



## GRADE 6 MATH: RATIOS AND PROPORTIONAL RELATIONSHIPS

### UNIT OVERVIEW

This 4-5 week unit focuses on developing an understanding of ratio concepts and using ratio reasoning to solve problems.

### TASK DETAILS

**Task Name:** Ratios and Proportional Relationship

**Grade:** 6

**Subject:** Mathematics

**Depth of Knowledge:** 2

**Task Description:** This sequence of tasks asks students to demonstrate and effectively communicate their mathematical understanding of ratios and proportional relationships. Their strategies and executions should meet the content, thinking processes and qualitative demands of the tasks.

#### **Standards:**

**6RP.1** Understand the concept of ratio and use ratio language to describe a ratio relationship between two quantities.

**6RP.2** Understand the concept of a unit rate  $a/b$  associated with a ratio  $a:b$  with  $b \neq 0$ , and use rate language in the context of a ratio relationship.

**6RP.3** Use ratio and rate reasoning to solve real-world and mathematical problems, i.e., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

**6.RP.3a** Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

**6.RP.3b** Solve unit rate problems including those involving unit pricing and constant speed.

**6.RP.3c** Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means  $30/100$  times the quantity); solve problems involving finding the whole, given a part and the percent.

**6.RP.3d** Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

#### **Standards for Mathematical Practice:**

**MP.1** Make sense of problems and persevere in solving them.

**MP.2** Reason abstractly and quantitatively.

**MP.3** Construct viable arguments and critique the reasoning of others.

**MP.6** Attend to precision.



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The task and instructional supports in the following pages are designed to help educators understand and implement tasks that are embedded in Common Core-aligned curricula. While the focus for the 2011-2012 Instructional Expectations is on engaging students in Common Core-aligned culminating tasks, it is imperative that the tasks are embedded in units of study that are also aligned to the new standards. Rather than asking teachers to introduce a task into the semester without context, this work is intended to encourage analysis of student and teacher work to understand what alignment looks like. We have learned through this year’s Common Core pilots that beginning with rigorous assessments drives significant shifts in curriculum and pedagogy. Universal Design for Learning (UDL) support is included to ensure multiple entry points for all learners, including students with disabilities and English language learners.

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# GRADE 6 MATH: RATIOS AND PROPORTIONAL RELATIONSHIPS PERFORMANCE TASK

Performance Based Assessment 1  
Proportional Reasoning – Grade 6

Student  
Name

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School

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Date

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Teacher

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1. Giovanni is visiting his grandmother who lives in an apartment building on the 25th floor. Giovanni enters the elevator in the lobby, which is the first floor in the building. The elevator stops on the 16th floor. What percentage of 25 floors does Giovanni have left to reach his grandmother's floor? Use pictures, tables or number sentences to solve the task. Explain your reasoning in words.

2. Pianos and pipe organs contain keyboards, a portion of which is shown below.



- a) What is the ratio of black keys to white keys in the picture above?
- b) If the pattern shown continues, how many black keys appear on a portable keyboard with 35 white keys?
- c) If the pattern shown continues, how many black keys appear on a pipe organ with a total of 240 keys?

3. a. Mr. Copper's class has a female student to male student ratio of 3:2. Mr. Copper's class has 18 girls, how many boys does he have? Show how you determined your answer. Explain your reasoning in words.

b) Ms. Green's class has the same number of students as Mr. Copper's class. Her female-to-male ratio is 2:1. Which class has the greater number of females? How do you know?

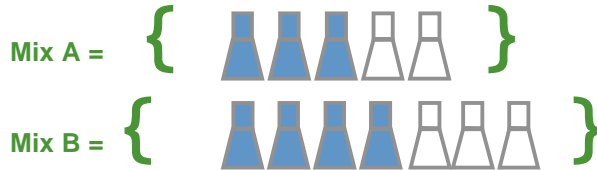
4. Use the recipe shown in the table to answer the questions below. Use pictures, tables or number sentences to solve the task.

Grandma's Recipe for Sugar Cookies
1 $\frac{1}{2}$ cups butter
2 cups sugar
4 eggs
$\frac{3}{4}$ teaspoon baking powder
1 $\frac{1}{4}$ cups flour
$\frac{1}{4}$ teaspoon salt

- a) How many cups of sugar are needed for each egg? How do you know?
- b) Your sister notices that she needs three times as much baking powder as salt in this recipe. What is the ratio of baking powder to salt? Explain your reasoning in words.



5. Fashion designers are trying to decide on just the right shade of blue for a new line of jeans. They have several bottles of fabric color, some with blue color and some with white color. They plan to mix these together to get the desired color.



- a) Will both mixes produce the same color jeans? Use mathematical reasoning to justify your answer.
- b) A designer uses the table below to think about her own special mix, Mix C. How many liters of blue color will she need to make a total of 40 liters? Explain your reasoning in words.

Liters of Blue Color	Liters of White Color	Total Liters
5	3	8
10	6	16



GRADE 6 MATH: RATIOS AND PROPORTIONAL  
RELATIONSHIPS  
UNIVERSAL DESIGN FOR LEARNING (UDL)  
PRINCIPLES

## **Math, Grade 6 – Ratios Universal Design for Learning**

The goal of using Common Core Learning Standards (CCLS) is to provide the highest academic standards to all of our students. Universal Design for Learning (UDL) is a set of principles that provides teachers with a structure to develop their instruction to meet the needs of a diversity of learners. UDL is a research-based framework that suggests each student learns in a unique manner. A one-size-fits-all approach is not effective to meet the diverse range of learners in our schools. By creating options for how instruction is presented, how students express their ideas, and how teachers can engage students in their learning, instruction can be customized and adjusted to meet individual student needs. In this manner, we can support our students to succeed in the CCLS.

Below are some ideas of how this Common Core Task is aligned with the three principles of UDL; providing options in representation, action/expression, and engagement. As UDL calls for multiple options, the possible list is endless. Please use this as a starting point. Think about your own group of students and assess whether these are options you can use.

**REPRESENTATION:** *The “what” of learning.* How does the task present information and content in different ways? How do students gather facts and categorize what they see, hear, and read. How are they identifying letters, words, or an author's style?

*In this task, teachers can...*

- ✓ **Pre-teach vocabulary and symbols, especially in ways that promote connection to the learners’ experience and prior knowledge** as applied to the understanding of concepts across the tasks by using common experiences, such as *sports*, hobbies, and *interests*) that can be annotated and calculated in numeric values.

**ACTION/EXPRESSION:** *The “how” of learning.* How does the task differentiate the ways that students can express what they know? How do they plan and perform tasks? How do students organize and express their ideas?

*In this task, teachers can...*

- ✓ **Compose in multiple media such as text, speech, drawing, illustration, design, film, music, dance/movement, visual art, sculpture or video** by allowing students choice and multiple forms with which to express solutions and mathematical thinking.

**ENGAGEMENT:** *The “why” of learning.* How does the task stimulate interest and motivation for learning? How do students get engaged? How are they challenged, excited, or interested?

*In this task, teachers can...*

- ✓ **Design activities so that learning outcomes are authentic, communicate to real audiences, and reflect a purpose that is clear to the participants** by looking at statistics that reflect areas of interests and common situations.

Visit <http://schools.nyc.gov/Academics/CommonCoreLibrary/default.htm> to learn more information about UDL.



# GRADE 6 MATH: RATIOS AND PROPORTIONAL RELATIONSHIPS

## BENCHMARK PAPERS WITH RUBRICS

This section contains benchmark papers that include student work samples for each of the tasks in the Ratios and Proportional Relationships assessment. Each paper has descriptions of the traits and reasoning for the given score point, including references to the Mathematical Practices.

NYC Grade 6 Assessment 1  
Elevator Task  
Benchmark Papers

1. Giovanni is visiting his grandmother who lives in an apartment building on the 25th floor. Giovanni enters the elevator in the lobby, which is the first floor in the building. The elevator stops on the 16th floor. What percentage of 25 floors does Giovanni have left to reach his grandmother's floor? Use pictures, tables or number sentences to solve the task. Explain your reasoning in words.

NYC Grade 6 Assessment 1  
Elevator Task  
Benchmark Papers

3 Points

The response accomplishes the prompted purpose and effectively communicates the student's mathematical understanding. The student's strategy and execution meet the content (including concepts, technique, representations, and connections), thinking processes and qualitative demands of the task. Minor omissions may exist, but do not detract from the correctness of the response.

Either verbally or symbolically, the strategy used to solve the problem is stated, as is the work used to find ratio and percent. Minor arithmetic errors may be present, but no errors of reasoning appear. Complete explanations are stated, based on work shown.

Accurate reasoning processes demonstrate the Mathematical Practices, (1) Make sense of problems and persevere in solving them, and (2) Reason abstractly and quantitatively (since students need to abstract information from the problem, create a mathematical representation of the problem, and correctly work with part-whole, ratio and percent). Evidence of the Mathematical Practice, (3) Construct viable arguments and critique the reasoning of others, is demonstrated by complete and accurate explanations. Evidence of the Mathematical Practice, (4) Model with mathematics, is demonstrated by representing the problem as a ratio and/or decimal and percent. Evidence of the Mathematical Practice, (6) Attend to precision, can include proper use of ratio notation and proper labeling of quantities.

