction while teaching arcs of lessons.

<b>INSTRUCTIONAL SUPPORTS</b> .....	<b>58</b>
<b>ARC OF LESSONS 1</b> .....	<b>64</b>
<b>ARC OF LESSONS 2</b> .....	<b>67</b>
<b>ARC OF LESSONS 3</b> .....	<b>70</b>
<b>LESSON GUIDES</b> .....	<b>72</b>
<b>TASK ANALYSIS GUIDE</b> .....	<b>108</b>
<b>SUPPORTS FOR ENGLISH LANGUAGE LEARNERS</b> .....	<b>109</b>
<b>SUPPORTS FOR STUDENTS WITH DISABILITIES</b> .....	<b>112</b>

# Unit Outline – Grade 7 Math

**INTRODUCTION:** This unit outline provides an example of how to integrate performance tasks into a unit. *Teachers may (a) use this unit outline as it is described below; (b) integrate parts of it into a currently existing curriculum unit; or (c) use it as a model or checklist for a currently existing unit on a different topic.*

## 7<sup>th</sup> Grade Math: Proportional Reasoning

### UNIT TOPIC AND LENGTH:

- This 3-4 week unit focuses on analyzing proportional relationships and using them to solve real-world problems. The unit deepens students' understanding and use of unit rates; develops their understanding of proportional relationships as represented in words, tables, equations, graphs and diagrams; and engages them in multi-step ratio and percent problem-solving.

### COMMON CORE LEARNING STANDARDS:

- 7.RP.1 Compute unit rates associated with ratios of fractions.
- 7.RP.2 Recognize and represent proportional relationships between quantities.
  - a. Decide whether two quantities are in a proportional relationship.
  - b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions.
  - c. Represent proportional relationships by equations.
  - d. Explain what a point  $(x, y)$  on the graph of a proportional relationship means in terms of the situation.
- 7.RP.3 Use proportional relationships to solve multistep ratio and percent problems.
- MP. 1 Make sense of problems and persevere in solving them.
- MP. 2 Reason abstractly and quantitatively.
- MP. 3 Construct viable arguments and critique the reasoning of others.
- MP. 4 Model with mathematics.
- MP. 6 Attend to precision.

### BIG IDEAS/ENDURING UNDERSTANDINGS:

- A ratio is a multiplicative comparison of two quantities, or it is a joining of two quantities in a composed unit.
- One representation may sometimes be more helpful than another. Multiple representations give a fuller understanding of a problem.
- Coordinate geometry can be used to represent and verify geometric/algebraic relationships.

### ESSENTIAL QUESTIONS:

- What makes a relationship “proportional”? How can I tell if a proportional relationship exists?
- How can I use tables, graphs or equations to determine whether a relationship is proportional?
- How can representing mathematical ideas in different ways (graphs, tables, equations, diagrams, words) help me solve problems?

# Unit Outline – Grade 7 Math

<p><b>CONTENT:</b></p> <p><b>Ratios &amp; Proportional Relationships:</b></p> <ul style="list-style-type: none"> <li>➤ Unit rates</li> <li>➤ Ratios of fractions including lengths, areas, and other quantities measured in like or different units</li> <li>➤ Proportional relationships between quantities (in tables, graphs, equations, diagrams)</li> <li>➤ Speed as a rate</li> <li>➤ Ratio tables</li> <li>➤ Scale as a ratio</li> <li>➤ Equivalent ratios</li> </ul> <p>-----</p> <p><b>Coordinate Geometry and Algebra</b></p> <ul style="list-style-type: none"> <li>➤ Graphing in the coordinate plane</li> <li>➤ Point of origin</li> <li>➤ <math>(x,y)</math> focus on <math>(0,0)</math> and <math>(1,r)</math> where <math>r</math> is the unit rate</li> <li>➤ Graph of a line</li> <li>➤ Linear relationships</li> <li>➤ Rate of change</li> <li>➤ Slope</li> <li>➤ Proportional and non-proportional relationships</li> </ul>	<p><b>SKILLS AND PRACTICES:</b></p> <p>The unit takes a holistic approach to the skills listed below and emphasizes the interdependence of the skills.</p> <ul style="list-style-type: none"> <li>➤ <b>Apply</b> concepts of rate and ratio to problem s solving tasks and situations</li> <li>➤ <b>Compute</b> unit rates associated with ratios of fractions</li> <li>➤ <b>Identify</b> the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships</li> <li>➤ <b>Decide</b> whether two quantities are in a proportional relationship by testing for equivalent ratios in a data table or by observing the graph of the relationship is a straight line through the origin.</li> <li>➤ <b>Write and solve</b> equations that represent proportional relationships</li> </ul> <p>-----</p> <ul style="list-style-type: none"> <li>➤ <b>Display</b> proportional relationships through equations and graphing</li> <li>➤ <b>Analyze</b> graphs to determine rate and proportional relationships</li> <li>➤ <b>Explain</b> what a point <math>(x, y)</math> on the graph of a proportional relationship means in terms of the situation, with special attention to the points <math>(0, 0)</math> and <math>(1, r)</math> where <math>r</math> is the unit rate</li> <li>➤ <b>Find</b> rate of change using Ordered Pairs in the coordinate plane</li> </ul> <p>In planning to address these skills and their interdependence, it is critically important also to consider the Standards for the Mathematical Practices, which require that the student:</p> <ul style="list-style-type: none"> <li>➤ <b>Make sense</b> of problems and persevere in solving them.</li> <li>➤ <b>Reason</b> abstractly and quantitatively.</li> <li>➤ <b>Construct</b> viable arguments</li> </ul>
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## Unit Outline – Grade 7 Math

	<ul style="list-style-type: none"><li>➤ <b>Model</b> with mathematics.</li><li>➤ <b>Critique</b> the reasoning of others.</li><li>➤ <b>Attend</b> to precision.</li></ul> <p>Together, these skills and practices should enable a student to achieve the objective of the unit.</p>
<p><b>VOCABULARY/KEY TERMS:</b></p> <ul style="list-style-type: none"><li>➤ Linear equations, rate, complex fraction, unit rate, per, proportion, proportional relationships, scale, ratio, composed unit, constant rate of change, multiplicative relationship, coordinate plane, origin</li></ul>	
<p><b>ASSESSMENT EVIDENCE AND ACTIVITIES:</b></p> <p>Please note: Each <i>Arc of Lessons</i> and the student tasks included therein can be used in sequence for individual and group exploration and discussion; and for ongoing formative assessment. The student tasks were designed to be cognitively demanding and to provide options in representation, action/expression, and engagement. See <i>The Mathematical Task Analysis Guide</i> and <i>Universal Design for Learning Principles</i> in this packet.</p> <p><b>FORMATIVE ASSESSMENT 1:</b></p> <p><i>Arc of Lessons 1</i> includes 4 student tasks as the basis for lessons. See the sample lesson guide “<i>Bicycle Shop</i>” for ideas on how to implement these tasks in ways that deepen student understanding while building procedural competence.</p>	
<p><b>FORMATIVE ASSESSMENT 2:</b></p> <p><i>Arc of Lessons 2</i> includes 5 student tasks as the basis for lessons. See the sample lesson guides “<i>Ounces of Coffee</i>” and “<i>Mixing Juices</i>” for ideas on how to implement these tasks in ways that deepen student understanding while building procedural competence.</p>	
<p><b>FORMATIVE ASSESSMENT 3:</b></p> <p><i>Arc of Lessons 3</i> includes 3 student tasks as the basis for lessons. See the sample lesson guide “<i>Melinda and Akira’s Walk</i>” and for ideas on how to implement these tasks in ways that deepen student understanding while building procedural competence.</p>	
<p><b>FINAL PERFORMANCE TASK:</b></p> <p>At the end of the unit the teacher should give the class the final assessment tasks to see</p>	

## Unit Outline – Grade 7 Math

how students have improved their thinking and mathematical skills over the course of the instructional unit. The final assessment contains 5 tasks. *Please see the assessment tasks for full details.*

### LEARNING PLAN & ACTIVITIES:

**In addition to the Arcs of Lessons and Lesson Guides included in this packet, please consider the following.**

*The problems listed below are examples that can be used for reinforcement of procedures after students participate in the problem-solving lessons.*

- Students will determine the best deal between two supermarkets. They will look over two weekly flyers and compare two items that are listed to determine the best deal.
- Students will use proportional reasoning to find the cost for an item they would like to purchase given the current exchange rate between US dollars and the currency of another country, the cost in one currency, and a calculator.
- Students will determine the rate at which each partner can stuff and address envelopes. (This is an important job in many organizations, including political campaigns.) One partner follows the procedure for stuffing an envelope. The other partner states when 5 minutes have passed and records the number of completed envelopes. Then partners trade roles and repeat the activity. Finally, partners compare their hourly rates for stuffing and addressing envelopes. They are then asked to consider how many envelopes could be completed in a set of number of hours for each partner individually and then if they worked together.
- **To Graph or not to Graph**  
Students will answer the question: "How can representing mathematical ideas graphically help me solve problems and check that my answers make sense?" in a journal entry.

Students must explain that graphing is a way to visually represent ratios and proportional relationships. This visual is a tool that mathematicians can use to check the logic of their equations and conclusions. The student should give a real life example for the context of their answer.

### RESOURCES:

#### *Suggested Resources for Teachers*

- Braunfeld, Peter, Carter, Ricky, Foster, Sydney, Lewis, Phil, Lucas, Joan, Manes, Michelle,

## Unit Outline – Grade 7 Math

Orrell, Cynthia, Palma, Melanie, & Watt, Daniel Lynn (2004). *Impact Mathematics*. McGraw Hill. New York, NY.

- Cai, J. & Sun, W. (2002). *Developing Students Proportional Reasoning*. Reston, VA; National Council of Teachers of Mathematics.
- Lobato, J. and Ellis, A. (2010). *Developing Essential Understanding of Ratios, Proportions & Proportional Reasoning: Grade 6-8*. National Council of Teachers of Mathematics. Reston, VA.
- Schielack, J., series advisor (2010). *Focus in Grade 7: Teaching with Curriculum Focus Points*. National Council of Teachers of Mathematics. Reston, VA.
- Schuster, L. and Anderson, N. (2005). *Good Questions for Math Teaching: Why Ask Them and What to Ask Grades 5-8*. Math Solutions Publications. Sausalito, CA.
- Van De Walle, J. (2007). *Elementary and Middle School Mathematics Teaching Developmentally*. Pearson Education Incorporated. New York, NY.
- Online resource for finding rate of change in the coordinate plane: see <http://www.slidermath.com/rpoly/Coord2.shtml>

### **Materials for problem-solving**

markers, paper, graph paper, rulers, adhesive tape, supermarket flyers, current exchange rates, cost of popular video games, hourly rates for stuffing and addressing envelopes.

- 7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.
- 7.RP.2 Recognize and represent proportional relationships between quantities.
- (a) Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
  - (b) Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
  - (c) Represent proportional relationships by equations.
  - (d) Explain what a point  $(x, y)$  on the graph of a proportional relationship means in terms of the situation.
- 7.RP.3 Use proportional relationships to solve multistep ratio and percent problems.

Essential Understandings of Ratios, Proportions & Proportional Reasoning (NCTM)

1. Reasoning with ratios involves attending to and coordinating two quantities.
2. A ratio is a multiplicative comparison of two quantities, or it is a joining of two quantities in a composed unit.
3. Forming a ratio as a measure of a real-world attribute involves isolating that attribute from other attributes and understanding the effect of changing each quantity on the attribute of interest.
4. A number of mathematical connections link ratios and fractions:
  - a. Ratios are often expressed in fraction notation, although ratios and fractions do not have identical meaning.
  - b. Ratios are often used to make “part-part” comparisons, but fractions are not.
  - c. Ratios and fractions can be thought of as overlapping sets.
  - d. Ratios can often be meaningfully reinterpreted as fractions.
5. Ratios can be meaningfully reinterpreted as quotients.
6. A proportion is a relationship of equality between two ratios. In a proportion, the ratio of two quantities remains constant as the corresponding values of the quantities change.
7. Proportional reasoning is complex and involves understanding that:
  - a. Equivalent ratios can be created by iterating and/or partitioning a composed unit;
  - b. If one quantity in a ratio is multiplied or divided by a particular factor, then the other quantity must be multiplied or divided by the same factor to maintain the proportional relationship; and
  - c. The two types of ratios – composed units and multiplicative comparisons – are related.
8. A rate is a set of infinitely many equivalent ratios.
9. Several ways of reasoning, all grounded in sense making, can be generalized into algorithms for solving proportion problems.
10. Superficial cues present in the context of a problem do not provide sufficient evidence of proportional relationships between quantities.



Flowers  
7 RP.2a, b, c  
maybe  
EU #1, 2, 3, 4,  
6, 8


The table below shows the price for buying bunches of mix-and-match flowers at a local supermarket.