The reasoning used to solve the parts of the problem may include:

- Understanding that the whole is 25 and either 16 or (25 – 16) floors is a part of the whole involved. Some students may consider a “missing 13<sup>th</sup> floor”. In that case, the whole is 24 and either 15 or (24 – 15) is a part of the whole involved.
- Recognition of the need to determine how many “out of 100”.
  - Using a fraction and its conversion to an equivalent fraction, possibly by simplifying first.
  - Converting to decimal then to percent.
  - Creating and reasoning from a grid representation of the context
- Recognition of the need either to work with 9/25 (or 9/24) or to subtract from 100% after changing 16/25 (or 15/24) to percent.

Handwritten student work showing calculations:

$$\frac{16}{24} = \frac{?}{100}$$

$$24? = \frac{1600}{24}$$

Long division:  $24 \overline{) 1600}$

50	
1200	-
400	
240	-
160	
120	-
40	
24	-
16	
0	

66.16%

$100 - 66.16 = 33.84\%$

rounded → 33.84%

Vertical list of numbers: 9, 10, 00, 67, 33, 24, 24, 24, 24, 24, 24, 20

NYC Grade 6 Assessment 1  
Elevator Task  
Benchmark Papers

2 Points

The response accomplishes the prompted purpose and effectively communicates the student's mathematical understanding. The student's strategy and execution meet the content (including concepts, technique, representations, and connections), thinking processes and qualitative demands of the task. Minor omissions may exist, but do not detract from the correctness of the response.

Either verbally or symbolically, the strategy used to solve the problem is stated, as is the work used to find ratio and percent. Reasoning may contain incomplete, ambiguous or misrepresentative ideas. Partial explanations are stated based on work shown.

Accurate reasoning processes demonstrate the Mathematical Practices, (1) Make sense of problems and persevere in solving them, and (2) Reason abstractly and quantitatively (since students need to abstract information from the problem, create a mathematical representation of the problem, and correctly work with part-whole, ratio and percent). Evidence of the Mathematical Practice, (3) Construct viable arguments and critique the reasoning of others, is demonstrated by complete and accurate explanations. Evidence of the Mathematical Practice, (4) Model with mathematics, is demonstrated by representing the problem as a ratio and/or decimal and percent. Evidence of the Mathematical Practice, (6) Attend to precision, can include proper use of ratio notation and proper labeling of quantities.

The reasoning used to solve the parts of the problem may include:

- a. Understanding that the whole is 25 (or 24) and either 16 (or 15) floors is a part of the whole but failing to recognize the need to use  $\frac{9}{25}$  (or  $\frac{9}{24}$ ) or to subtract from 100% after changing  $\frac{16}{25}$  (or  $\frac{15}{24}$ ) to percent.
- b. Recognition of the need to work with  $\frac{9}{25}$  (or  $\frac{9}{24}$ ), but failing to change the ratio into a percent.

The image shows handwritten student work. At the top, there is a crossed-out fraction  $\frac{16}{24} = \frac{100}{100}$  with  $24x$  written to the right. Below this, the number 1600 is written. In the middle, the percentage 77.7% is written, followed by a long string of 7s: 77.77777. To the right of this is a long division problem:  $24 \overline{) 1600}$ . The student has written 144 below 1600, subtracted it to get 160, and then written 144 below 160. At the bottom, the student has written  $24 \times 5 = 120, 144, 168$ . There is also a small note "100-20" written above the multiplication.

NYC Grade 6 Assessment 1  
Elevator Task  
Benchmark Papers

1 Point

The response demonstrates some evidence of mathematical knowledge that is appropriate to the intent of the prompted purpose. An effort was made to accomplish the task, but with little success. Evidence in the response demonstrates that, with instruction, the student can revise the work to accomplish the task.

Some evidence of reasoning is demonstrated either verbally or symbolically, but may be based on misleading assumptions, and/or contain errors in execution. Some work is used to find ratio and percent or partial answers are evident. Explanations are incorrect, incomplete or not based on work shown. Accurate reasoning processes demonstrate the Mathematical Practices, (1) Make sense of problems and persevere in solving them, and (2) Reason abstractly and quantitatively (since students need to abstract information from the problem, create a mathematical representation of the problem, and correctly work with part-whole, ratio and percent). Evidence of the Mathematical Practice, (3) Construct viable arguments and critique the reasoning of others, is demonstrated by complete and accurate explanations. Evidence of the Mathematical Practice, (4) Model with mathematics, is demonstrated by representing the problem as a ratio and/or decimal and percent. Evidence of the Mathematical Practice, (6) Attend to precision, can include proper use of ratio notation and proper labeling of quantities.

The reasoning used to solve the parts of the problem may include:

- a. Forming the ratio 16/25 (or 15/24), and trying but failing to change the ratio into a percent.
- b. Forming the ratio 9/16 (or 9/24) and changing that ratio to a percent.

$$\frac{16}{24}$$

Answer: 15%

$$\begin{array}{r} 116 \\ * 2 \\ \hline 32 \end{array}$$

$$\begin{array}{r} 1.5x \\ 16 \overline{) 24.0} \\ \underline{-16} \downarrow \\ 80 \\ \underline{-80} \\ 0 \end{array}$$

$$\begin{array}{r} 416 \\ * 7 \\ \hline 112 \end{array}$$

$$\begin{array}{r} 316 \\ * 5 \\ \hline 80 \end{array}$$



NYC Grade 6 Assessment 1  
Piano Task  
Benchmark Papers

2. Pianos and pipe organs contain keyboards, a portion of which is shown below.



- a) What is the ratio of black keys to white keys in the picture above?
- b) If the pattern shown continues, how many black keys appear on a portable keyboard with 35 white keys?
- c) If the pattern shown continues, how many black keys appear on a pipe organ with a total of 240 keys?

NYC Grade 6 Assessment 1

Piano Task

Benchmark Papers

3 Points

The response accomplishes the prompted purpose and effectively communicates the student's mathematical understanding. The student's strategy and execution meet the content (including concepts, technique, representations, and connections), thinking processes and qualitative demands of the task. Minor omissions may exist, but do not detract from the correctness of the response.

Either verbally or symbolically, the strategy used to solve each part of the problem is stated, as is the work used to find ratios, proportions, and partial answers to problems. Minor arithmetic errors may be present, but no errors of reasoning appear.

Accurate reasoning processes demonstrate the Mathematical Practice, (1) Make sense of problems and persevere in solving them, and (2) Reason abstractly and quantitatively (since students need to abstract information from the problem, create a mathematical representation of the problem, and correctly work with both part-part and part-whole situations). Evidence of the Mathematical Practice, (4) Model with mathematics, is demonstrated by representing the problem with tables and/or ratios. Evidence of the Mathematical Practice, (6) Attend to precision, can include proper use of ratio notation, proper symbolism and proper labeling of quantities. Repeated use of the same structure to solve parts b and c can be seen as evidence of the Mathematical Practice, (8) Look for and express regularity in repeated reasoning.

The reasoning used to solve the parts of the problem may include:

- Scaling up the 5:7 ratio in fraction form to a denominator of 35, signifying 35 white keys.
- Scaling up the 5:7 ratio in tabular form.
- Using a proportion or proportional reasoning (e.g., 35 white keys is 5 times 7 white keys, so I can multiply 5 by 5 black keys to find the number of black keys).
- Recognizing the part-to-whole nature of part c as 5:12 and employing any of the above techniques.

5	10	15	20	25
7	14	21	28	35

a) What is the ratio of black keys to white keys in the picture above?



b) If the pattern shown continues, how many black keys appear on a portable keyboard with 35 white keys

B	W
7	14
21	21
28	28
35	35

Handwritten notes: 25:35, 25 black keys

c) If the pattern shown continues, how many black keys appear on a pipe organ with a total of 240 keys?

Black (10)	White (14)	Total
10	14	24
20	28	48
30	42	72
40	56	96
50	70	120
60	84	144
70	98	168
80	112	192
90	126	216
100	140	240

Handwritten notes: 48 x 5 = 240, 48 x 7 = 336, 100 Black keys

NYC Grade 6 Assessment 1  
Piano Task  
Benchmark Papers

2 Points

The response demonstrates adequate evidence of the learning and strategic tools necessary to complete the prompted purpose. It may contain overlooked issues, misleading assumptions, and/or errors in execution. Evidence in the response demonstrates that the student can revise the work to accomplish the task with the help of written feedback or dialogue.

Either verbally or symbolically, the strategy used to solve each part of the problem is stated, as is the work used to find ratios, proportions, and partial answers to problems. Minor arithmetic errors may be present. Reasoning may contain incomplete, ambiguous or misrepresentative ideas. Accurate reasoning processes demonstrate the Mathematical Practice, (1) Make sense of problems and persevere in solving them, and (2) Reason abstractly and quantitatively (since students need to abstract information from the problem, create a mathematical representation of the problem, and correctly work with both part-part and part-whole situations). Evidence of the Mathematical Practice, (4) Model with mathematics, is demonstrated by representing the problem with tables and/or ratios. Evidence of the Mathematical Practice, (6) Attend to precision, can include proper use of ratio notation, proper symbolism and proper labeling of quantities. Repeated use of the same structure to solve parts b and c can be seen as evidence of the Mathematical Practice, (8) Look for and express regularity in repeated reasoning.