Number of Bunches	3	6	9	12	15
Price	9	18	27	36	45

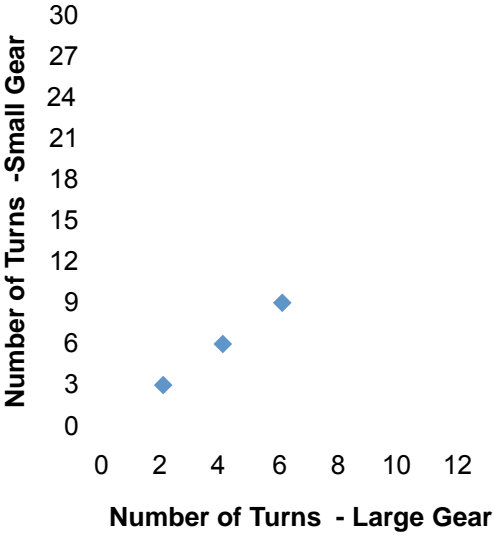
1. Generate several ratios of price to number of bunches. Describe any patterns you see.
2. Predict the price for buying 100 bunches. Explain how you made your prediction.
3. Predict how many bunches you can buy with \$63. Explain how you made your decisions.

*(Use to define proportional relationships)*

Gears  
7 RP.1 maybe  
7 RP.2a, b, d  
EU #1, 2, 3, 4,  
6, 8



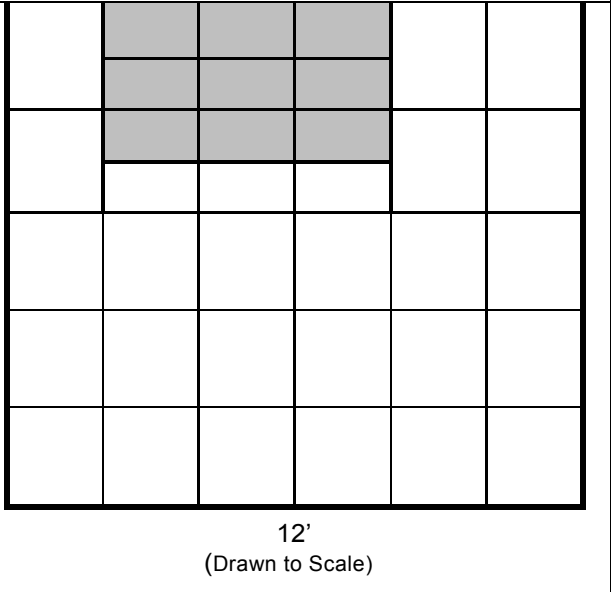
Aaron collected data on the number of times a pair of interlocking gears like those shown here revolved each minute. The graph below shows his data.



Number of Turns - Large Gear	Number of Turns - Small Gear
2	3
4	6
6	9

1. Generate several ratios comparing the number of times the large gear turned compared to number of times the small gear turned. Describe any patterns you see.
2. Write the inverse of any ratio from question 1 and explain what it tells you.
3. Add three points to the graph that the gears will generate and explain how you know they belong on the graph.

*(Use to test for proportional relationships)*

<p>Bicycle Shop 7 RP.2, 3</p> <p>EU #1, 2, 3, 4, 6, 8, 9</p>	<p>Two bicycle shops build custom-made bicycles. Bicycle City charges \$160 plus \$80 for each additional day that it takes to build the bicycle. Bike Town charges \$120 for each day that it takes to build the bicycle. For what number of days will the charge be the same at each store?</p> <p><i>(Use to distinguish proportional relationships from those that are not.)</i></p>	
<p>Bedroom 7 RP.1 7 RP.2 b 7 RP.3</p> <p>EU #1, 2, 3, 5, 6, 7b</p>	<p>The picture to the right represents a 10' by 12' rectangular bedroom.</p> <p>a. A rectangle representing Shantia's bed is located at the top of the picture. Determine the actual size of her bed and explain how you made your decisions.</p> <p>b. The rectangle representing the bedroom measures 3 inches in width and 2 ½ inches in height. What is the actual length of the line segments representing Shantia's bed?</p> <p>c. Draw a rectangular dresser in Shantia's room. Describe how to determine the length of the line segments representing the dresser and the actual length and width of the dresser.</p>	

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  - (c) Represent proportional relationships by equations.
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  - c. The two types of ratios – composed units and multiplicative comparisons – are related.
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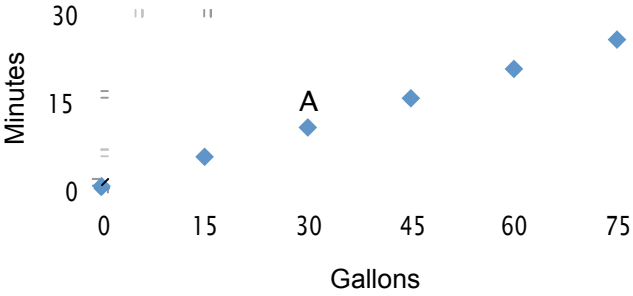
<p>Weight on the Moon          7 RP.1          7 RP.2 c          7 RP.3          EU #1, 2, 3, 4, 8</p>	<p>Physics tells us that weights of objects on the moon are proportional to their weights on Earth.          Suppose a 180-pound man weighs 30 pounds on the moon. What will a 60-pound boy weigh on the moon?</p>								
<p>Ounces of Coffee          7 RP.1          7 RP.2 a, b          EU #1, 2, 3, 4, 8, 9</p>	<p>Julia made observations about the selling price of a new coffee that sold in three different-sized bags. She recorded those observations in the following table:</p> <table border="1" data-bbox="654 823 1195 936"> <tr> <td>Ounces of Coffee</td> <td>6</td> <td>8</td> <td>16</td> </tr> <tr> <td>Price in Dollars</td> <td>2.40</td> <td>3.20</td> <td>6.40</td> </tr> </table> <p>a) Is there a proportional relationship between the amount of coffee and the price? Why or why not?          b) Find the unit rates associated with this problem.          c) Explain in writing what the unit rates mean in the context of this problem.          d) Explain in writing why it is helpful for Julia to determine if the relationship between the amount of coffee and the price is proportional before she buys a bag of the new coffee.</p>	Ounces of Coffee	6	8	16	Price in Dollars	2.40	3.20	6.40
Ounces of Coffee	6	8	16						
Price in Dollars	2.40	3.20	6.40						

<p>Mixing Juice</p> <p>7 RP.1 7 RP.3</p> <p>EU #1, 2, 3, 4, 6, 7, 8</p>	<p>Julia and Mariah attend summer camp. Everyone at the camp helps with the cooking and cleanup at meal times.</p> <p>One morning, Julia and Mariah make orange juice for all the campers. They plan to make the juice by mixing water and frozen orange juice concentrate. To find the mix that tastes best, they decide to test some mixes.</p> <div style="display: flex; justify-content: space-around; margin: 10px 0;"> <div style="border: 1px solid #ccc; background-color: #ffffcc; padding: 5px; width: 150px;"> <p style="text-align: center; margin: 0;"><b>Mix A</b></p> <hr style="border: 0; border-top: 1px solid #ccc; margin: 2px 0;"/> <p style="text-align: center; margin: 0;">2 cups      3 cups concentrate    cold water</p> <hr style="border: 0; border-top: 1px solid #ccc; margin: 2px 0;"/> </div> <div style="border: 1px solid #ccc; background-color: #ffffcc; padding: 5px; width: 150px;"> <p style="text-align: center; margin: 0;"><b>Mix B</b></p> <hr style="border: 0; border-top: 1px solid #ccc; margin: 2px 0;"/> <p style="text-align: center; margin: 0;">5 cups      9 cups concentrate    cold water</p> <hr style="border: 0; border-top: 1px solid #ccc; margin: 2px 0;"/> </div> </div> <div style="display: flex; justify-content: space-around; margin: 10px 0;"> <div style="border: 1px solid #ccc; background-color: #ffffcc; padding: 5px; width: 150px;"> <p style="text-align: center; margin: 0;"><b>Mix C</b></p> <hr style="border: 0; border-top: 1px solid #ccc; margin: 2px 0;"/> <p style="text-align: center; margin: 0;">1 cup      2 cups concentrate    cold water</p> <hr style="border: 0; border-top: 1px solid #ccc; margin: 2px 0;"/> </div> <div style="border: 1px solid #ccc; background-color: #ffffcc; padding: 5px; width: 150px;"> <p style="text-align: center; margin: 0;"><b>Mix D</b></p> <hr style="border: 0; border-top: 1px solid #ccc; margin: 2px 0;"/> <p style="text-align: center; margin: 0;">3 cups      5 cups concentrate    cold water</p> <hr style="border: 0; border-top: 1px solid #ccc; margin: 2px 0;"/> </div> </div> <p><b>Developing Comparison Strategies</b></p> <p>A. Which mix will make juice with the strongest orange taste? Explain.</p> <p>B. Which mix will make juice with the strongest orange taste? Explain.</p> <p>C. Which comparison statement is correct? Explain.</p> <div style="display: flex; justify-content: space-around; margin: 10px 0;"> <p>a. <math>\frac{5}{9}</math> of Mix B is concentrate</p> <p>b. <math>\frac{5}{14}</math> of Mix B is concentrate</p> </div> <p>D. Assume that each camper will get <math>\frac{1}{2}</math> cup of juice.</p> <ol style="list-style-type: none"> <li>1. For each mix, how many batches are needed to make juice for 240 campers?</li> <li>2. For each mix, how much concentrate and how much water are needed to make juice for 240 campers?</li> </ol>
<p>Investing Money</p> <p>7 RP.2 c 7 RP.3</p> <p>EU #5, 6, 7, 9</p>	<p>Ray and Crystal buy and sell bicycle parts in their neighborhood. Because they invested money in this small business in a ratio of 2:3, they will split the profit in a ratio of 2:3. If the profit from the business is \$1000, how much money will each person receive? Show how to use a model to solve the problem.</p>
<p>Light Bulbs</p> <p>7 RP.1 7 RP.2 c 7 RP.3</p> <p>EU #5, 6, 7, 9</p>	<p>Alazar Electric Company sells light bulbs to major outlets like Home Depot, Sears, Walmart and other big chains. They sample their bulbs for defects routinely.</p> <ol style="list-style-type: none"> <li>a. A sample of 96 light bulbs included 4 defective ones. Assume that today's batch of 6,000 light bulbs has the same proportion of defective bulbs as the sample. Determine the total number of defective bulbs made today.</li> <li>b. The big businesses Alazar Electric Company sells to accept no larger than a 4% rate of defective bulbs. Does today's batch meet that expectation? Explain how you made your decisions.</li> </ol>

- 7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.
- 7.RP.2 Recognize and represent proportional relationships between quantities.
- (a) Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
  - (b) Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
  - (c) Represent proportional relationships by equations.
  - (d) Explain what a point  $(x, y)$  on the graph of a proportional relationship means in terms of the situation.
- 7 RP.3 Use proportional relationships to solve multistep ratio and percent problems.

Essential Understandings of Ratios, Proportions & Proportional Reasoning (NCTM)

1. Reasoning with ratios involves attending to and coordinating two quantities.
2. A ratio is a multiplicative comparison of two quantities, or it is a joining of two quantities in a composed unit.
3. Forming a ratio as a measure of a real-world attribute involves isolating that attribute from other attributes and understanding the effect of changing each quantity on the attribute of interest.
4. A number of mathematical connections link ratios and fractions:
  - a. Ratios are often expressed in fraction notation, although ratios and fractions do not have identical meaning.
  - b. Ratios are often used to make “part-part” comparisons, but fractions are not.
  - c. Ratios and fractions can be thought of as overlapping sets.
  - d. Ratios can often be meaningfully reinterpreted as fractions.
5. Ratios can be meaningfully reinterpreted as quotients.
6. A proportion is a relationship of equality between two ratios. In a proportion, the ratio of two quantities remains constant as the corresponding values of the quantities change.
7. Proportional reasoning is complex and involves understanding that:
  - a. Equivalent ratios can be created by iterating and/or partitioning a composed unit;
  - b. If one quantity in a ratio is multiplied or divided by a particular factor, then the other quantity must be multiplied or divided by the same factor to maintain the proportional relationship; and
  - c. The two types of ratios – composed units and multiplicative comparisons – are related.
8. A rate is a set of infinitely many equivalent ratios.
9. Several ways of reasoning, all grounded in sense making, can be generalized into algorithms for solving proportion problems.
10. Superficial cues present in the context of a problem do not provide sufficient evidence of proportional relationships between quantities.

<p>Trading Cards 7 RP.2 7 RP.3 EU #1, 2, 3, 4, 6, 7, 8</p>	<p>If the cost of trading cards is two packs for \$6, how much will it cost to buy 10 packages? <i>Teaching with Curriculum Focal Points, Focus in Grade 7, NCTM.</i> See book for lesson suggestions.</p>
<p>Melinda and Akira's Walk 7 RP.1 7 RP.2 c 7 RP.3 EU #1, 2, 3, 4, 6, 7, 8, 9</p>	<p>Melinda and her sister Akira are walking around the track at school. Melinda and Akira walk at a steady rate and Melinda walks 5 feet in the same time that Akira walks 2 feet.</p> <p>a) Set up a table and draw a graph to represent this situation. Let the x-axis represent the number of feet that Melinda walks and the y-axis represent the number of feet that Akira walks.</p> <p>b) When Melinda walks 45 feet, how far will Akira walk? Explain in writing or show how you found your answer.</p>
<p>Taking a Shower 7 RP.1 7 RP.2 a c d 7 RP.3 EU #1, 2, 3, 4, 5, 6, 7, 8, 9</p>	<p>The graph below shows the amount of time a person can shower with a certain amount of water.</p>  <p>a) Does the graph represent a proportional relationship? Why or why not?</p> <p>b) How long can a person shower with 15 gallons of water? With 60 gallons of water?</p> <p>c) What information does point A on the graph tell us about the amount of time a person can shower with a certain amount of water?</p> <p>d) Explain how to find a unit rate associated with this graph.</p> <p>e) Write the equation of the line relating the time in minutes, <math>m</math>, a person can shower with a given amount of water, <math>g</math>.</p>

## BICYCLE SHOP SEVENTH GRADE LESSON GUIDE

### LESSON OVERVIEW:

The Bicycle Shop task asks students to identify the constant of proportionality and identify graphically, in a table or algebraically the solution to a system of linear relationships.

Note: Expand -- what is the purpose of the task – i.e. what mathematical ideas will students grapple with via engaging in the task?

### COMMON CORE STATE STANDARDS:

- **7.RP.2** Recognize and represent proportional relationships between quantities.
  - a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
  - b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
  - c. Represent proportional relationships by equations. *For example, if total cost  $t$  is proportional to the number  $n$  of items purchased at a constant price  $p$ , the relationship between the total cost and the number of items can be expressed as  $t = pn$ .*
    1. Explain what a point  $(x, y)$  on the graph of a proportional relationship means in terms of the situation, with special attention to the points  $(0, 0)$  and  $(1, r)$  where  $r$  is the unit rate.
- **7.RP.3** Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

### NCTM ESSENTIAL UNDERSTANDINGS<sup>1</sup>:

1. Reasoning with ratios involves attending to and coordinating two quantities.
2. A ratio is a multiplicative comparison of two quantities, or it is a joining of two quantities in a composed unit.
3. Forming a ratio as a measure of a real-world attribute involves isolating that attribute from other attributes and understanding the effect of changing each quantity on the attribute of interest.
4. A number of mathematical connections link ratios and fractions:
  - a. Ratios are often expressed in fraction notation, although ratios and fractions do not have identical meaning.
  - b. Ratios are often used to make “part-part” comparisons, but fractions are not.
  - c. Ratios and fractions can be thought of as overlapping sets.
  - d. Ratios can often be meaningfully reinterpreted as fractions.
6. A proportion is a relationship of equality between two ratios. In a proportion, the ratio of two quantities remains constant as the corresponding values of the quantities change.
8. A rate is a set of infinitely many equivalent ratios.

<sup>1</sup> NCTM (2010) Developing Essential Understandings of Ratios, Proportions & Proportional Reasoning: Grades 6 -8.



<p><b>DRIVING QUESTIONS:</b></p> <ul style="list-style-type: none"> <li>• How can we decide if two quantities are in a proportional relationship using a context, table, graph and equation?</li> <li>• How can we find the solution to a system of linear equations using a table or a graph?</li> <li>• What does the solution to a system of linear equation mean in the context of a problem?</li> </ul>	<p><b>SKILLS DEVELOPED:</b></p> <p><i>Students will be able to:</i></p> <ul style="list-style-type: none"> <li>• <i>determine whether or not two quantities are in a proportional relationship using a variety of representations</i></li> <li>• <i>identify the constant of proportionality in a table, context, graph and equation.</i></li> <li>• <i>find the solution to a system of linear equations using a table or a graph.</i></li> <li>• <i>Interpret the meaning of the solution to a system of linear equations within the context of a problem.</i></li> </ul>
<p><b>MATERIALS:</b> “Bicycle Shop” Task, Document Projector or Chart Paper</p>	<p><b>GROUPING:</b> Students will begin their work individually, but will then work in pairs or triads.</p>

<p><b>SET-UP</b></p>
<p><b>Instructions to Students:</b></p> <p>Using either a document reader or overhead projector present the task to the class. Have one student read the question that follow the graph and tabular representations.</p> <p>Ask the students: “What do you know?” “What is the question asking you?”</p> <p>Inform the students that there are several ways to get the answers to the questions asked. First each individual must work alone for at least 5 minutes after which they will share their initial findings with their group. Then they will continue to work out a common solution.</p> <p>Expectations that all students must adhere to: explain their thinking and reasoning, use correct mathematical language and symbols in their explanations or solutions, justify their solutions, make sense of other students’ explanations; seek help from the teacher or students when they do not understand.</p>
<p><b>EXPLORE PHASE: Supporting Students’ Exploration of the Mathematical Ideas</b></p>
<p><b>Private Think Time:</b> Allow students to work individually for 3-5 minutes without intervening, though you might want to circulate quickly to get an idea of the strategies that they are using.</p> <p><b>Small-Group Work:</b> After 3-5 minutes, ask students to work with their partner or in their small groups. As students are working, circulate around the room. Be persistent in:</p> <ul style="list-style-type: none"> <li>• asking questions related to the mathematical ideas, problem-solving strategies, and connections between representations.</li> <li>• asking students to explain their thinking and reasoning.</li> <li>• asking students to explain in their own words, and build onto, what other students have said.</li> </ul> <p>As you circulate, identify solution paths that you will have groups share during the Share, Discuss, Analyze Phase, and decide on the sequence that you would like for them to be shared. Give groups a “heads up” that you will be asking them to come to the front of the room. If a document reader is not</p>

available, give selected groups an OVH transparency or chart paper to write their solution on.

**Possible Solution Paths**

1. Making a table

Number of Days	Bike City	Bike Town
0	160	0
1	240	120
2	320	240
3	400	360
4	480	480
5	560	600
6	640	720

**Possible Assessing and Advancing Questions**

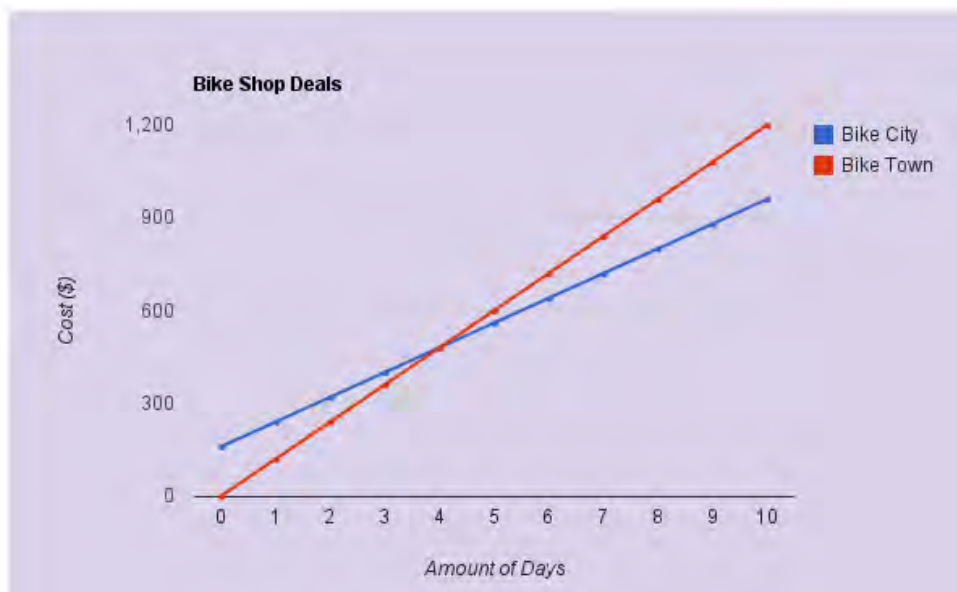
**Assessing Questions**

- What do the numbers represent in your table?
- How did you determine the numbers in your table?
- How does the table help you to solve the problem?

**Advancing Questions**

- Will there be another day at which two stores will charge the same amount? How do you know?
- Is there a proportional relationship between the number of days and the charge for either of the bike stores? How do you know?
- How do we see the daily rate for each of the bike shops in the table?

2. Drawing a graph



**Assessing Questions**

- What do the two lines represent in your graph?
- How does the graph help you to solve the problem?

**Advancing Questions**

- What do the points (0, 0) and (0, 160) mean the context of the problem?
- Is there a proportional relationship between the number of days and the charge for either of the bike stores? How do you know?
- What does the point (4,480) mean in the context of the problem?
- How do we see the daily rate for each of the bike shops in the graph?
- Will there be another day at which two stores will charge the same amount? How do you know?

3. Algebraic Solution ( IF NO GROUPS ATTEMPT AN ALGEBRAIC SOLUTION, IT IS NOT NECESSARY TO PRESS FOR IT AT THIS TIME)

Bike City:  $80x + 160 = y$   
 Bike Town:  $120x = y$

$$\begin{array}{r} 80x + 160 = 120x \\ -80x \quad \quad -80x \\ \hline 160 = 40x \end{array} \quad \text{Step 1: Subtract } 80x \text{ from both sides of the equal sign}$$

$$\frac{160}{40} = \frac{40x}{40} \quad \text{Step 2: Divide both sides by } 40$$

$4 = x$       *It will cost the same at Day 4.*

**NOTE:** The algebraic solution will not be discussed during the SDA phase since this was not a standard identified for this task. Explain to the students that you will just ask them to share, and explain, their equations.

Return to the algebraic solution for this task when you move to solving systems of linear equations.

**Assessing Questions**

- What do the two equations mean in the context of the problem? What does  $x$  represent?  $y$ ? What do the 80, 160, and 120 represent?
- Why did you make the equations equal?
- What does the solution  $4 = x$  mean in the context of the problem?

**Advancing Questions**

- Do either of these equations represent a proportional relationship? How do you know?
- How can we find the rate of change in the equations?
- Are either of the rates of change also a constant of proportionality? How do you know?

**Possible Errors and Misconceptions**

Graphing Errors:

- Inconsistent intervals
- plotting coordinates incorrectly

Mistaking Bike City as proportional since it has a constant rate of change.

Thinking that any rate of change is also a constant of proportionality

Incorrect equations

**Possible Questions to Address Errors and Misconceptions**

**Assessing Questions**

- What have you done so far?

**Advancing Questions**

- What elements do graphs need to have in place in order to be accurate?
- Why does it help to plot this data on the same graph?
- Is there another way you can show the relationship between the  $x$  and  $y$  values for Bike City? Bike Town?
- Can you describe some similarities and differences between the two graphs?
- What's the rate of change (constant of proportionality)? What is value of  $y$  when  $x$  equals 0?
- We've looked at graphs that are proportional. How is Bike City's graph different from these? (Have graphs ready that show proportionality)

**SHARE DISCUSS ANALYZE PHASE: Deepening and Connecting Mathematical Understanding**

**General Considerations:**

- Orchestrate the class discussion so that it builds on, extends, and connects the thinking and reasoning of students
- Sequence the solution paths so that you will be able to press students to make comparisons and connections across, and between, the various solution paths.

**Possible Sequence of Solution Paths**

**Possible Questions and Possible Student Responses**

1. Start with a solution using a table

**Explain your group's solution.**

- *We started with one day for Bicycle City and found out how much it would cost for that day. Then we decided to continue adding up to 6 days. After that we did the same thing for Bike Town only stopping when Bike Town. We found that on day 4 they both had the same charge.*

**Are either of these relationships proportional? If so, how can you tell that by looking at the table? At the context?**

- *The charge for Bike Town is a proportional relationship because on day one it doesn't cost anything.*
- *You can also tell that bike town is proportional because when you double the number of days you double the charge. That doesn't happen for Bike City.*
- *Bike City isn't proportional because you start out with a \$160 charge that gets added on.*

2. Have students share their graphical solution

**What does your graph represent?**

- *We thought that it would be an easy solution and we would be able to see when the two bike companies have the same day and charge.*

**How can you tell when the two bike shops charge the same charge by looking at the graph?**

- *When the two lines cross that's when the two shops charge the same amount for the same number of days.*

**How can you tell if either of the relationships is proportional by looking at the graph?**

- *I can tell that Bike Town's charge is a proportional relationship because the graph starts at (0,0).*

**From the graph can you tell if the two companies will ever have the same day and charge again?**

- *The lines won't ever meet again. The lines are going at different slants because Bike Town charges more for each day than Bike City.*

<p>3. Have students share their equations</p> <p><b>NOTE:</b> For the purpose of this lesson, focus ONLY on the equations themselves during the SDA phase, not the algebraic solution of systems of linear equations.</p>	<p><b>Explain how you arrived at the two equations.</b></p> <ul style="list-style-type: none"> <li>• <i>Since Bike City starts with a \$160 charge and then adds \$80 for each day, I came up with the equation <math>80x + 160 = y</math>. <math>x</math> is the number of days it takes to build the bike. Bike Town just charges \$120 per day so their equation is <math>120x = y</math>.</i></li> </ul> <p><b>How can we tell which relationship is proportional by looking at the equations?</b></p> <ul style="list-style-type: none"> <li>• <i>We can tell that Bike Town is a proportional relationship because there's nothing added. The charge will always be 120 times the number of days.</i></li> </ul> <p><b>So what is the constant of proportionality for Bike Town?</b></p> <ul style="list-style-type: none"> <li>• <i>The constant of proportionality is 120. The charge will always be 120 times the number of days.</i></li> </ul> <p><b>Bike City charges \$80 per day. Why isn't that also a constant of proportionality?</b></p> <ul style="list-style-type: none"> <li>• <i>You can't just multiply the number of days by 80 to find Bike City's charge. You also have to add \$120. A constant of proportionality is always a multiple.</i></li> </ul>
<p>4. Look across the different representations</p>	<p><b>We have seen three representations – tables, graphs, and equations. How do they each help us to determine if a relationship is proportional?</b></p> <ul style="list-style-type: none"> <li>• <i>We can see in all of them that Bike Town charges \$0 for 0 days. We can see that in the first row of the table. We see that on the graph because the line goes through (0,0). We see that in the equation because when you multiply a number by zero you end up with zero, and you're not adding anything else.</i></li> <li>• <i>We can also see that the charge for Bike Town is always \$120 times the number of days. In the table you can multiply to check. In the equation it's <math>120x</math>. It's a little harder in the graph, but you see the line always goes up the same amount. It doesn't curve.</i></li> <li>• <i>You don't see any of these for Bike City.</i></li> </ul>

<p><b>CLOSURE</b></p>
<p><b>Quickwrite:</b> How can you tell if a relationship is proportional by looking at the context, table, graph and equation?</p>
<p><b>Possible Assessment:</b></p> <ul style="list-style-type: none"> <li>• .Look at different graphs and identify which are proportional and which are not with explanations.</li> </ul>
<p><b>Homework:</b></p> <ul style="list-style-type: none"> <li>• .Similar problem with different numbers</li> </ul>

## Bicycle Shop

Two bicycle shops build custom-made bicycles. Bicycle City charges \$160 plus \$80 for each day that it takes to build the bicycle. Bike Town charges \$120 for each day that it takes to build the bicycle.