The reasoning used to solve the parts of the problem may include:

- Scaling up the 5:7 ratio in fraction form to some denominator, but failing to stop at 35, signifying 35 white keys.
- Scaling up the 5:7 ratio in tabular form, but failing to stop at 35.
- Reversing white and black keys, but maintaining the scaling to 35 white keys as indicated by labels.
- Using a proportion or proportional reasoning (e.g., 35 white keys is 5 times 7 white keys), but failing to then multiply 5 by 5 black keys to find the total number of black keys.
- Recognizing the part-to-whole nature of part c as 5:12 and employing any of the above techniques or errors.

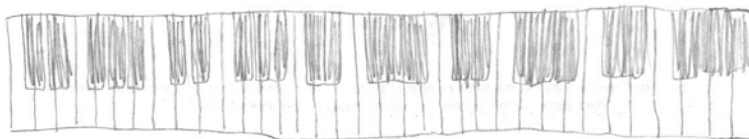
5	10	15	20	25
7	14	21	28	35

a) What is the ratio of black keys to white keys in the picture above?

5 to 7

The ratio is  
25 to 35

b) If the pattern shown continues, how many black keys appear on a portable keyboard with 35



c) If the pattern shown continues, how many black keys appear on a pipe organ with a total of 240 keys?

$$\begin{array}{r} 80 \\ 37240 \overline{) 80} \\ - 240 \\ \hline 000 \end{array}$$

The ratio would be  
200 Black to  
40 white

$$\begin{array}{r} 80 \\ \times 3 \\ \hline 240 \end{array}$$

$$\begin{array}{r} 120 \\ + 80 \\ \hline 200 \end{array}$$

$$\begin{array}{r} 120 \\ 27240 \overline{) 90} \\ - 180 \\ \hline 60 \\ - 60 \\ \hline 00 \end{array}$$

NYC Grade 6 Assessment 1

Piano Task

Benchmark Papers

1 Point

The response demonstrates some evidence of mathematical knowledge that is appropriate to the intent of the prompted purpose. An effort was made to accomplish the task, but with little success. Evidence in the response demonstrates that, with instruction, the student can revise the work to accomplish the task.

Some evidence of reasoning is demonstrated either verbally or symbolically, is often based on misleading assumptions, and/or contains errors in execution. Some work is used to find ratios or proportions or partial answers to portions of the task are evident. Accurate reasoning processes demonstrate the Mathematical Practice, (1) Make sense of problems and persevere in solving them, and (2) Reason abstractly and quantitatively (since students need to abstract information from the problem, create a mathematical representation of the problem, and correctly work with both part-part and part-whole situations). Evidence of the Mathematical Practice, (4) Model with mathematics, is demonstrated by representing the problem with tables and/or ratios. Evidence of the Mathematical Practice, (6) Attend to precision, can include proper use of ratio notation, proper symbolism and proper labeling of quantities. Repeated use of the same structure to solve parts b and c can be seen as evidence of the Mathematical Practice, (8) Look for and express regularity in repeated reasoning.

The reasoning used to solve the parts of the problem may include:

- a. Identification of the 5:7 ratio in some form (including tabular).
- b. Some attempt to scale, but failure to maintain the ratio, typically by reverting to addition.
- c. Failure to attempt at least two parts of the problem.

a) What is the ratio of black keys to white keys in the picture above?

w	b
7	5

b) If the pattern shown continues, how many black keys appear on a portable keyboard with 35 white keys

w	b
7	5
14	10
21	15
<del>28</del>	<del>20</del>
35	20

c) If the pattern shown continues, how many black keys appear on a pipe organ with a total of 240 keys?

w	b
7	5
14	10
21	15
<del>28</del>	<del>20</del>
35	25
<del>42</del>	<del>30</del>
49	35
56	40
63	45
70	50
77	55
84	60
91	65
98	70
105	75
112	80
119	85
126	90
133	95
140	100
147	105
154	110
161	115
168	120
175	125
182	130
189	135
196	140
203	145
210	150
217	155
224	160
231	165
238	170
245	175

- 3.
- a) Mr. Copper's class has a female student-to-male student ratio of 3:2. If Mr. Copper's class has 18 girls, how many boys does he have? Show or explain in writing how you determined your answer.
  - b) Ms. Green's class has the same number of students as Mr. Copper's class. Her female-to-male ratio is 2:1. Which class has the greater number of females? How do you know?

NYC Grade 6 Assessment 1  
Boys and Girls Task  
Benchmark Papers

3 Points

The response accomplishes the prompted purpose and effectively communicates the student's mathematical understanding. The student's strategy and execution meet the content (including concepts, technique, representations, and connections), thinking processes and qualitative demands of the task. Minor omissions may exist, but do not detract from the correctness of the response.

Either verbally or symbolically, the strategy used to solve the problem is stated, as is the work used to find ratio and scaling. Minor arithmetic errors may be present, but no errors of reasoning appear. Complete explanations are stated, based on work shown.

Accurate reasoning processes demonstrate the Mathematical Practice (1) Make sense of problems and persevere in solving them and (2) Reason abstractly and quantitatively (since students need to abstract information from the problem, create a mathematical representation of the problem, and correctly work with both part-part and part-whole situations). Evidence of the Mathematical Practice, (3) Construct viable arguments and critique the reasoning of others, is demonstrated by complete and accurate explanations. Evidence of the Mathematical Practice, (4) Model with mathematics, is demonstrated by representing the problem with a table and/or ratios. Evidence of the Mathematical Practice, (6) Attend to precision, can include proper use of ratio and/or proportion notation, proper symbolism and proper labeling of quantities.

The reasoning used to solve the parts of the problem may include:

- a. Scaling up the 3:2 ratio in fraction form to a numerator of 18, signifying 18 girls. Recognizing 18 + 12 as the whole, and 2:1 as the ratio of girls-to-whole in the other class, and following another scaling-up process.
- b. Scaling up the 3:2 ratio in tabular form; scaling up the 2:1 ratio in a similar fashion, to a total of 30 students.
- c. Using a proportion or proportional reasoning (e.g., 18 girls is 6 times 3 girls, so I can multiply 6 by 2 boys to find the number of boys). Using a part-whole fraction to form a proportion for the second class.
- d. Recognizing that a 3:2 ratio is being compared with a 2:1 ratio for the same number of students. As a result, the ratio that represents a larger fraction also represents the class with the larger number of girls, since the ratio is female-to-male in both instances.

3	6	9	12	15	18
2	4	6	8	10	12

- a) Mr. Copper's class has a female student-to-male student ratio of 3:2. If Mr. Copper's class has 18 girls, how many boys does he have? Show or explain in writing how you determined your answer.

$\frac{3F}{2M} = \frac{18F}{12M}$   
 IF you multiply 2m to 6 it will equal 12m  
 IF you multiply 3F your gain to get  
 30 students

- b) Ms. Green's class has the same number of students as Mr. Copper's class. Her female-to-male ratio is 2:1. Which class has the greater number of females? How do you know?

$\frac{2M \times 10}{1M \times 10} = \frac{20F}{10M}$   
 20 Females and 10 equals 30  
 you multiply 2 x 10 which equals 20

NYC Grade 6 Assessment 1  
Boys and Girls Task  
Benchmark Papers

2 Points

The response demonstrates adequate evidence of the learning and strategic tools necessary to complete the prompted purpose. It may contain overlooked issues, misleading assumptions, and/or errors in execution. Evidence in the response demonstrates that the student can revise the work to accomplish the task with the help of written feedback or dialogue.

Either verbally or symbolically, the strategy used to solve the problem is stated, as is the work used to find ratio and scaling. Reasoning may contain incomplete, ambiguous or misrepresentative ideas. Partial explanations are stated based on work shown.

Accurate reasoning processes demonstrate the Mathematical Practice (1) Make sense of problems and persevere in solving them, and (2) Reason abstractly and quantitatively (since students need to abstract information from the problem, create a mathematical representation of the problem, and correctly work with both part-part and part-whole situations). Evidence of the Mathematical Practice, (3) Construct viable arguments and critique the reasoning of others, is demonstrated by complete and accurate explanations. Evidence of the Mathematical Practice, (4) Model with mathematics, is demonstrated by representing the problem with a table and/or ratios. Evidence of the Mathematical Practice, (6) Attend to precision, can include proper use of ratio and/or proportion notation, proper symbolism and proper labeling of quantities.