For what number of days will the charge be the same at each shop?

## MIXING JUICE TASK LESSON GUIDE

### LESSON OVERVIEW

In the *Mixing Juice* task, students encounter an open-ended problem where they are asked to compare the “orangeyness” of four drink mixes. Students will likely approach the task using a range of different strategies, making comparisons among the mixes with ratios, percents, and fractions. Students will investigate how ratios can be formed and scaled up to find equivalent ratios. In addition, students will use proportional reasoning to decide how to use the different mixes to make juice for 240 people.

The strategies for comparing the mixes will be compared and connected during the whole-group discussion. Students should be able to see how each form, ratios, percents, and fraction, provides information needed to derive one of the other forms.

Before working on the *Mixing Juice* task, the Warm-Up task *Comparing by Using Ratios* will focus students’ attention on different ways to form ratios and different notations for ratios. Allow 15 to 20 minutes for students to engage in and discuss the Warm-Up task. The *Mixing Juice* task is a challenging problem for students and at least 1.5 class periods should be allowed for students to explore and have a whole-class discussion on the task.

### COMMON CORE STATE STANDARDS

- **6.RP.1** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.
- **6.RP.2** Understand the concept of a unit rate  $a/b$  associated with a ratio  $a:b$  with  $b \neq 0$ , and use rate language in the context of a ratio relationship.
- **6.RP.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
  - c) Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity).
- **7.RP.1** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units.
- **7.RP.3** Use proportional relationships to solve multistep ratio and percent problems. *Example: percent increase.*

**ESSENTIAL QUESTIONS:**

- What are different types of ratios and how are ratios used to make comparisons?
- What strategies can be used to compare ratios?
- How are ratios related to fractions?

**NCTM ESSENTIAL UNDERSTANDINGS<sup>2</sup>:**

- Reasoning with ratios involves attending to and coordinating two quantities.
- A ratio is a multiplicative comparison of two quantities, or it is a joining of two quantities in a composed unit.
- Forming a ratio as a measure of a real-world attribute involves isolating that attribute from other attributes and understanding the effect of changing each quantity on the attribute of interest.
- A number of mathematical connections link ratios and fractions:
  - Ratios are often expressed in fraction notation, although ratios and fractions do not have identical meaning.
  - Ratios are often used to make “part-part” comparisons, but fractions are not.
  - Ratios can often be meaningfully reinterpreted as fractions.
- Ratios can be meaningfully reinterpreted as quotients.
- Proportional reasoning is complex and involves understanding that:
  - Equivalent ratios can be created by iterating and/or partitioning a composed unit;
  - If one quantity in a ratio is multiplied or divided by a particular factor, then the other quantity must be multiplied or divided by the same factor to maintain the proportional relationship.

**SKILLS DEVELOPED:**

- Use different representations to form ratios and make comparisons with ratios.
- Use visual and numerical strategies to compare ratios.
- Form equivalent ratios and use equivalent ratios to solve problems.

**MATERIALS:**

Warm-Up task  
*Comparing by Using Ratios* and *Mixing Juice*  
 task sheet, Calculators,  
 Chart paper.

**GROUPING:**

Students will begin their work individually, but will then work in groups of three or four to discuss the task and arrive at a common solution.

<sup>2</sup> NCTM (2010) *Developing Essential Understandings of Ratios, Proportions & Proportional Reasoning: Grades 6 -8.*



## Warm-Up Task (*Comparing by Using Ratios*)

### SET-UP

Tell students that a useful way to compare numbers is to form ratios. With students working in groups of three, give them the *Comparing by Using Ratios* task sheet and ask them to take turns reading the ratio statements to each other. Tell them they should form and interpret the ratios and think about different ways ratios can be written. They should also look for similarities and differences in the ratio statements.

### EXPLORE PHASE

#### Monitoring Student Work:

Circulate while students are reading and interpreting the ratios. Focus students' attention on the ratio comparisons. Listen to and make note of students' debating and deciding the types of ratios in Statements A-G. Tell students you will want them to share their reasons during the whole-class discussion.

#### Possible Solution Paths

Each ratio is a part-to-part ratio, a part-to-whole ratio, or a ratio comparing different kinds of measures or counts (also called a rate). Statement D compares a part to a whole. Statements C and F compare two different kinds of measures; this type of ratio is called a *rate*. The remaining statements compare parts to parts. Note that statement E can be interpreted as part-to-part or part-to-whole. Make note of students who are arguing either interpretation and highlight this during the Share Discuss Analyze Phase of the Warm Up.

Ratios are often written in the form 5:6 or 5 to 6 to help students separate the ideas of ratios from fraction arithmetic.

- *How are these statements similar?*
- *How are they different?*

### SHARE DISCUSS ANALYZE PHASE

#### General Considerations:

Start by prompting students to focus on ratio statements that they think are similar, and ask them to explain why. You might start by asking them, "How are statements A and B similar?" Once they have identified part-part, you might then ask them if D is also part-part, thus distinguishing part-whole and part-part. You might then have them decide whether G fits one of these classes. Next, ask whether C and F fit into either of these groups. Name the groups as they are formed. You could then have a discussion of E – saying that you don't know which it is. Through this discussion you should separate the statements into the three types, and introduce the terminology – part-part, part-whole, and rate. If you allow 7-10 min for small-group discussion, that will leave only 10-13 minutes for the Share, Discuss, Analyze Phase.

## SET-UP (Mixing Juice Task)

Make sure students understand the context.

Suggested Questions:

- *How many of you have made juice by adding water to a mix before?*
- *What was involved in making it?*

You may want to bring in a can of frozen orange juice (thawed) and, with your class, make juice following the instructions on the can. You can discuss the fact that you have one container of concentrated juice and to this you add three containers of water (or whatever it says on the container of concentrate). Point out that the recipes given in the problem are different from the one on the can. At camp, the juice concentrate comes in a very large container without mixing proportions given.

You might let students begin to explore the juice recipes C in small-group, and then reassemble in whole-group to discuss the groups' initial ideas about the different mixes. This approach gives groups a chance to consider several representations and comparison strategies. You might discuss parts A, B and C in whole-group and then challenge groups to solve part D. Make sure that students have solved D for at least two mixes before calling them together to discuss it C in whole-group.

Remind students that they will be expected to: justify their solutions; explain their thinking and reasoning to others; make sense of other students' explanations; ask questions of the teacher or other students when they do not understand; and use correct mathematical language, and symbols.

## EXPLORE PHASE: Supporting Students' Exploration of the Mathematical Ideas

**Private Think Time:** Allow students to work individually for 3 – 5 minutes without intervening, though you might want to circulate quickly to get an idea of the strategies that they are using.

**Small-Group Work:** After 3 – 5 minutes, ask students to work with their partner or in their small groups. As students are working, circulate around the room. Be persistent in:

- asking questions related to the mathematical ideas, problem-solving strategies, and connections between representations.
- asking students to explain their thinking and reasoning.
- asking students to explain in their own words, and build onto, what other students have said.

As you circulate, identify solution paths that you will have groups share during the Share, Discuss, Analyze Phase, and decide on the sequence that you would like for them to be shared. Give groups a “heads up” that you will be asking them to put their solutions on chart paper to share during the whole-class discussion. Having the various strategies on chart paper will allow you to arrange the work in the room in a way that supports analyzing and making connections between and among them.

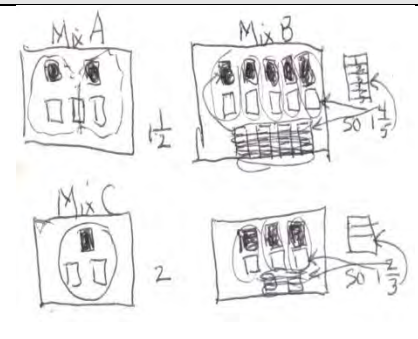
Some students will start with naïve strategies such as simply finding the difference between the number of cans of concentrate and ignore the water. Challenge this idea by asking: *Can I keep adding cans of water without making the juice less orangey?*

Questions A and B will allow misconceptions (additive strategies) as well as alternate multiplicative approaches for comparing ratios to emerge.

Question C is designed to raise the issue that the phrase “of Mix B” signals that this is a part-whole statement, thus  $\frac{5}{9}$  is not correct. Though students will discuss it during the Explore Phase, you will focus on questions A, B and D as you circulate during the Explore Phase.

**Possible Solution Paths for Parts A and B**

**Possible Assessing and Advancing Questions**



Draw pictures to show how much water there is for each cup of concentrate in each mix. The goal is to partition the water squares so that each cup of concentrate gets the same amount of water. In this way, you can see that Mix C has the most water for each cup of concentrate (least orangey) and Mix A has the least amount of water (most orangey).

*Assessing Questions*

- Tell me about your diagram. What does it show?
- How does this help to decide which mix is most or least orangey?
- What kind of ratios does this visual represent?

*Advancing Questions*

- If you wanted to write numerical ratios to represent what you have in this visual strategy, what would they look like? How would they help you decide which is most or least orangey?

Mix C	Mix B	Mix D	Mix A
$\frac{1}{3}$	$\frac{5}{14}$	$\frac{3}{8}$	$\frac{2}{5}$

Use part-to-whole ratios written in fraction form to express the relationships of concentrate to total liquid in a batch. Using prior knowledge about fractions, students may represent the fractions as decimals or percents. There are a variety of strategies that can then be used to order the fractions, i.e., benchmark comparisons or common denominators.

*Assessing Questions*

- Tell me about your work.
- Tell me, for example, what  $\frac{3}{8}$  means in the context of the problem. What does the 3 represent? The 8? Is there any way you can let me know that in your explanation?

*Advancing Questions (if no comparisons are being made)*

- How are you going to compare the mixes? Which is most orangey and how do you know?
- What are some ways that you have used in the past to make comparisons? See if you can think about some of these and then try to make comparisons, in one or two ways.

*Advancing Questions (if comparisons are being made)*

- How does knowing that  $\frac{1}{3}$  is the ratio of concentrate to mix help you to answer Question D?

A.  $\frac{1\frac{1}{2}}{1}$ ;  $\frac{1\frac{4}{5}}{1}$ ;  $\frac{2}{1}$ ;  $\frac{1\frac{2}{3}}{1}$

B.  $\frac{2}{1}$ ;  $\frac{5}{1}$ ;  $\frac{1}{1}$ ;  $\frac{3}{1}$

A. Figure out how much water goes with each cup of concentrate. Notice that with these ratios we focus on most and least water.

B. Figure out how much concentrate goes with each cup of water. Notice that with these ratios we focus on most and least concentrate.


*Assessing Questions*

- Tell me about your work. What did you do and why?
- What kind of ratios have you created? What do they represent? Is there any way you can let me know that in your explanation?

*Advancing Questions*

- How are you using your ratios to decide which is most orangey?
- How are you going to compare the mixes? Which is most orangey and how do you know?
- What strategy are you using to order your ratios? What does the order mean in the context of the problem?

Mix A $\frac{2}{30} : \frac{3}{45}$ Mix B $\frac{5}{30} : \frac{9}{54}$	Use part-to-part ratios and make the number of cups of concentrate the same. Notice that with these ratios we focus on most and least water.	<i>Assessing Questions</i> <ul style="list-style-type: none"> <li>• <i>Tell me about your work.</i></li> <li>• <i>What kind of ratios have you created? What do they represent? Is there any way you can let me know that in your explanation?</i></li> <li>• <i>How did you decide that 30 cups of concentrate would be a helpful amount?</i></li> <li>• <i>How did you make the new ratios? What strategy did you use?</i></li> <li>• <i>How does making the amount of concentrate the same help you to reason about the problem?</i></li> </ul>
Mix C $\frac{1}{30} : \frac{2}{60}$ Mix D $\frac{3}{30} : \frac{5}{50}$		

Possible Errors and Misconceptions for Parts A and B	Possible Questions to Address Errors and Misconceptions
<p>Comparing the amount of water using absolute differences approach:</p>  <p>A. The juice that will taste most orangey is Mix C because it does not have as much water as mixes A, B, and D.</p> <p>B. The juice that will taste least orangey is Mix B because it has more water than mixes A, C, and D.</p>	<p>Assessing Question</p> <ul style="list-style-type: none"> <li>• <i>Tell me about your work. Explain your thinking.</i></li> </ul> <p>Advancing Questions</p> <ul style="list-style-type: none"> <li>• <i>What would 2 batches of Mix C look like? How would the “orangeyness” of this new one compare to Mix A? Why?</i></li> <li>• <i>For every one cup of water in Mix A how many cups of concentrate would I have?</i></li> </ul>
Possible Solution Paths for Part D	Possible Assessing and Advancing Questions
<p>Find the number of batches needed to make 120 cups of juice from each recipe. Then multiply to find the amount of water and concentrate. For example, one batch of Mix A makes 5 cups of juice and since 120 cups are needed, <math>120 \div 5</math> yields 24 batches. So, <math>2 \times 24</math> or 48 cups of concentrate and <math>3 \times 24</math> or 72 cups of cold water are needed. 48 cups of concentrate plus 72 cups of water yields 120 cups of juice for the 240 campers.</p>	<p>Assessing Questions</p> <ul style="list-style-type: none"> <li>• <i>Why did you decide to work with 120 cups of juice?</i></li> <li>• <i>How did you use 120 to help you solve the problem?</i></li> <li>• <i>What does the 24 mean in the context of the problem and why did you multiply the 2 and 3 by it?</i></li> </ul> <p>Advancing Questions</p> <ul style="list-style-type: none"> <li>• <i>How can you check to see that the amounts you calculated are correct?</i></li> <li>• <i>So the ratio of cups of juice in the big batch to cups of juice in the recipe is 120:5 (pointing to <math>120 \div 5</math> on the paper). What is the ratio of cups of concentrate in the big batch to cups of concentrate in the recipe? What about the ratio of cups of water in the big batch to cups of water in the recipe? Do you think that will happen with the other mixes, too?</i></li> <li>• <i>Write those ratios in equation form and study the way they look. Is there a way to decide by looking at the statement that two ratios are equal</i></li> </ul>

Make a rate or ratio table to scale up:

**Mix D** (for example)

Concentrate in cups	Water in cups	Total in cups
3	5	8
6	10	16
9	15	24
12	20	32
15	25	40
45	75	120

Students may continue to add 3:5:8 to each row until they reach 45:75:120, or they may notice that they can multiply a row by a scale factor to get to their result more quickly as is shown in the last 2 rows of this table.

Assessing Questions

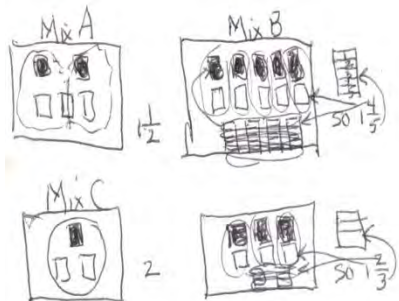
- *Why did you decide to organize your work in a table? How is that helpful?*
- *How are you getting from one row to the next in your table?*
- *How did you know when to stop making new rows?*
- *What patterns do you see in the table?*

Advancing Questions

- *Can you show what you did in the last two rows so we don't have to guess? Use either an numerical expression or a written explanation. Then say WHY you knew you could do what you did.*

Possible Errors and Misconceptions for Part D	Possible Questions to Address Errors and Misconceptions																		
<p>If students are stuck on the question of making a recipe for 240 people, ask them to consider Mix A to start.</p>	<ul style="list-style-type: none"> <li>• <i>How much total juice does one batch of Mix A make? How can we figure out how many people one batch of this juice will serve?</i></li> <li>• <i>What if each serving is one cup? What if each serving is ½ cup?</i></li> <li>• <i>If you were going to serve juice to 50 people, how many cups of juice would you have to make if each person gets ½ cup of juice? How many batches of juice would this be?</i></li> <li>• <i>What are different strategies you might use to answer this question?</i> (Students might divide 50 people by 10 servings per batch to determine that 5 batches are needed. Alternatively, some students may reason that if 1 batch makes 10 servings, then 2 batches make 20 servings, 3 batches makes 30 servings, etc. Students may make a table from which to reason.</li> </ul> <table border="1" data-bbox="1304 448 1913 532"> <thead> <tr> <th colspan="6">Servings of Juice</th> </tr> <tr> <th>Number of Batches</th> <th>1</th> <th>2</th> <th>3</th> <th>4</th> <th>5</th> </tr> </thead> <tbody> <tr> <td>Servings</td> <td>10</td> <td>20</td> <td>30</td> <td>40</td> <td>50</td> </tr> </tbody> </table>	Servings of Juice						Number of Batches	1	2	3	4	5	Servings	10	20	30	40	50
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<p><b>SHARE DISCUSS ANALYZE PHASE: Deepening and Connecting Mathematical Understanding</b></p>																			
<p><b>General Considerations:</b></p> <ul style="list-style-type: none"> <li>• Orchestrate the class discussion so that it builds on, extends, and connects the thinking and reasoning of students.</li> <li>• Sequence the solution paths so that you will be able to press students to make comparisons and connections across, and between, the various solution paths.</li> </ul> <p>It is recommended that you have groups share and discuss Parts A and B, then discuss C and D. For Parts A and B, the point you want students to think about is: <i>What does it mean to be most orangey tasting? To be least orangey tasting? Why do we have to consider BOTH the amount of concentrate AND the amount of water?</i></p>																			
Possible Sequence of Solution Paths	Possible Questions and <i>Possible Student Responses</i>																		
<p>Begin with the visual strategy under Possible Misconceptions and Errors for Parts A and B.</p>	<p>While the reasoning that was used to decide which mix is most orangey is faulty, the visual can be used to make connections to some of the other strategies. You might decide to have the group share their drawing and come back to them after they have heard some of the other strategies described to see if they will reconsider their reasoning and their answer about which is most orangey.</p>																		
<p>Have a group who used a part-to-whole ratio share their strategy. If there are a variety of strategies that were used to reason about the part-to-whole ratios, start with reasoning from the ratio itself, then move to those that converted the ratio to a decimal, then move to percents, and finally connect to the common denominator approach.</p>	<p><b>How did your group decide to compare the mixes?</b></p> <ul style="list-style-type: none"> <li>• <i>We made our comparisons using part-to-whole ratios. We compared the part of the mix that is concentrate to the whole mix. Then we made these fractions into decimals and looked for the largest because that would tell us which mix was most orangey.</i></li> </ul> <p><b>How is your strategy related to the diagrams we saw in the first strategy?</b></p> <ul style="list-style-type: none"> <li>• <i>The numerator in our fraction is the number of pieces that are shaded and these represent the concentrate. The denominator in our fraction is the total number of pieces in the diagram for a particular mix.</i></li> </ul> <p><b>How do your decimals relate to the diagrams?</b></p> <ul style="list-style-type: none"> <li>• <i>The decimal (percent) would represent the portion of the whole diagram that is shaded. In this case, the whole mix would have a value of 1 or 100%.</i></li> </ul>																		

Use a visual that shows how much water there is for each cup of concentrate in each mix. This is a unit rate approach.



**Tell us what your drawing means.**

- The solid squares represent cups of concentrate and the empty squares represent cups of water. We divided the water squares so that each cup of concentrate gets the same amount of water.

**How does your drawing help you decide which mix is most orangey?**

- We can see that Mix A has the least water,  $1 \frac{1}{2}$  cups, for each cup of concentrate, so it is the most orangey.

**How does your drawing help you decide which mix is least orangey?**

- We can see that Mix C has the most water, 2 cups, for each cup of concentrate, so it is the least orangey.

**How does your strategy compare to the ones we saw earlier?**

- We were using part-to-part comparisons and they were using part-to-whole comparisons.
- For them to say which mix was most orangey they had to look for the mix that had the most concentrate. For us to say which mix was most orangey, we looked for the mix that has the least water.

Have a group that used a unit rate approach with unit ratios expressed numerically share next.

**Tell us about the ratios your group made. How did you calculate them?**

- We compared cups of water to cups of concentrate. We wanted to figure out how much water goes with each cup of concentrate, so we divided the number of cups of water by the number of cups of concentrate.

**How does this compare to the last groups' strategy?**

- They divided up the area of the squares and we used ratios, unit rates, but they both convey similar information. Like for Mix B, our ratio was  $\frac{5}{4}$  which is water to concentrate and that's what their picture shows.

Have a group that used part-to-part ratios and made the number of cups of concentrate (or water) the same.

*(This strategy is helpful to scaffold student thinking for Part D.)*

**Tell us about your approach. What kind of ratios did you use to compare the mixes?**

- We used part-to-part ratios. We thought if we had 4 big pots and used each pot to make many batches of each mix we could compare them. We wanted each pot to have the same amount of concentrate so we could think about how much water is in each pot.

**How will the multiple batches of juice in the pots compare to the original mix?**

- The juice in each pot will taste the same as the original mix because we kept the ratio of concentrate to water the same.

**How does your strategy compare to the other strategies we have seen?**

- Our strategy is probably most like the part-to-whole ratios where they found common denominators. Even though we used part-to-part ratios to reason, their strategy was similar because, like us, they ended up with a lot more juice than was in the original mix.



## CLOSURE

### Quickwrite:

- Why is a ratio a useful way to make comparisons?
- Which of the following will taste the most orangey? 2 cups of concentrate and 3 cups of water; 4 cups of concentrate and 6 cups of water; or 10 cups of concentrate and 15 cups of water? Explain your reasoning.

### Possible Assessment:

- Provide some additional contexts where they need to compare quantities. Ask them to explain their thinking in writing.

### Homework:

- Find items from the current curriculum that will allow them to apply these ideas and understandings.

## References

Lappan, Fey, Fitzgerald, Friel, Phillips (2009). Teacher's Guide: Connected Mathematics 2. *Comparing and Scaling: Ratio, Proportion, and Percent*, Pearson.

## Comparing by Using Ratios

A useful way to compare numbers is to form *ratios*. Talk to your classmates about what is the same and what is different about these ratio statements.

- A. In taste tests, people who preferred Bolda Cola outnumbered those who preferred Cola Nola by a ratio of 3 to 2.
- B. The ratio of boys to girls in our class is 12 boys to 15 girls.
- C. For every four tents there are 12 scouts.
- D. The ratio of boys to students in our class is 12 boys to 27 students.
- E. The ratio of kittens to cats in our neighborhood is  $\frac{1}{4}$ .
- F. The sign in the hotel lobby says:  
1 dollar Canadian : 0.85 dollars U.S.
- G. A paint mixture calls for 5 parts blue paint to 2 parts yellow paint.

## MIXING JUICE

Julia and Mariah attend summer camp. Everyone at the camp helps with the cooking and cleanup at meal times.

One morning, Julia and Mariah make orange juice for all the campers. They plan to make the juice by mixing water and frozen orange juice concentrate. To find the mix that tastes best, they decide to test some mixes.

Mix A		Mix B	
2 cups	3 cups	5 cups	9 cups
concentrate	cold water	concentrate	cold water

Mix C		Mix D	
1 cup	2 cups	3 cups	5 cups
concentrate	cold water	concentrate	cold water

### Developing Comparison Strategies

- A. Which mix will make juice that is the most "orangey"? Explain.
- B. Which mix will make juice that is the least "orangey"? Explain.
- C. Which comparison statement is correct? Explain.

a.  $\frac{5}{9}$  of Mix B is concentrate

$\frac{5}{14}$  of Mix B is concentrate

- D. Assume that each camper will get  $\frac{1}{2}$  cup of juice.
- For each mix, how many batches are needed to make juice for 240 campers?
  - For each mix, how much concentrate and how much water are needed to make juice for 240 campers?

## OUNCES OF COFFEE TASK SEVENTH GRADE LESSON GUIDE

### LESSON OVERVIEW:

Students will be presented with the Ounces of Coffee problem. They will be asked to determine whether or not there is a proportional relationship between the ounces of coffee to the price. Students then will be asked to find the unit price and explain in writing what the unit price means in the context of the problem. Finally, students will explain why it is helpful to determine if the relationship between the amount of coffee and price is proportional.

Students should be able to see the proportional relationship between the ounces of coffee and the price. They should be able to find the unit price and to see that the cost per ounce is the same.

Before working on the Ounces of Coffee task, students will do a Warm-Up task identifying ratios and the appropriate representations of unit rates. Students will be allowed 15-20 minutes to engage in and discuss the Warm-Up task. At least 2-3 periods will be allotted for students to explore and share their work.

### COMMON CORE STATE STANDARDS:

- **6.RP.1** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.
- **6.RP.2** Understand the concept of a unit rate  $a/b$  associated with a ratio  $a:b$  with  $b \neq 0$ , and use rate language in the context of a ratio relationship.
- **6.RP.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
  - a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
  - b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*
  - c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
  - d. Use ratio reasoning to convert measurement units to manipulate and transform units appropriately when multiplying or dividing quantities.
- **7.RP.1** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks  $1/2$  mile in each  $1/4$  hour, compute the unit rate as the complex fraction  $1/2/1/4$  miles per hour, equivalently 2 miles per hour.*
- **7.RP.2** Recognize and represent proportional relationships between quantities.
  - d. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
  - e. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

f. Represent proportional relationships by equations. *For example, if total cost  $t$  is proportional to the number  $n$  of items purchased at a constant price  $p$ , the relationship between the total cost and the number of items can be expressed as  $t = pn$ .*

2. Explain what a point  $(x, y)$  on the graph of a proportional relationship means in terms of the situation, with special attention to the points  $(0, 0)$  and  $(1, r)$  where  $r$  is the unit rate.