The reasoning used to solve the parts of the problem may include:

- a. Scaling up the 3:2 ratio in fraction or tabular form, but failing to stop at a numerator of 18, signifying 18 girls. Recognizing 2:1 as the ratio of girls to whole in the other class, and following another scaling up process (possibly incorrectly).
- b. Recognizing the need for a whole, but determining it with faulty reasoning or scaling part b to 18 girls rather than 30 students.
- c. Using a proportion or proportional reasoning (e.g., 18 girls is 6 times 3 girls, so I can multiply 6 by 2 boys to find the number of boys). Incorrectly using a part-part fraction to form a proportion for the second class.
- d. Recognizing that a 3:2 ratio is being compared with a 2:1 ratio for the same number of students; failing to determine the ratio that represents the larger fraction.

- a) Mr. Copper's class has a female student-to-male student ratio of 3:2. If Mr. Copper's class has 18 girls, how many boys does he have? Show how you determined your answer. Explain your reasoning in words.

	F	M	T
	3	2	5
x6	18	12	30

12 boys

- b) Ms. Green's class has the same number of students as Mr. Copper's class. Her female-to-male ratio is 2:1. Which class has the greater number of females? How do you know?

	F	M	T
	2	1	3
x9	18	9	27

NYC Grade 6 Assessment 1  
Boys and Girls Task  
Benchmark Papers

1 Point

The response demonstrates some evidence of mathematical knowledge that is appropriate to the intent of the prompted purpose. An effort was made to accomplish the task, but with little success. Evidence in the response demonstrates that, with instruction, the student can revise the work to accomplish the task.

Some evidence of reasoning is demonstrated either verbally or symbolically, but may be based on misleading assumptions, and/or contain errors in execution. Some work is used to find ratio and scaling or partial answers are evident. Explanations are incorrect, incomplete or not based on work shown. The student may make some attempt at using some mathematical practices.

Accurate reasoning processes demonstrate the Mathematical Practice (1) Make sense of problems and persevere in solving them and (2) Reason abstractly and quantitatively (since students need to abstract information from the problem, create a mathematical representation of the problem, and correctly work with both part-part and part-whole situations). Evidence of the Mathematical Practice, (3) Construct viable arguments and critique the reasoning of others, is demonstrated by complete and accurate explanations. Evidence of the Mathematical Practice, (4) Model with mathematics, is demonstrated by representing the problem with a table and/or ratios. Evidence of the Mathematical Practice, (6) Attend to precision, can include proper use of ratio and/or proportion notation, proper symbolism and proper labeling of quantities.

The reasoning used to solve the parts of the problem may include:

- Identification of the 3:2 ratio in some form (including tabular).
- Some attempt to scale, but failure to maintain the ratio, typically by reverting to addition.
- Failure to recognize that ratios are being compared in part b; attempting instead to compare by using the 3 from Mr. Copper's class with the 2 from Ms. Green's class (or 2 vs. 1).
- Failure to attempt at least one part of the problem
  - Mr. Copper's class has a female student-to-male student ratio of 3:2. If Mr. Copper's class has 18 girls, how many boys does he have? Show how you determined your answer. Explain your reasoning in words.

	3F	2M
18	18	12
12	12	8
6	6	4
2	2	1.33

= 12



NYC Grade 6 Assessment 1  
Boys and Girls Task  
Benchmark Papers

- b) Ms. Green's class has the same number of students as Mr. Copper's class. Her female-to-male ratio is 2:1. Which class has the greater number of females? How do you know?

Mr. Copper has more female  
because he has a bigger number  
2 multiply

NYC Grade 6 Assessment 1  
 Sugar Cookies Task  
 Benchmark Papers

4. Use the recipe shown in the table to answer the questions below. Use pictures, tables or number sentences to solve the task.

Grandma's Recipe for Sugar Cookies
1 $\frac{1}{2}$ cups butter
2 cups sugar
4 eggs
$\frac{3}{4}$ teaspoon baking powder
1 $\frac{1}{4}$ cups flour
$\frac{1}{4}$ teaspoon salt

- a) How many cups of sugar are needed for each egg? How do you know?
- b) Your sister notices that she needs three times as much baking powder as salt in this recipe. What is the ratio of baking powder to salt? Explain your reasoning in words.

NYC Grade 6 Assessment 1

Sugar Cookies Task

Benchmark Papers

3 Points

The response accomplishes the prompted purpose and effectively communicates the student's mathematical understanding. The student's strategy and execution meet the content (including concepts, technique, representations, and connections), thinking processes and qualitative demands of the task. Minor omissions may exist, but do not detract from the correctness of the response.

Either verbally or symbolically, the strategy used to solve each part of the problem is stated, as is the work used to find ratios, unit rates, scaling and partial answers to problems. Minor arithmetic errors may be present, but no errors of reasoning appear. Complete explanations are stated, based on work shown.

Accurate reasoning processes demonstrate the Mathematical Practices, (1) Make sense of problems and persevere in solving them and (2) Reason abstractly and quantitatively (since students need to abstract information from the problem, create a mathematical representation of the problem, and correctly work with the tables and/or ratios). Evidence of the Mathematical Practice, (3) Construct viable arguments and critique the reasoning of others, is demonstrated by complete and accurate explanations. Evidence of the Mathematical Practice, (4) Model with mathematics, is demonstrated by representing the problem as a table, ratio or proportion. Evidence of the Mathematical Practice, (6) Attend to precision, can include proper use of ratio notation and proper labeling of quantities. Evidence of the Mathematical Practice, (7) Look for and make use of structure, may be demonstrated by student recognition, without actually calculating, that, "three times as much as" can be interpreted as a 3/1 ratio.

The reasoning used to solve the parts of the problem may include:

- a. A ratio of cups of sugar to each egg is formed and simplified or scaled down to a denominator of 1.
- b. Scaling down the 2:4 ratio in tabular form.
- c. Recognizing the phrase "three times as much baking powder as salt" as a 3:1 ratio.
- d. Forming a  $\frac{3}{4} : \frac{1}{4}$  ratio; possibly scaling that ratio up to  $2\frac{3}{4} : \frac{3}{4}$  on the basis of the "three times as much" language.
- e. Drawing a picture and reasoning from the picture for either or both parts.

2	1	$\frac{1}{2}$
4	2	1

$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

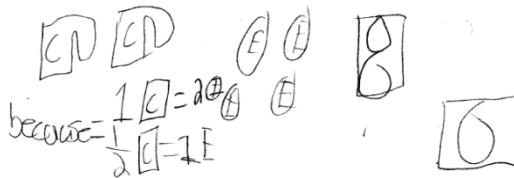
Baking Powder

$\frac{1}{4}$
$\frac{1}{4}$
$\frac{1}{4}$
$\frac{1}{4}$

Salt

3 teaspoons baking powder : 1 teaspoon salt

- a. How many cups of sugar are needed for each egg? How do you know?



- b. Your sister notices that she needs three times as much baking powder as salt in this recipe. What is the ratio of baking powder to salt? Explain your reasoning in words.

$3\frac{3}{4} : \frac{1}{4}$

I know because baking powder comes first then salt then you just put the recipe with baking powder first then salt.

NYC Grade 6 Assessment 1  
Sugar Cookies Task  
Benchmark Papers

2 Points

The response demonstrates adequate evidence of the learning and strategic tools necessary to complete the prompted purpose. It may contain overlooked issues, misleading assumptions, and/or errors in execution. Evidence in the response demonstrates that the student can revise the work to accomplish the task with the help of written feedback or dialogue.

Either verbally or symbolically, the strategy used to solve each part of the problem is stated, as is the work used to find ratios, unit rates, scaling, and partial answers to problems. Minor arithmetic errors may be present. Reasoning may contain incomplete, ambiguous or misrepresentative ideas. Partial explanations are stated, based on work shown.

Accurate reasoning processes demonstrate the Mathematical Practices, (1) Make sense of problems and persevere in solving them, and (2) Reason abstractly and quantitatively (since students need to abstract information from the problem, create a mathematical representation of the problem, and correctly work with the tables and/or ratios). Evidence of the Mathematical Practice, (3) Construct viable arguments and critique the reasoning of others, is demonstrated by complete and accurate explanations. Evidence of the Mathematical Practice, (4) Model with mathematics, is demonstrated by representing the problem as a table, ratio or proportion. Evidence of the Mathematical Practice, (6) Attend to precision, can include proper use of ratio notation and proper labeling of quantities. Evidence of the Mathematical Practice, (7) Look for and make use of structure, may be demonstrated by student recognition, without actually calculating, that, “three times as much as” can be interpreted as a 3/1 ratio.

The reasoning used to solve the parts of the problem may include:

- A ratio of cups of sugar to each egg is formed but not scaled down to a denominator of 1.
- A ratio of eggs to cups of sugar is formed and scaled down to a denominator of 1.
- Forming a  $\frac{3}{4} : \frac{1}{4}$  ratio; but incorrectly attempting to multiply by 3 on the basis of the “three times as much” language.
- Drawing a picture for both parts, but reasoning incorrectly from the picture for one part.

- How many cups of sugar are needed for each egg? How do you know?

2 cups because if you divide 4 eggs into 2 cups of sugar it will be 2 eggs per cup.

- Your sister notices that she needs three times as much baking powder as salt in this recipe. What is the ratio of baking powder to salt? Explain your reasoning in words.

$\frac{3}{4}$  teaspoon :  $\frac{1}{4}$  teaspoon.