- **7.RP.3** Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*
- **8.EE.5** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.
- **8.EE.6** Use similar triangles to explain why the slope  $m$  is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation  $y = mx$  for a line through the origin and the equation  $y = mx + b$  for a line intercepting the vertical axis at  $b$ .

**DRIVING QUESTION:**

- What are different types of ratios?
- How can ratios be used to make comparisons?
- How are ratios related to fractions?

**NCTM ESSENTIAL UNDERSTANDINGS<sup>3</sup>:**

1. Reasoning with ratios involves attending to and coordinating two quantities
2. A ratio is a multiplicative comparison of two quantities, or it is a joining of two quantities in a composed unit.
3. Forming a ratio as a measure of a real world attribute involves isolating that attribute from other attributes and understanding the effect of changing each quantity on the attribute of interest.
4. Ratios can be meaningfully reinterpreted as quotients.
5. Proportional reasoning is complex and involves understanding that:
  - Equivalent ratios can be created by iterating and/or partitioning a composed unit;
  - If one quantity in a ratio is multiplied or divided by a particular factor, then the other quantity must be multiplied or divided by the same factor to maintain the proportional relationship.

**SKILLS DEVELOPED:**

- Use different representations to form ratios and make comparisons with ratios.
- Form equivalent ratios and use equivalent ratios to solve problems.
- Find the Unit Rate.

**MATERIALS:**

Warm up task, Ounces of Coffee sheet, calculators, Smartboard

**GROUPING:**

Students will work alone and then in groups of three to four.

<sup>3</sup> NCTM (2010) Developing Essential Understandings of Ratios, Proportions & Proportional Reasoning: Grades 6 -8.

SET-UP													
<p>Instructions to Students: Students will discuss their understanding of ratio, unit rate, rate and proportion. A student will be told to read the problem while others follow along silently. Explain to students that they will have to justify their solutions and explain their reasoning. Students will be told to work alone for 7 to 10 minutes and then in small groups.</p>													
EXPLORE PHASE: Supporting Students' Exploration of the Mathematical Ideas													
<p><b>Private Think Time:</b> Allow students to work individually for 3-5 minutes without intervening, though you might want to circulate quickly to get an idea of the strategies that they are using.</p> <p><b>Small-Group Work:</b> After 3-5 minutes, ask students to work with their partner or in their small groups. As students are working, circulate around the room. Be persistent in:</p> <ul style="list-style-type: none"> <li>• asking questions related to the mathematical ideas, problem-solving strategies, and connections between representations.</li> <li>• asking students to explain their thinking and reasoning.</li> <li>• asking students to explain in their own words, and build onto, what other students have said.</li> </ul> <p>As you circulate, identify solution paths that you will have groups share during the Share, Discuss, Analyze Phase, and decide on the sequence that you would like for them to be shared. Give groups a “heads up” that you will be asking them to come to the front of the room. If a document reader is not available, give selected groups an OVH transparency or chart paper to write their solution on.</p>													
Possible Solution Paths	Possible Assessing and Advancing Questions												
<p>If a group is unable to start: Focus students on the table.</p> <p>5. <i>What does the task ask us to figure out?</i></p> <p>6. <i>What is being compared?</i></p> <p>7. <i>What are the items on the table being compared?</i></p> <p><i>We are comparing ounces of coffee to price in dollars.</i></p>	<p>Assessing</p> <ul style="list-style-type: none"> <li>• What are we trying to figure out in this problem?</li> <li>• What can you tell me about the ounces of coffee and the price in dollars?</li> </ul>												
<table border="1" style="width: 100%; text-align: center;"> <tr> <td style="padding: 5px;">OZ.</td> <td style="padding: 5px;">OZ.</td> <td style="padding: 5px;">OZ.</td> <td style="padding: 5px;">OZ.</td> <td style="padding: 5px;">OZ.</td> <td style="padding: 5px;">OZ.</td> </tr> <tr> <td style="padding: 5px;">.40</td> <td style="padding: 5px;">.40</td> <td style="padding: 5px;">.40</td> <td style="padding: 5px;">.40</td> <td style="padding: 5px;">.40</td> <td style="padding: 5px;">.40</td> </tr> </table>	OZ.	OZ.	OZ.	OZ.	OZ.	OZ.	.40	.40	.40	.40	.40	.40	<p>Assessing Questions</p> <ul style="list-style-type: none"> <li>• Tell about your work.</li> <li>• How did you figure out that each ounce would cost 40¢?</li> <li>• How did you know that each ounce of coffee would cost forty cents?</li> </ul> <p>Advancing Questions</p> <ul style="list-style-type: none"> <li>• Is there another way to show the relationship between the amount of coffee and the price?</li> </ul>
OZ.	OZ.	OZ.	OZ.	OZ.	OZ.								
.40	.40	.40	.40	.40	.40								

$\begin{array}{r} \underline{0.40} \\ 6 \overline{)2.40} \\ \underline{2.40} \end{array}$ $\begin{array}{r} \underline{0.40} \\ 8 \overline{)3.20} \\ \underline{3.20} \end{array}$ $\begin{array}{r} \underline{0.40} \\ 16 \overline{)6.40} \end{array}$	<p>Assessing Questions</p> <ul style="list-style-type: none"> <li>• Tell us about your work.</li> <li>• Why are you dividing the price by the ounces?</li> <li>• What does the .40 tell us? What do we call it?</li> <li>• How do you know that dividing the ounces by the dollars will give you the unit rate?</li> </ul> <p>Advancing Questions (Not all of these would be asked at the same time.)</p> <ul style="list-style-type: none"> <li>• How does knowing the unit price benefit you?</li> <li>• How can you represent the quantities from the table in a ratio? Can you represent the ratio in fraction notation?</li> <li>• How can you compare the ratios? Are they the same or equivalent?</li> <li>• Is there a proportional relationship?</li> </ul>														
<table border="1" data-bbox="178 738 420 982"> <tr><td>1</td><td>.40</td></tr> <tr><td>2</td><td>.80</td></tr> <tr><td>3</td><td>1.20</td></tr> <tr><td>4</td><td>1.60</td></tr> <tr><td>5</td><td>2.00</td></tr> <tr><td>6</td><td>2.40</td></tr> <tr><td>8</td><td>3.20</td></tr> </table>	1	.40	2	.80	3	1.20	4	1.60	5	2.00	6	2.40	8	3.20	<p>Assessing Questions</p> <ul style="list-style-type: none"> <li>• Tell me about your table and what you noticed. Do you see a pattern?</li> <li>• What patterns do you see in your table?</li> </ul> <p>Advancing Questions</p> <ul style="list-style-type: none"> <li>• If students have not noticed a pattern then, tell me about the pattern in the table?</li> <li>• What does the pattern tell you?</li> <li>• So if you have 32 ounces of coffee, how much will that cost?</li> <li>• Is there a proportional relationship between the ounces of coffee and the cost? Why or why not?</li> <li>• Will this method for solving for proportion always work? How do you know?</li> <li>• Can you think of a proportion that this method of solving will not work with?</li> </ul>
1	.40														
2	.80														
3	1.20														
4	1.60														
5	2.00														
6	2.40														
8	3.20														

Possible Errors and Misconceptions	Possible Questions to Address Errors and Misconceptions																
<p>Students may ignore the decimal point in the \$2.40, \$3.20 and \$6.40.</p>	<p>Assessing Questions</p> <ul style="list-style-type: none"> <li>• What is being compared?</li> <li>• Where did you get the 240, 320, and 640 from? Or devils advocate: Wow is that 240 dollars?</li> <li>• How is it the same as the quantities in the table?</li> </ul> <p>Advancing Questions</p> <ul style="list-style-type: none"> <li>• What are the quantities being compared in this problem?</li> <li>• How are you going to use ratios to help you see if there is a proportional relationship?</li> </ul>																
<p><b>SHARE DISCUSS ANALYZE PHASE: Deepening and Connecting Mathematical Understanding</b></p>																	
<p><b>General Considerations:</b></p> <ul style="list-style-type: none"> <li>• Orchestrate the class discussion so that it builds on, extends, and connects the thinking and reasoning of students</li> <li>• Sequence the solution paths so that you will be able to press students to make comparisons and connections across, and between, the various solution paths.</li> </ul>																	
Possible Sequence of Solution Paths	Possible Questions and <i>Possible Student Responses</i>																
<p><i>A focus on Pattern Finding and Describing the Proportional Relationship</i></p> <p><i>ORTIZ</i></p> <table border="1" data-bbox="178 1084 415 1367"> <tbody> <tr><td>1</td><td>.40</td></tr> <tr><td>2</td><td>.80</td></tr> <tr><td>3</td><td>1.20</td></tr> <tr><td>4</td><td>1.60</td></tr> <tr><td>5</td><td>2.00</td></tr> <tr><td>6</td><td>2.40</td></tr> <tr><td>8</td><td>3.20</td></tr> <tr><td>16</td><td>6.40</td></tr> </tbody> </table> <p>I looked at the possible ratio as a fraction and simplified it to its lowest terms. I found that the ounces of coffee to price in dollars were a ratio of 1 to .40 for each given quantity.</p>	1	.40	2	.80	3	1.20	4	1.60	5	2.00	6	2.40	8	3.20	16	6.40	<ul style="list-style-type: none"> <li>• Tell us about your work.</li> <li>• Do you see a pattern?</li> <li>• What patterns do you see in your table?</li> <li>• What made you create this table?</li> <li>• What does the pattern tell you?</li> <li>• Is there a proportional relationship here? What is it?</li> <li>• What was the method that this group used to figure out if there was a proportional relationship? Will this method for solving for proportion always work? How do you know?</li> <li>• So, if you have 32 ounces of coffee, how much will that cost?</li> <li>• Can you think of a proportion that this method of solving will not work with?</li> </ul>
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2	.80																
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8	3.20																
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- A) There is a proportional relationship between ounces of coffee and price in dollars because when you divide ounces of coffee to the price in dollars, it gives you 1/.40 or 1 divided by .40 and this is for each quantity given. For example,  $6/2.40 = 1/.40$  and  $8/3.20 = 1/.40$  and  $16/6.40 = 1/.40$  so the relationship is proportional.
- B) 1/.40 is the unit rate for this problem. When I simplified 6 over \$2.40; 8 over \$3.20 and 16 over \$6.40, they all resulted in 1 over .40.
- C) For every ounce of coffee the price is 40¢.
- D) The reason why Julia has to find out if it is proportional is the lowest price per ounce would be the best bag to buy. If it is proportional then Julia will know that it is okay to purchase any bag because the coffee price will always remain the same per ounce of coffee. If not, Julia will need to find the lowest price per ounce of coffee.

Miraj A Focus on using ratios to see if the different price and amount are proportional.

Since we have three different sized bags and three different prices, I will use ratios to compare the amount of coffee expressed in ounces to the price in dollars. Based on the table given, I will write the following ratios:

\$2.40 : 6 or      \$2.40 to 6 or      \$2.40/6  
 \$3.20 : 8 or      \$3.20 to 8 or      \$3.20/8  
 \$6.40 :16 or      \$6.40 to 16 or      \$6.40/16

I am trying to see if there is any proportional relationship between the amount of coffee and the price. To do that, I had to compare to see if the ratios I created are the same.

To see if the ratios are proportional, I cross multiplied and I found out that the first two ratios are in a proportional.

The evidence for that is the equation

$$\frac{\$2.40}{6} = \frac{\$3.20}{8} = \frac{\$6.40}{16}$$

Explain your group's solution.

- How did you find out whether or not the ratios were the same or proportional?
- How did you know that they were in a proportional relationship?
- If you had \$19.20 how many ounces of coffee can you buy?
- How can you represent this information in another way?

<p>B) The unit rate will be determined by one of the ratios since all of the ratios are the same I will take \$2.40 divided by 6 equals \$.40 or forty cents.</p> <p>C) The unit rate means the price for coffee for each ounce in each bag is the same, forty cents.</p> <p>D) It is important for Julia to know that the amount of coffee and the price of the coffee is proportional so she can calculate how much money she will need for the new bag of coffee.</p>	
<p>Boatright: A Focus on using quotient to get the price of</p> <p>Ounces/Price in Dollars      <math>6/2.40 \div 6/6 = 1/.40</math></p> <p>Ounces/Price in Dollars      <math>8/3.20 \div 8/8 = 1/.40</math></p> <p>Ounces/Price in Dollars      <math>16/6.40 \div 16/16 = 1/.40</math></p> <p>I looked at the possible ratio as fractions and simplified it to its lowest terms. I found that the ounces of coffee to price in dollars were a ratio of 1 to .40 for each given quantity.</p> <p>The unit rate is one ounce per \$ 0.40</p> <p>a. Yes, the proportion is equal to 1/.40, for each quantity given. Therefore, it is proportional.</p> <p>b. 1/.40, is the unit rate because when simplified the ratio is one to 40 hundredths.</p> <p>c. Ounce of coffee/price of coffee is 1/.40. This means that one-ounce cost 40 cents.</p> <p>d. It is helpful for Julia to find the unit price because this way she assures herself that each coffee package has the same cost per ounce.</p>	<p>Explain your group's solution.</p> <ul style="list-style-type: none"> <li>• What is a ratio?</li> <li>• What is a proportion?</li> <li>• Why are you simplifying?</li> <li>• What is a unit rate? What does it mean in this context?</li> <li>• Can a unit rate be simplified?</li> <li>• Can a unit rate be negative?</li> </ul>

## CLOSURE

**Quick Write:** Choose one of the questions below depending on your students' understanding.

- **WHEN SOMETHING IS CHANGING PROPORTIONALLY, WHAT INFORMATION CAN WE GET FROM THE RATIO TO DESCRIBE THE CHANGE?**
- **What does it mean if there is a proportional relationship? Refer to the coffee problem in your explanation. (I wonder if this will permit you to see if they refer to the ratio.)**

**Possible Assessment:**

- Continue to provide similar problems in which various solution paths can be used.

**Homework:**

- Jose and Russell jogging problem, weight on the moon problem and the light bulb problem.

## OUNCES OF COFFEE

Name \_\_\_\_\_ Date \_\_\_\_\_

Julia made observations about selling price of a new coffee that sold in three different sized bags. She recorded those observations in the following table:

<b>Ounces of Coffee</b>	<b>6</b>	<b>8</b>	<b>16</b>
<b>Price in Dollars</b>	<b>\$2.40</b>	<b>\$3.20</b>	<b>\$6.40</b>

**a)** Is there a proportional relationship between the amount of coffee and the price? Why or why not?

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**b)** Find the unit rates associated with the problem.

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**c)** Explain in writing what the unit rates mean in the context of this problem.

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**d)** Explain in writing why is it helpful for Julia to determine if the relationship between the amount of coffee and the price is proportional before she buys a new bag of coffee.

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## MELINDA AND AKIRA'S WALK SEVENTH GRADE LESSON GUIDE

### LESSON OVERVIEW:

Melinda and her sister Akira are walking around the track at school. Melinda and Akira walk at a steady rate and Melinda walks 5 feet in the same time that Akira walks 2 feet.

- a) Set up a table and draw a graph to represent this situation. Let the x-axis represent the number of feet that Melinda walks and the y-axis represent the number of feet that Akira walks.
- b) When Melinda walks 45 feet, how far will Akira walk? Explain in writing or show how you found your answer.

(Expand on this section in the future: explain the purpose of the lesson.)

### COMMON CORE STATE STANDARDS:

- **6.RP.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
  - e. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
  - f. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*
  - g. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
  - h. Use ratio reasoning to convert measurement units to manipulate and transform units appropriately when multiplying or dividing quantities.
- **7.RP.1** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks  $\frac{1}{2}$  mile in each  $\frac{1}{4}$  hour, compute the unit rate as the complex fraction  $\frac{1/2}{1/4}$  miles per hour, equivalently 2 miles per hour.*
- **7.RP.2** Recognize and represent proportional relationships between quantities.
  - g. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
  - h. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
  - i. Represent proportional relationships by equations. *For example, if total cost  $t$  is proportional to the number  $n$  of items purchased at a constant price  $p$ , the relationship between the total cost and the number of items can be expressed as  $t = pn$ .*
    3. Explain what a point  $(x, y)$  on the graph of a proportional relationship means in terms of the situation, with special attention to the points  $(0, 0)$  and  $(1, r)$  where  $r$  is the unit rate.
- **7.RP.3** Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

<b>DRIVING QUESTION:</b>	<p><b>NCTM ESSENTIAL UNDERSTANDINGS<sup>4</sup>:</b></p> <ol style="list-style-type: none"> <li>6. Reasoning with ratios involves attending to and coordinating two quantities.</li> <li>7. A ratio is a multiplicative comparison of two quantities, or it is a joining of two quantities in a composed unit.</li> <li>8. Forming a ratio as a measure of a real-world attribute involves isolating that attribute from other attributes and understanding the effect of changing each quantity on the attribute of interest.</li> <li>9. A number of mathematical connections link ratios and fractions: <ol style="list-style-type: none"> <li>a) Ratios are often expressed in fraction notation, although ratios and fractions do not have identical meaning.</li> <li>b) Ratios are often used to make “part-part” comparisons, but fractions are not.</li> <li>c) Ratios and fractions can be thought of as overlapping sets.</li> <li>d) Ratios can often be meaningfully reinterpreted as fractions.</li> </ol> </li> <li>10. Ratios can be meaningfully reinterpreted as quotients.</li> <li>11. A proportion is a relationship of equality between two ratios. In a proportion, the ratio of two quantities remains constant as the corresponding values of the quantities change.</li> <li>12. Proportional reasoning is complex and involves understanding that: <ol style="list-style-type: none"> <li>a) Equivalent ratios can be created by iterating and/or partitioning a composed unit;</li> <li>b) If one quantity in a ratio is multiplied or divided by a particular factor, then the other quantity must be multiplied or divided by the same factor to maintain the proportional relationship; and</li> <li>c) The two types of ratios – composed units and multiplicative comparisons – are related.</li> </ol> </li> <li>13. A rate is a set of infinitely many equivalent ratios.</li> <li>14. Several ways of reasoning, all grounded in sense making, can be generalized into algorithms for solving proportion problems.</li> </ol>	<b>SKILLS DEVELOPED:</b>
<b>MATERIALS:</b>	<b>GROUPING:</b>	

<sup>4</sup> NCTM (2010) Developing Essential Understandings of Ratios, Proportions & Proportional Reasoning: Grades 6 -8.

**SET-UP**

**Instructions to Students:**

**EXPLORE PHASE: Supporting Students' Exploration of the Mathematical Ideas**

**Private Think Time:** Allow students to work individually for 3-5 minutes without intervening, though you might want to circulate quickly to get an idea of the strategies that they are using.

**Small-Group Work:** After 3-5 minutes, ask students to work with their partner or in their small groups. As students are working, circulate around the room. Be persistent in:

- asking questions related to the mathematical ideas, problem-solving strategies, and connections between representations.
- asking students to explain their thinking and reasoning.
- asking students to explain in their own words, and build onto, what other students have said.

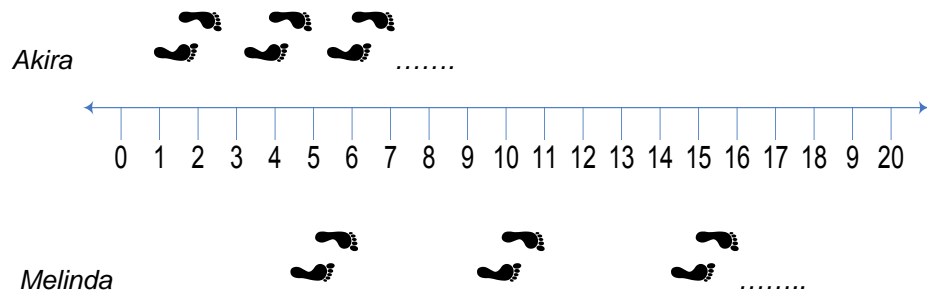
As you circulate, identify solution paths that you will have groups share during the Share, Discuss, Analyze Phase, and decide on the sequence that you would like for them to be shared. Give groups a “heads up” that you will be asking them to come to the front of the room. If a document reader is not available, give selected groups an OVH transparency or chart paper to write their solution on.

**Possible Solution Paths**

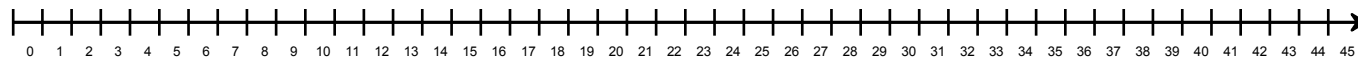
**Possible Assessing and Advancing Questions**

If a group is unable to start:

8. Make a number line picture or diagram of the problem:



*Below is a number line extended to 45:*



Assessing Questions

- Tell me what you are thinking.
- What is the task asking you to do?

Advancing Questions

- Are Melinda and Akira walking at the same rate? Can you make a diagram of the problem?
- Tell us what your diagram shows.

Create a Table

Melinda	0	5	10	15	20	25	30	35	40	45
Akira	0	2	4	6	8	10	12	14	16	18

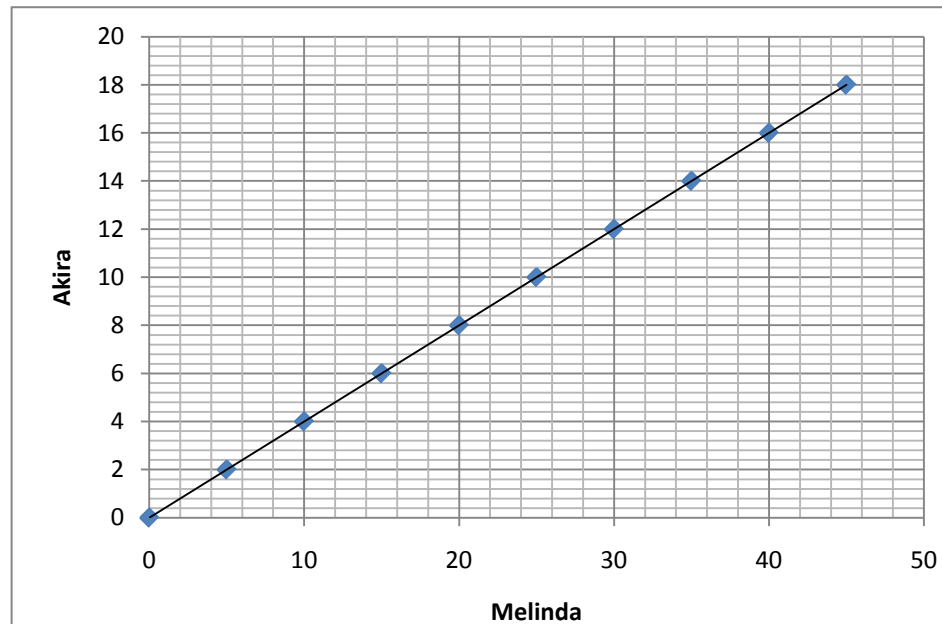
Assessing Question

- Tell me what the table tells you about Melinda and Akira's walk.

Advancing Questions

- What patterns do you see in the table? Explain the pattern.
- Can you describe a pattern in the table that uses multiplication?

9. Make a Graph  
Melinda and Akira's Walk



Assessing Question

- What does this point mean in this context? (*Pointing to the next to uppermost point on the graph.*)

Advancing Questions

- Does your graph represent a proportional relationship? Why or why not?
- Can you predict how far Akira will walk if Melinda walks 1000 feet?

10. Write and Solve a Proportion:

$$\frac{5}{2} = \frac{45}{x}$$

$$5x = 90$$

$$\frac{5x}{5} = \frac{90}{5}$$

$$x = 18$$

Assessing Question

- How do you know that the relationship between Melinda and Akira's pace is proportional? Explain.

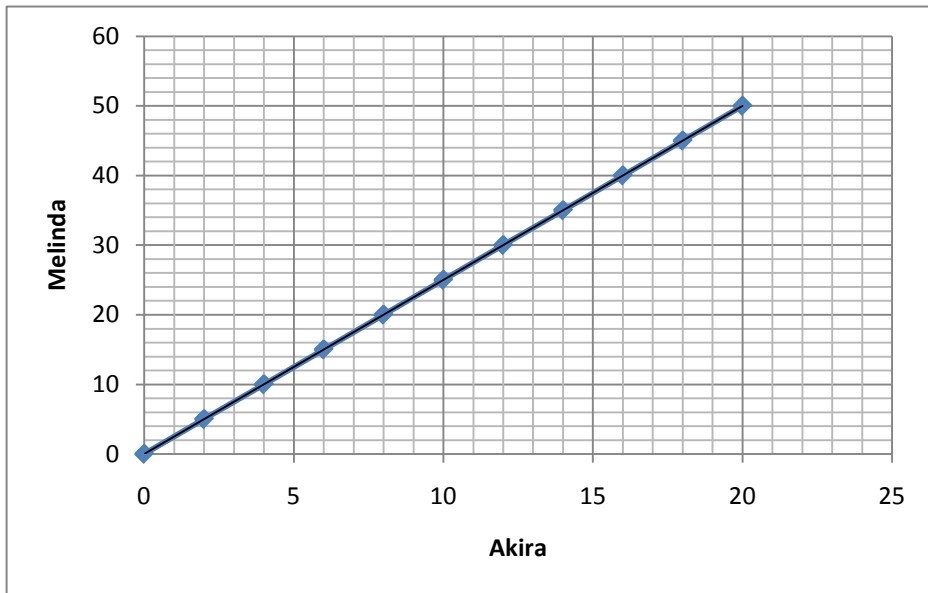
Advancing Question

- When Melinda walks 450 feet, how far will Akira walk?



**Possible Errors and Misconceptions**

1. Reversing the Axes on the graph (Melinda is supposed to be represented by the x-axis and Akira by the y-axis):



- a) focusing only on the difference in the paces – Melinda is always 3 feet ahead of Akira.

**Possible Questions to Address Errors and Misconceptions****Assessing Question**

- Can you explain what your graph means?

**SHARE DISCUSS ANALYZE PHASE: Deepening and Connecting Mathematical Understanding****General Considerations:**

- Orchestrate the class discussion so that it builds on, extends, and connects the thinking and reasoning of students.
- Sequence the solution paths so that you will be able to press students to make comparisons and connections across, and between, the various solution paths.