NYC Grade 6 Assessment 1  
Sugar Cookies Task  
Benchmark Papers

1 Point

The response demonstrates some evidence of mathematical knowledge that is appropriate to the intent of the prompted purpose. An effort was made to accomplish the task, but with little success. Evidence in the response demonstrates that, with instruction, the student can revise the work to accomplish the task.

Some evidence of reasoning is demonstrated either verbally or symbolically, is often based on misleading assumptions, and/or contains errors in execution. Some work is used to find ratios or unit rates or partial answers to portions of the task are evident. Explanations are incorrect, incomplete or not based on work shown.

Accurate reasoning processes demonstrate the Mathematical Practices, (1) Make sense of problems and persevere in solving them and (2) Reason abstractly and quantitatively (since students need to abstract information from the problem, create a mathematical representation of the problem, and correctly work with the tables and/or ratios). Evidence of the Mathematical Practice, (3) Construct viable arguments and critique the reasoning of others, is demonstrated by complete and accurate explanations. Evidence of the Mathematical Practice, (4) Model with mathematics, is demonstrated by representing the problem as a table, ratio or proportion. Evidence of the Mathematical Practice, (6) Attend to precision, can include proper use of ratio notation and proper labeling of quantities. Evidence of the Mathematical Practice, (7) Look for and make use of structure, may be demonstrated by student recognition, without actually calculating, that, "three times as much as" can be interpreted as a 3/1 ratio.

The reasoning used to solve the parts of the problem may include:

- a. Expressing a correct ratio, with no explanation.
- b. Forming the ratio of eggs to cups of sugar, but not scaling down to a denominator of 1.
- c. Forming a ratio from two ingredients, only one of which is baking powder or salt; however, correctly placing the baking powder or salt in the ratio.
- d. Drawing a picture, but reasoning incorrectly from the picture for both parts.

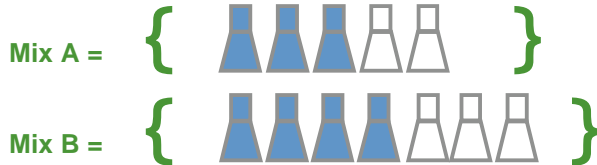
a) How many cups of sugar are needed for each egg? How do you know?

$\frac{1}{2}$  a cup of  
Sugar

b) Your sister notices that she needs three times as much baking powder as salt in this recipe. What is the ratio of baking powder to salt? Explain your reasoning in words.

~~1/12~~  $1 \frac{1}{2} : 1 \frac{1}{4}$

5. Fashion designers are trying to decide on just the right shade of blue for a new line of jeans. They have several bottles of fabric color, some with blue color and some with white color. They plan to mix these together to get the desired color.



- a) Will both mixes produce the same color jeans? Justify your reasoning.
- b) A designer uses the table below to think about her own special mix, Mix C. How many liters of blue color will she need to make a total of 40 liters? Explain your reasoning.

Liters of Blue Color	Liters of White Color	Total Liters
5	3	8
10	6	16

NYC Grade 6 Assessment 1  
Mixture Task  
Benchmark Papers

3 Points

The response accomplishes the prompted purpose and effectively communicates the student's mathematical understanding. The student's strategy and execution meet the content (including concepts, technique, representations, and connections), thinking processes and qualitative demands of the task. Minor omissions may exist, but do not detract from the correctness of the response.

Either verbally or symbolically, the strategy used to solve the problem is stated, as is the work used to find ratios, unit rates, scaling, and partial answers to problems. Minor arithmetic errors may be present, but no errors of reasoning appear. Complete explanations are stated, based on work shown.

Accurate reasoning processes demonstrate the Mathematical Practice, (1) Make sense of problems and persevere in solving them, and (2) Reason abstractly and quantitatively (since students need to abstract information from the problem, create a mathematical representation of the problem, and correctly work with both part-part and part-whole situations). Evidence of the Mathematical Practice, (3) Construct viable arguments and critique the reasoning of others, is demonstrated by complete and accurate explanations. Evidence of the Mathematical Practice, (4) Model with mathematics, is demonstrated by representing the problem tables, ratios and/or proportions. Evidence of the Mathematical Practice, (6) Attend to precision, can include proper use of ratio and/or proportion notation, proper symbolism and proper labeling of quantities.

Justification may include reasoning as follows:

- Forming unit rates and indicating that the unit rates differ or are not equivalent.
- Forming ratios and indicating that the ratios differ or are not equivalent.
- Building a table of values to a common number and noting, e.g., that one mix uses 9 blue and 6 white while the other differs, using 8 blue and 6 white, so the first is "bluer".

3	6	9
2	4	6

4	8
3	6

- Extending the table to a total of 40 and correctly identifying the number of blue liters.
- Using a proportion or proportional reasoning (e.g., 40 total liters is 5 times 8, so I can multiply 8 by 5 to find the number of blue liters).



- a) Will both mixes produce the same color jeans? Justify your reasoning.

*No Mix B has more white than Mix A*

- b) A designer uses the table below to think about her own special mix, Mix C. How many liters of blue color will she need to make a total of 40 liters? Explain your reasoning.

Liters of Blue color	Liters of White color	Total Liters
5	3	8
10	6	16
15	12	24
20	15	32
25	15	40

*When you use a grid if you continue the pattern the mix will remain the same except the total will get bigger*

*25*

NYC Grade 6 Assessment 1  
Mixture Task  
Benchmark Papers

2 Points

The response demonstrates adequate evidence of the learning and strategic tools necessary to complete the prompted purpose. It may contain overlooked issues, misleading assumptions, and/or errors in execution. Evidence in the response demonstrates that the student can revise the work to accomplish the task with the help of written feedback or dialogue.

Either verbally or symbolically, the strategy used to solve the problem is stated, as is the work used to find ratios, unit rates, scaling, and partial answers to problems. Reasoning may contain incomplete, ambiguous or misrepresentative ideas. Partial explanations are stated, based on work shown.

Accurate reasoning processes demonstrate the Mathematical Practice (1) Make sense of problems and persevere in solving them and (2) Reason abstractly and quantitatively (since students need to abstract information from the problem, create a mathematical representation of the problem, and correctly work with both part-part and part-whole situations). Evidence of the Mathematical Practice, (3) Construct viable arguments and critique the reasoning of others, is demonstrated by complete and accurate explanations. Evidence of the Mathematical Practice, (4) Model with mathematics, is demonstrated by representing the problem with tables, ratios and/or proportions. Evidence of the Mathematical Practice, (6) Attend to precision, can include proper use of ratio and/or proportion notation, proper symbolism and proper labeling of quantities.

Justification may include reasoning as follows:

- Incorrectly forming or comparing unit rates or ratios for part a.
- Building a table of values to a common number, but incorrectly interpreting results for part a.
- Extending the table to 40 blue liters and/or a total of 40 liters, and possibly attempting to scale white liters to 40; therefore, incorrectly identifying the number of blue liters.
- Using a proportion or proportional reasoning incorrectly (e.g., 40 blue liters is 5 times 8, so I can multiply 8 by 5 to find the total number of liters).

a) Will both mixes produce the same color jeans? Justify your reasoning.

Yes because they are equivalent ratios

b) A designer uses the table below to think about her own special mix, Mix C. How many liters of blue color will she need to make a total of 40 liters? Explain your reasoning.

Liters of Blue color	Liters of White color	Total Liters
5	3	8
10	6	16
25	15	40

Handwritten annotations: A circle around the 25 in the 'Liters of Blue color' column. A circle around the 40 in the 'Total Liters' column. An arrow from the 40 to the 25. A box containing the number 25 below the table. A handwritten '5' with an arrow pointing to the 5 in the first row, and another '5' with an arrow pointing to the 15 in the third row.



NYC Grade 6 Assessment 1  
Mixture Task  
Benchmark Papers

1 Point

The response demonstrates some evidence of mathematical knowledge that is appropriate to the intent of the prompted purpose. An effort was made to accomplish the task, but with little success. Evidence in the response demonstrates that, with instruction, the student can revise the work to accomplish the task.

Some evidence of reasoning is demonstrated either verbally or symbolically, is often based on misleading assumptions, and/or contains errors in execution. Some work is used to find ratios, unit rates, scaling or partial answers to portions of the task are evident. Explanations are incorrect, incomplete or not based on work shown. The student may make some attempt at using some mathematical practices. Accurate reasoning processes demonstrate the Mathematical Practice, (1) Make sense of problems and persevere in solving them, and (2) Reason abstractly and quantitatively (since students need to abstract information from the problem, create a mathematical representation of the problem, and correctly work with both part-part and part-whole situations). Evidence of the Mathematical Practice, (3) Construct viable arguments and critique the reasoning of others, is demonstrated by complete and accurate explanations. Evidence of the Mathematical Practice, (4) Model with mathematics, is demonstrated by representing the problem tables, ratios and/or proportions. Evidence of the Mathematical Practice, (6) Attend to precision, can include proper use of ratio and/or proportion notation, proper symbolism and proper labeling of quantities.