Possible Sequence of Solution Paths	Possible Questions and <i>Possible Student Responses</i>
<b>CLOSURE</b>	
<b>Quick Write:</b> <ul style="list-style-type: none"><li>•</li></ul>	
<b>Possible Assessment:</b> <ul style="list-style-type: none"><li>•</li></ul>	
<b>Homework:</b> <ul style="list-style-type: none"><li>•</li></ul>	



# The Mathematical Task Analysis Guide

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## Lower-Level Demands

### Memorization Tasks

- Involves either producing previously learned facts, rules, formulae, or definitions OR committing facts, rules, formulae, or definitions to memory.
- Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure.
- Are not ambiguous – such tasks involve exact reproduction of previously seen material and what is to be reproduced is clearly and directly stated.
- Have no connection to the concepts or meaning that underlie the facts, rules, formulae, or definitions being learned or reproduced.

### Procedures Without Connections Tasks

- Are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task.
- Require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it.
- Have no connection to the concepts or meaning that underlie the procedure being used.
- Are focused on producing correct answers rather than developing mathematical understanding.
- Require no explanations, or explanations that focus solely on describing the procedure that was used.

## Higher-Level Demands

### Procedures With Connections Tasks

- Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas.
- Suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts.
- Usually are represented in multiple ways (e.g., visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations helps to develop meaning.
- Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.

### Doing Mathematics Tasks

- Requires complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example).
- Requires students to explore and to understand the nature of mathematical concepts, processes, or relationships.
- Demands self-monitoring or self-regulation of one's own cognitive processes.
- Requires students to access relevant knowledge and experiences and make appropriate use of them in working through the task.
- Requires students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions.
- Requires considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process required.

Stein, Smith, Henningsen, & Silver (2000 and 2009). Implementing standards-based mathematics instruction: A casebook for professional development. New York: Teachers College Press.





GRADE 7 MATH:  
PROPORTIONAL REASONING  
SUPPORTS FOR ENGLISH  
LANGUAGE LEARNERS

# GRADE 7 MATH: PROPORTIONAL REASONING

## Supports for ELLS

**Title:** [Proportional Reasoning](#)

**Grade:** 7

### **Linguistic Access:**

#### A. Performance Task Section

In this summative assessment, a distinction between the vocabulary and the language functions is needed to provide entry points to the math content. Both need to be clarified to ensure comprehension of the tasks. This can be done by introducing the most essential vocabulary and language functions before beginning the assessment. The following vocabulary/language functions are suggested:

#### Vocabulary words:

Tier I (Nonacademic language): race (as in a competition), drove, cross-country, gas (gasoline), grid, actual (false cognate)

Tier II (General academic language): diagram, context, represent, “as noted below”

Tier III (Math technical language and concepts that the students must be familiar with prior to assessment): rate, unit rate, centimeters (cm), patterns, miles (mi), gallons, line segment, average, proportional relationships, equation, coordinate pairs, slope.

Language Functions: explain, identify, describe, compute, assume

#### B. Instructional Support Section

Because some of the words used in the Arcs problems might present difficulties for ELLs, especially for newcomers, an annotated definition of key words (with visual representation when possible) in the margin of the page will be useful. Such challenging words can include the following: custom-made, interlocking gears, bunches, outlets (business).

### **Content Access:**

For this performance assessment, a clear understanding of ratio and proportion is required.

In question 1a, the student is expected to state a unit rate for the situation represented by the graph. The connection between unit rate as used in this context and a ratio can be made explicit for ELLs.

In question 2, the concept of a scale map may be a barrier. Clarify what is meant by a scale in a map and how to apply it to find relative distances.

In question 3b, students are expected to relate the concept of a slope of a line to the graph. The relationship between the slope and speed may not be obvious to all students and some clarifications may

be necessary.

**Scaffolds:**

- On page 3 of the document, the performance task “Proportional Reasoning” is a summative assessment and no scaffolds are appropriate at this time.
- On page 42, Instructional Supports, there are a series of optional math problems (Arcs) that can be a resource for ELLs to develop a conceptual understanding of proportionality. The Frayer Model can be used to elicit from the student what they know about ratio and proportion prior to engaging in these tasks. Another strategy for scaffolding the concept of ratio and proportion can be the use of visuals, for example, the Turning Wheel problem (page 44).
- As with all the Arcs, it is recommended that teachers use think-alouds to verbalize their thinking as they solve the problems.
- Teachers should gather from the class and make available ideas about how to tackle these problems (make students’ thinking available).
- For the Arcs Tasks, the technique of reciprocal teaching can be very useful for ELLs because it involves four cognitive strategies: questioning, summarizing, clarifying, and predicting during the reading of the math text.



## GRADE 7 MATH: PROPORTIONAL REASONING

### SUPPORTS FOR STUDENTS WITH DISABILITIES



# GRADE 7 MATH: PROPORTIONAL REASONING

Instructional Supports for Students with Disabilities using UDL Guidelines

Provide Multiple Means of Representation

- *Offer ways of customizing the display of information :*

The diagram is enclosed in a black rectangular border. At the top left is an analog clock showing 11:15. At the top right is an analog clock showing 1:15. Below each clock is a photograph of a red car. At the bottom, a horizontal line with vertical end caps represents a distance of 80 miles. The left end is labeled 'Point 1' and the right end is labeled 'Point 2'.

And/or

**11:15 AM**

**1:15 PM**

- **Display information in a flexible format** by varying the layout of visuals, the size of pictures, and the contrast between the background and image.
- **Offer alternatives for auditory information** by reading aloud and/or recording Assessment Questions 1-5, in order to provide students with multiple opportunities to hear and comprehend the task requirements.
- **Offer alternatives for visual information** by providing both images of clocks in both digital and analog form. Ensure that alternative representations are provided not only for accessibility, but for clarity and comprehensibility across all learners.

### Provide Background Information

- Information is more accessible and likely to be assimilated by learners when it is presented in a way that primes, activates, or provides any prerequisite knowledge. **Activate or supply background knowledge by anchoring instruction.** Prepare students to determine whether a particular relationship is proportional or not by applying prior knowledge to connect to important mathematical concepts.

1. Did you know that you use proportions and ratios every day, whether you realize it or not? Do any of members of your family drive a car? If so, do you ever hear them mention driving a certain number of **miles per hour**, or do you ever hear them say that they “are driving at the speed limit of 60 miles per hour?”

When you go shopping with your family, have you ever seen prices like \$2.98 per pound, \$1.50 a gallon, or \$8.00 a yard? You are using ratios, rates, and proportions.

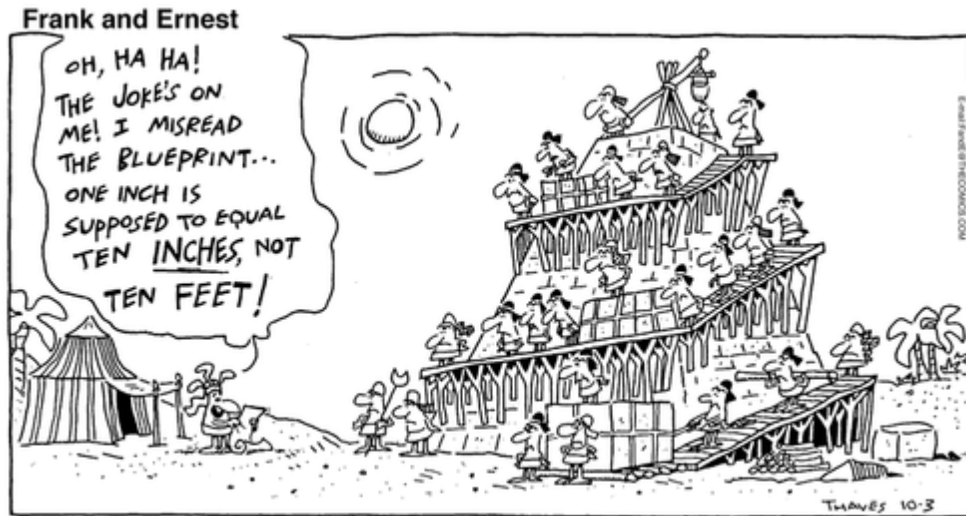
2. Consider this problem: Robert reads 60 pages of a book in 30 minutes. How long should it take him to read 150 pages? What would you have to do to solve this problem?

<b>Pages</b>	<b>60</b>	<b>150</b>	
<b>Minutes</b>	<b>30</b>	<b>X</b>	

Or, consider this problem: What if a car travels **90 miles** in **3 hours**, how long could it travel in **4 hours? 5 hours? 8 hours?**

<b>Miles</b>			<b>90</b>	<b>X</b>	<b>X</b>			<b>X</b>
<b>Hours</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>

3. Or consider a cartoon: What happened?



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- **Highlight patterns, critical features, big ideas, and relationships** by articulating and reinforcing conceptual understandings: *In proportion problems, you have two things that both change at the same rate. For example, you have dollars and gallons in the one situation:*

2 gallons - 7.00 dollars  
6 gallons - X dollars

90 miles - 3 hours  
X miles - 4 hours

*In both examples, there are two things that both change at the same rate.*

<b>Dollars</b>		<b>7.00</b>				<b>X</b>
<b>Gallons</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>

<b>Miles</b>			<b>90</b>	<b>X</b>
<b>Hours</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>

**Provide Support for Conceptual Understandings**

The semantic elements through which information is presented – the words, symbols, numbers, and icons – are differently accessible to learners with varying backgrounds, languages, and lexical knowledge. To ensure accessibility for all, key vocabulary, labels, icons, and symbols should be linked to, or associated with, alternate representations of their meaning: an embedded glossary or definition, a graphic equivalent, or a chart or map.

- **Embed support for vocabulary and symbols** by providing online tools, such as *Math is Fun* and *Purplemath* which use multiple means of representation to explain concepts:

<http://www.mathsisfun.com/>

<http://www.purplemath.com/modules/ratio2.htm>

- **Embed support for unfamiliar references within the text** by defining academic vocabulary, such as *proportional reasoning*, *unit rate*, *ratio*, *proportion*, *rate*, *qualitative*, *quantitative*, *comparison*, and *equivalence*.

A ratio is two things compared to each other. For example “3 dollars per gallon” is a ratio. Or, 40 miles per 1 hour. Or, 15 girls versus 14 boys. Or, 569 words in 2 minutes. Or, 23 green balls to 41 blue balls. A ratio is a comparison of two things.

Proportion is when you have two ratios set to be equal to each other. For example, “3 dollars per gallon” equals “6 dollars per two gallons.” Or “40 miles per hour “equals 80 miles in two hours” Or, 2 teachers per 20 students equals 3 teachers per \_\_\_\_\_ students?

<b>Teachers</b>	<b>2</b>	<b>3</b>
<b>Students</b>	<b>20</b>	<b>X</b>

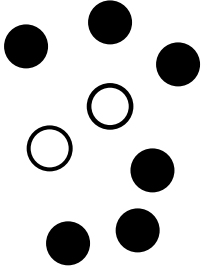
Note: A *ratio* is a *multiplicative comparison* of two quantities or measures. A key developmental milestone is the ability of a student to begin to think of a ratio as a distinct entity, different from the two measures that made it up.

*Ratios and proportions involve **multiplicative rather than additive** comparisons. Equal ratios result from **multiplication or division**, not from addition or subtraction.*

*Proportional reasoning is developed through activities involving comparing and determining the equivalence of ratios and solving proportions in a wide variety of situations and contexts.*

- **Provide multiple examples of novel solutions to authentic problems** by allowing students to see their peers’ perspectives on proportional reasoning.

- **Provide alternatives in the permissible tools and scaffolds to optimize challenges** by providing calculators or *designing customized mini-lessons* to activate prior knowledge of the concept of proportional reasoning.
- **Present key concepts by illustrating through multiple media.** Use ratio cards where students compare various visual representations of ratios intuitively.

	<p>Proportion of black is</p> <p>Error! Bookmark not defined.<math>\frac{6}{8}</math></p>	<p>Proportion of black is</p> <p>Error! Bookmark not defined.<math>\frac{3}{4}</math></p>	<p>Ratio of black to white is</p> <p>6:2</p>	<p>Ratio of black to white is</p> <p>3:1</p>
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- **Facilitate managing information and resources.** Provide graphic organizers and templates for data collection and organizing information in order for students to explain how they solved a problem and how they know their answers make sense.

<b>Planning Sheet for Assessment 1</b>		
<p>What questions am I being asked to answer? What problems am I being asked to solve?</p>	<p>What strategy or strategies can I use to solve the math word problem and answer <u>all</u> the questions?</p>	<p>Show how I made my decisions and arrived at my answers.</p>
<p>Did use a table or graph? Which one or both?</p>		
<p>Did I use a Math rule? Which one?</p>		

**Guide appropriate goal –setting.** Review rubric and provide a checklist to support students’ mathematical thinking and problem-solving strategies:

<b>Student Checklist</b>	
Did I read the math problem several times?	
Do I know what the problem is asking me to solve?	
Did I answer <b>all</b> the questions?	
Did I <b>label</b> my work correctly?	
Did I check all my computations and are they correct?	
Did I show how I solved the problem? Did I use a graph or table?	
Did I show <b>all</b> my mathematical thinking?	
Did I show how I made my decisions and arrived at my answers?	
Did I <u>underline</u> , circle, or draw a box around all my answers?	
Do I need draw a <b>diagram</b> to explain my answer? Did I label and include a <b>key</b> in my diagram?	