The reasoning used to solve the parts of the problem may include:

- a. No attempt at mathematical reasoning to respond to part a.
- b. Some attempt to scale in either or both parts, but failure to maintain the ratio, typically by reverting to addition.
- c. Incorrectly forming and comparing unit rates or ratios for part a; also, scaling blue, white and total in part b.

I think no because mix b is darker and lighter.

- b) A designer uses the table below to think about her own special mix, Mix C. How many liters of blue color will she need to make a total of 40 liters? Explain your reasoning.

Liters of Blue color	Liters of White color	Total Liters
5	3	8
10	6	16
15	9	24
20	12	32
25	15	40
30	18	
35	21	
40	24	
	27	
	30	
	36	
	39	
	42	



# GRADE 6 MATH: RATIOS AND PROPORTIONAL RELATIONSHIPS ANNOTATED STUDENT WORK

## Assessment 1: Question 1

1. Giovanni is visiting his grandmother who lives in an apartment building on the 24th floor. Giovanni enters the elevator in the lobby, which is the first floor in the building. The elevator stops on the 16th floor. What percentage of 24 floors does Giovanni have left to reach his grandmother's floor? Use pictures, tables or number sentences to solve the task. Explain your reasoning in words.

### CCSS (Content) Addressed by this Task:

- 6.RP.1 Understand the concept of ratio and use ratio language to describe a ratio relationship between two quantities.
- 6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g. by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
- 6.RP.3c Find a percent of a quantity as a rate per 100; solve problems involving finding the whole, given a part and the percent.

### CCSS for Mathematical Practice Addressed by this Task:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics (possible)
6. Attend to precision

NYC Grade 6 Assessment 1  
Elevator Task  
Annotated Student Work

Annotation of Student Work With a Score of 3

The percentage of floors Giovanni have left is  $33\frac{1}{3}\%$  because I simplified  $\frac{16}{24}$  to  $\frac{2}{3}$   
 $\frac{16}{24} = \frac{2}{3} = 66\frac{2}{3}\%$  which equals  $66\frac{2}{3}\%$  and I subtracted 100 and  $66\frac{2}{3}$  which gave me  $33\frac{1}{3}$ .

$$\begin{array}{r} 99 \\ 100 \\ - 66\frac{2}{3} \\ \hline 33\frac{1}{3} \end{array}$$

**Content Standards:** The student received a score of 3 because s/he recognizes that the context can be translated into a ratio of the number of floors traveled to number floors remaining (6.RP.1); however, the numbers are not explicitly labeled. The student then simplifies  $16/24$  to the equivalent ratio  $2/3$ , which s/he appears to know to equal  $66\frac{2}{3}\%$  (6.RP.3c). The student is able to explain the reasoning that s/he uses to find the solution to the problem. The student also correctly reasons that the percent remaining can be determined by subtracting the percent traveled from 100%.

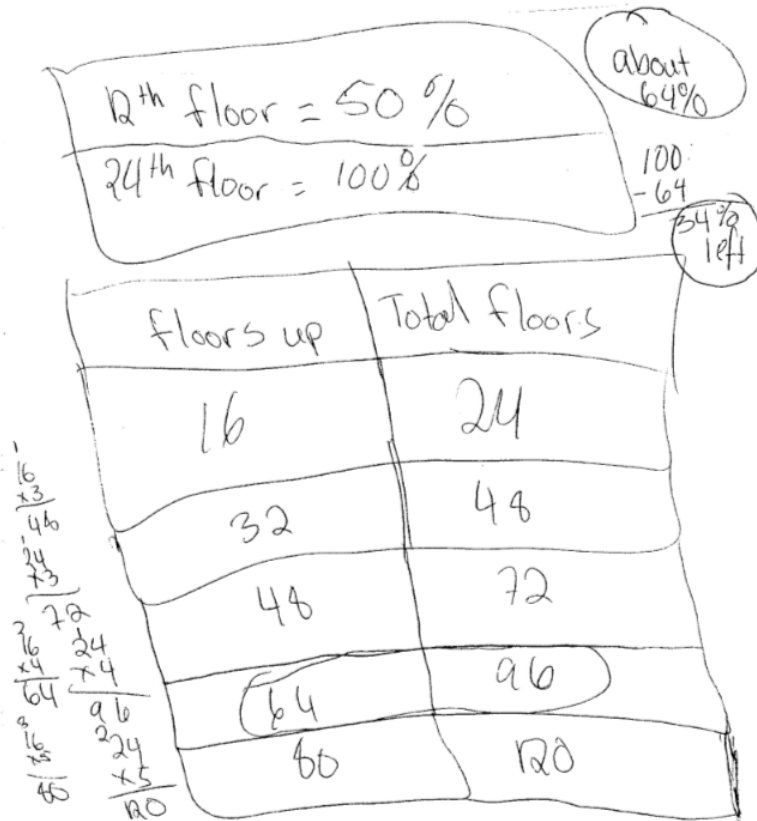
**Mathematical Practices:** This student makes sense of the problem (Practice 1) and is able to model the situation abstractly and quantitatively and relate the answer back to the context "percentage of floors left" (Practices 2 & 4). The student correctly expresses ratio equivalence ( $16/24 = 2/3$ ), and uses the percent symbol when expressing the results (Practice 6). S/he presents a coherent explanation of the rationale for the procedure used (Practice 3).

**Next Instructional Steps:** Ask the student to explain how s/he knows that  $2/3$  is equivalent to  $66\frac{2}{3}\%$ .

Ask the student how s/he can determine the percent of floors traveled if there were 25 floors instead of 24 floors. Ask him/her how this can be determined if the ratio is  $15/23$ .

NYC Grade 6 Assessment 1  
Elevator Task  
Annotated Student Work

Annotation of Student Work With a Score of 2



**Content Standards:** The student received a score of 2 because s/he shows an understanding of ratio as a relationship between two quantities (6.RP.1) by creating a table with labeled columns. The student also shows an understanding that percent is a rate per 100 (6.RP.3c) when s/he uses a table to scale up 16/24 (6.RP.3) to find that it is equivalent to a rate of 64/96, which is approximated as 64%. The student also correctly reasons that the percent remaining can be determined by subtracting the percent traveled from 100%. However, the student chooses a strategy that, while it can be used in all situations, will generally not yield an exact answer.

**Mathematical Practices:** This student makes sense of the problem (Practice 1) and is able to model the situation abstractly and quantitatively but does not explicitly relate the answer back to the context “percentage of floors left” (Practices 2 & 4). The student correctly expresses ratio equivalence, uses the percent symbol when expressing his results and correctly labels the columns in the table (Practice 6). While the symbolic work is well-organized and easy to follow, the student does not present a written explanation of the rationale for the procedure used (Practice 3).

**Next Instructional Steps:** Ask the student to justify how s/he knows that Giovanni had traveled approximately 64% of the floors, and ask whether the exact percent would be more, or less, than 64%. Ask the student how s/he knows that 12 floors is equivalent 50%. Finally, ask the student to think of a way that s/he can find the exact percent of floors remaining. If the student is unable to make progress, suggest that the student join a group who used a different procedure, and have the students explain their strategy.

**Annotation of Student Work With a Score of 1**

$24 = 100\%$   $12 = 50\%$   $8\%$

$16 = 92\%$

I think it's 8% percent  
because 24 would be the same 100%.  
Also from 16 to 24 is 8.  
So I think he has 8% to  
go.

**Content Standards:** The student received a score of 1 because the student shows an emerging understanding of percent as a ratio when s/he explicitly notes that 24 floors is 100% and 12 floors = 50% (6.RP.3 and 6.RP.3c). This understanding is incomplete, however, and there is a major error in reasoning (thinking that the answer is 8% because there are 8 floors to go).

**Mathematical Practices:** This student's work shows some aspects of precision with the consistent use of the % symbol; however, the student fails to label the numbers, e.g., in the statement "from 16 to 24 is 8" (Practice 6). The student shows some ability to express his/her reasoning in writing (Practice 3). The student is able to model one aspect of the task – that s/he is being asked to find the percent of floors remaining, and thus uses subtraction to determine this amount (Practices 1 & 4). While the student abstracts the quantities from the problem, the student fails to re-contextualize the numbers in terms of the context (Practice 2).

**Next Instructional Steps:** Ask the student to explain the statement "from 16 to 24 is 8". Ask him/her to label the numbers – "what do they represent?" Give the student a 10 x 10 grid and have him/her represent the statements "24 floors = 100%" and "12 floors = 50%" on the 10 x 10 grid. The student should also shade the portion that they think represents 8 floors, and also shade 8% of another 10 x 10 grid to determine if 8% is a reasonable answer. Give the student additional problems where s/he works to represent percents, fractions and decimals using a 10 x 10 grid in order to develop an understanding of equivalence across these three forms of rational numbers.

## Assessment 1: Question 2

2. Pianos and pipe organs contain keyboards, a portion of which is shown below.



- What is the ratio of black keys to white keys in the picture above?
- If the pattern shown continues, how many black keys appear on a portable keyboard with 35 white keys?
- If the pattern shown continues, how many black keys appear on a pipe organ with a total of 240 keys?

### CCSS (Content) Addressed by this Task:

- 6.RP.1 Understand the concept of ratio and use ratio language to describe a ratio relationship between two quantities.
- 6.RP.2 Understand the concept of a unit rate  $a/b$  associated with a ratio  $a:b$  with (possible)  $b \neq 0$ , and use rate language in the context of a ratio relationship.
- 6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g. by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
- 6.RP.3a Make tables of equivalent ratios relating quantities with whole-number (potential) measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

### CCSS for Mathematical Practice Addressed by this Task:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
4. Model with mathematics (possible)
6. Attend to precision
8. Look for and express regularity in repeated reasoning

NYC Grade 6 Assessment 1  
Piano Task  
Annotated Student Work

Annotation of Student Work With a Score of 3

a)

W	B
5	7

$5:7$

b)

B	W
5	7
10	14
15	21
20	28
25	35
30	
35	

$25:35$

25 black keys

c)

	Black (10)	White (14)	Total
10	10	14	24
20	20	28	48
30	30	42	72
40	40	56	96
50	50	70	120
60	60	84	144
70	70	98	168
80	80	112	192
90	90	126	216
100	100	140	240
110			

$48 \times 5 = 240$

$48 \times 7 = 336$

118
14
132
14
146

100 Black keys

**Content Standards:** The student received a score of 3 because s/he shows an understanding of the ratio concept by setting up tables, as well as expressing the ratio using the  $a:b$  notation (6.RP.1) The student uses tables to form equivalent ratios between black and white keys (6.RP.3) and these tables are used to find missing values (6.RP.3a.) The student correctly represents both the part-part ratios and the part-whole ratios to solve this real-world problem (6.RP.3).

**Mathematical Practices:** This student is able to represent the problem quantitatively, find a quantitative solution, and express the result in terms of the context (Practice 2). The student makes sense of the problem (Practice 1) in parts b) and c) by representing the problem with tables and persisting until solutions to the questions are found. The student consistently labels the quantities (Practice 6), but does not use any information from the table in part b) as a shortcut to finding an answer to part c) (Practice 8).

**Next Instructional Steps:** Ask the student to figure out a way to determine the number of black keys if the organ had 480 total keys without extending the table, and to explain why it works. Ask the student if s/he sees other patterns within his/her table in part c).



**Annotation of Student Work With a Score of 2**

a) The ratio of black keys to white keys is 5:7.

b) There will be 25 black keys with 35 white keys.  
 $\frac{5}{7} = \frac{25}{35}$

c) ~~6/7~~ black keys  
 $\frac{25}{35} = \frac{5}{7} = \frac{66}{240}$

**Content Standards:** The student received a score of 2 because the student correctly represents the ratios involved (6.RP.1) using labeled quantities and the  $a:b$  notation. The student uses an equivalent ratio to answer the question (part b) though s/he doesn't indicate how it is formed (6.RP.3). In part c) the student fails to show an understanding that 240 represents a part-whole ratio, since s/he again begins with the part-part ratio of black to white keys. The answer given is also not a reasonable answer, given the part-part ratio indicated.

**Mathematical Practices:** This student is able to represent the problem quantitatively, find a quantitative solution, and express the result in terms of the context (Practices 1 & 2). The student consistently labels the quantities (Practice 6). The student provides a written response/explanation (Practice 3) even though this is not explicitly asked for in the prompt. The student attempts to use repeated reasoning (Practice 8) in each part of the problem, but fails to recognize the difference between the part-part reasoning of parts a) and b) versus the part-whole reasoning of part c).

**Next Instructional Steps:** Ask the student to explain what 240 represents in terms of the context. Ask the student whether his/her answer of  $6 \frac{6}{7}$  black keys seems reasonable. Suggest that the student construct a table and use that to try to figure out how many black keys there will be if there are 240 total keys.

**Annotation of Student Work With a Score of 1**

- a) 5 black keys to 7 white keys
- b) 33 black keys to 35 white keys
- c) 238 black keys to 240 white keys

**Content Standards:** The student received a score of 1 because a beginning understanding of ratio is apparent (6.RP.1) in the student's response to part a), and the student attempts to use a method of scaling up to produce equivalent ratios (6.RP.3) to answer parts b) and c); however, the student uses an additive, rather than a multiplicative, approach. The student maintains the constant *difference* between black and white keys, instead of a constant *ratio*. The student also fails to recognize that part c) is asking for a part-whole relationship.

**Mathematical Practices:** This student labels all of the quantities (Practice 6), and abstracts the quantities from the context, works with those quantities, and re-contextualizes the quantities in terms of the problem (Practice 2).

**Next Instructional Steps:** In order to help the student to move from an additive, to a multiplicative, understanding of equivalent ratios, it will be helpful to have the student use manipulatives or diagrams to represent the problem. It will also be helpful to have him/her use a table to represent the equivalent ratios of black to white keys as the pattern is extended. The student can then be asked to explore the patterns that emerge in both the model and the table, and to try to anticipate how the pattern will grow beyond what is shown.

### Assessment 1: Question 3

- 3 a) Mr. Black's class has a female student-to-male student ratio of 3:2. If Mr. Black's class has 18 girls, how many boys does he have? Show or explain in writing how you determined your answer.
- b) Ms. Green's class has the same number of students as Mr. Black's class. Her female-to-male ratio is 2:1. Which class has the greater number of females? How do you know?

#### CCSS (Content) Addressed by this Task:

- 6.RP.1 Understand the concept of ratio and use ratio language to describe a ratio relationship between two quantities.
- 6.RP.2 Understand the concept of a unit rate  $a/b$  associated with a ratio  $a:b$  with  $b \neq 0$ , and use rate language in the context of a ratio relationship.
- 6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g. by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
- 6.RP.3a Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. (possible)

#### CCSS for Mathematical Practice Addressed by this Task:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
6. Attend to precision
7. Look for and express regularity in repeated reasoning

NYC Grade 6 Assessment 1  
Boys and Girls Task  
Annotated Student Work

Annotation of Student Work With a Score of 3

3. a)

$$3:2 \quad 3 \rightarrow 2 \quad \frac{3}{2}$$

$$\frac{3}{2} \times 6 = 18$$

$$\frac{2}{2} \times 6 = 12$$

12 boys I figured this by using  
an equivalent fractions since 3 times 6 is 18 <sup>which</sup> are the girls  
then 2 times 6 is 12 for boys

$$\begin{array}{r} 12 \\ + 18 \\ \hline 30 \end{array}$$

b)  $\frac{2 \cdot 10 = 20}{1 \cdot 10 = 10}$

$$\frac{2 \cdot 9 = 18}{1 \cdot 9 = 9}$$

Ms Green's class

has more females because since she has 30 students like

Ms Black so Ms Black has  $\frac{3}{2} = 18$  girls to make 30  
12 boys  $\frac{18}{30}$

so then  $\frac{2}{1} = \frac{20}{10}$  so Ms Green  
has more  
females

$$\begin{array}{r} 20 \\ + 10 \\ \hline 30 \end{array}$$

**Content Standards:** The student received a score of 3 because the work shows a clear understanding of scaling up ratios to form equivalent ratios and find missing values (6.RP.3 & 6.RP.3a). This student does this both with the part-part and part-whole ratios (girls to boys in Mr. Black's class and females to class in Ms. Green's class). S/he scales up the ratios by multiplying both parts of the ratio (expressed in fraction form) by the same factor. The student also shows an understanding of the multiple ways to express ratios (6.RP.1), and explains that s/he uses equivalent fractions to find the missing value. The student gives complete explanations of his/her reasoning, using both words and symbols. There are no arithmetic errors.

**Mathematical Practices:** This student's work is strengthened by all of the identified mathematical practices – the student shows the ability to model the situation mathematically (Practice 4) in the process of developing a solution strategy and carrying it out successfully (Practice 1). The student moves from the context to the abstraction and back again to the context (Practice 2) and the work is made clear by the use of appropriate labels and ratio notation (Practice 6). The student clearly discerns and uses the structure of proportional relationships, whether part-part or part-whole (Practice 7). Finally the student gives a complete explanation of his/her reasoning and the meaning of the results (Practice 3).

**Next Instructional Steps:** Ask the student to explain what multiplying the numerator and denominator of the ratio  $\frac{3}{2}$  by 6 (or the numerator and denominator of the ratio  $\frac{2}{1}$  by 10 or 9) means in the context of

the problem. Ask him/her "why" you have to multiply both by the same number, and why that is important in problems like this. Ask him/her if it would have been possible to do the same thing if the ratios were in the form 3:2 or 2:1.

**Annotation of Student Work With a Score of 2**

3 a)

3	2
6	4
9	6
12	8
15	10
18	12

$18:12$

18
712
30

18 | 12 = Answer

b)

2	1
4	2
6	3
8	4
10	5
12	6

12	6
14	7
16	8
18	9
20	10
22	11
24	12
26	13
28	14
30	15

$30:15$

**Content Standards:** This student received a score of 2 because the work in both parts of the problem shows evidence of scaling up ratios, using a table, to form equivalent ratios and find missing values (6.RP.3 & 6.RP.3a). Although a multiplicative approach is suggested, it is not clear whether this is done additively or multiplicatively. The student uses appropriate notation to express the ratios (6.RP.1). The student shows some understanding that part b) refers to a part-whole relationship, since s/he first finds the total number of students in Mr. Black's class, and scales up the table of ratios in part b) until 30 is reached; however, the student fails to attend to the fact that 2:1 is a part-part ratio. The student fails to give a written explanation of his/her reasoning, though the symbolic work suggests the nature of the reasoning that occurred. The student fails to explicitly answer the questions in parts a) and b), though numbers are present which could be used to answer the questions (incorrectly in part b). There are no arithmetic errors.

**Mathematical Practices:** One of the difficulties in the student's reasoning might be due to the lack of Practice 6 (attend to precision), since none of the columns in the tables are labeled; thus, the student may have lost track of what the ratios refer to. There are some practices present – the student shows the ability to partially model the situation mathematically (Practice 4) in the process of developing a solution strategy and carrying it through, though not completely successfully (Practice 1). The student moves from the context to the abstraction but fails to refer back again to the context (Practice 2). Finally the student fails to give an explanation of his/her reasoning or the meaning of the results (Practice 3).

**Next Instructional Steps:** Ask the student to explain orally the strategies that were used, and to explain what the numbers in the columns of the table mean. Ask the student also to explain how s/he got from one row to the next, and what that means in the context of the problem. If the student uses an additive approach (i.e. adding 3 to the number of females and 2 to the number of males, etc.) ask the student if there is another way s/he can get from 3:2 to 18:15 without adding repeatedly.

**Annotation of Student Work With a Score of 1**

Handwritten student work for part a). On the left, a table with two columns labeled 'g' and 'b'. The 'g' column contains the number 3 repeated seven times. The 'b' column contains the number 2 repeated seven times. To the right of the table, the ratio  $18:12$  is circled. Below this, a vertical addition shows 18 plus 12 equals 30.

Handwritten student work for part b). On the left, a table with two columns labeled 'g' and 'b'. The 'g' column contains the number 2 repeated seven times. The 'b' column contains the number 1 repeated seven times. To the left of the table, the ratio  $20:10$  is boxed. To the right, a vertical addition shows 20 plus 10 equals 30.

**Content Standards:** This student received a score of 1 because, although the student uses a table, the work does not show evidence of scaling up ratios to form equivalent ratios and find missing values (6.RP.3a). Instead, the student uses an additive approach in both parts of the problem to find the solution to the problem. The given ratio (3:2 or 2:1) is added until the desired value (18 or 20) is reached). The student shows an understanding that part b) refers to a part-whole relationship, since s/he first finds the total number of students in Mr. Black's class and then understands that a 2:1 ratio with a total of 30 students would yield a ratio of 20:10. The student fails to provide a written explanation of his/her reasoning, though it is clear symbolically what was done. The student fails to explicitly answer the questions in parts a) and b), though numbers are present which could be used to answer the questions. There are no arithmetic errors.

**Mathematical Practices:** The student correctly labels the columns in parts a) and b) (Practice 6). The student shows the ability to model the situation mathematically with tables (Practice 4) and develops a solution strategy and carries it through in a way that could provide an answer to the questions (Practice 1). The student moves from the context to the abstraction but fails to refer back again to the context (Practice 2). Finally, the student fails to give an explanation of his/her reasoning or the meaning of the results (Practice 3).

**Next Instructional Steps:** This student will benefit from being grouped with other students who use multiplicative approaches to find equivalent ratios, and who are able to articulate their reasoning. S/he can also be encouraged to draw pictures to represent the situation, and to focus on the total number of students in each step of the table. The student can be encouraged to look for patterns and to explain what the patterns mean in the context of the problem.

### Assessment 1: Question 4

4. Use the recipe shown in the table to answer the questions below. Use pictures, tables or number sentences to solve the task.

Grandma's Recipe for Sugar Cookies
1 $\frac{1}{2}$ cups butter
2 cups sugar
4 eggs
$\frac{3}{4}$ teaspoon baking powder
1 $\frac{1}{4}$ cups flour
$\frac{1}{4}$ teaspoon salt

- a) How many cups of sugar are needed for each egg? How do you know?
- b) Your sister notices that she needs three times as much baking powder as salt in this recipe. What is the ratio of baking powder to salt? Explain how you know.

#### CCSS (Content) Addressed by this Task:

- 6.RP.1 Understand the concept of ratio and use ratio language to describe a ratio relationship between two quantities.
- 6.RP.2 Understand the concept of a unit rate  $a/b$  associated with a ratio  $a:b$  with  $b \neq 0$ , and use rate language in the context of a ratio relationship.
- 6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
- 6.RP.3b Solve unit rate problems including those involving unit pricing and constant speed.

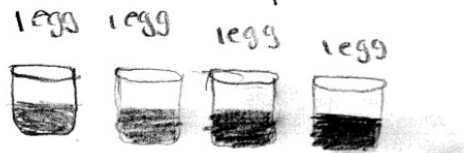
#### CCSS for Mathematical Practice Addressed by this Task:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics (possible)
6. Attend to precision
7. Look for and make use of structure.

NYC Grade 6 Assessment 1  
Sugar Cookies Task  
Annotated Student Work

Annotation of Student Work With a Score of 3

a) One half cup is needed for each egg.



b) there is a 3:1 ratio baking powder to salt. She said she needs three times as much.

3/4 teaspoon baking powder is three times as much as 1/4 teaspoon salt.

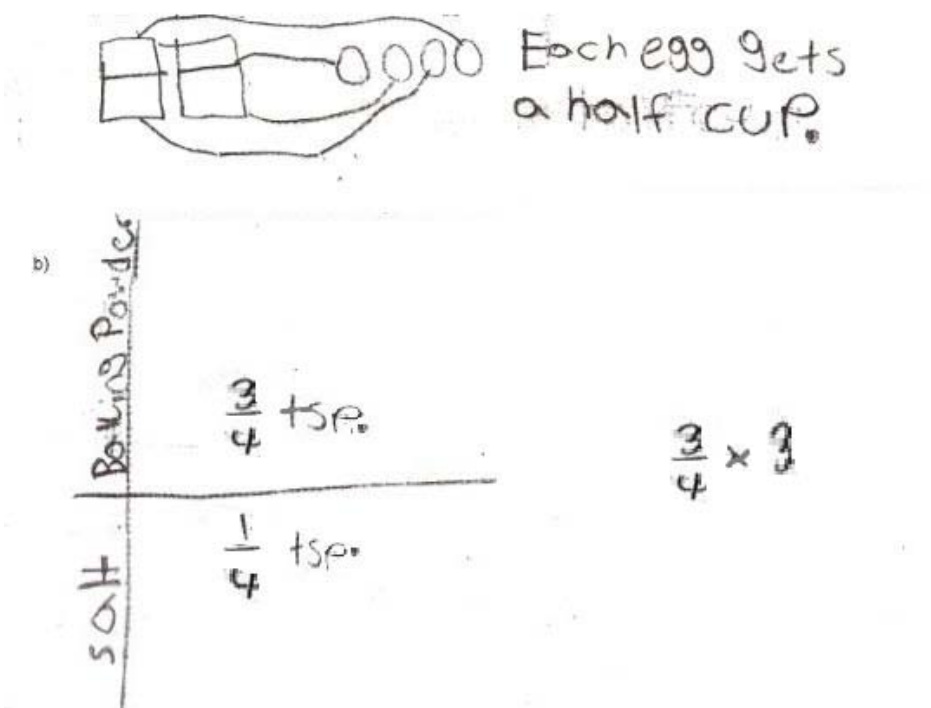
**Content Standards:** The student received a score of 3 because s/he uses a diagram to determine amount of sugar needed for each egg (unit rate) (6.RP.1, 6.RP.2, 6.RP.3). The student also expresses the ratio of baking powder to salt using ratio notation (6.RP.1) and shows that s/he understands the connection of the ratio to “three times as much”. The student is able to correctly solve a unit rate problem based on a real-world context (6.RP.3b).

**Mathematical Practices:** This student makes sense of the problem (Practice 1) and models it using a diagram (Practice 4). The student uses this model to justify his/her response (Practice 3) though a written explanation is not provided. The student abstracts the quantities in the problem, reasons with them, and relates the answers back to the context (Practice 2). The student correctly labels the quantities involved (Practice 6).

**Next Instructional Steps:** Ask the student to explain his/her diagram, and ask him/her what other ways could s/he use to represent the ratio. Ask the student what this type of ratio is called (unit rate). Ask the student to show you how s/he can figure out “how much butter is needed for each egg”.



**Annotation of Student Work With a Score of 2**



**Content Standards:** The student received a score of 2 because s/he correctly uses a diagram to determine and explain the unit rate (6.RP.2) of a half-cup per egg. Although the student uses informal language to express the ratio and unit rate, she does not use standard ratio notation or language (6.RP.1). The student also fails to form a ratio for part b), though by setting up a horizontal table and inserting the correct quantities, s/he appears to clearly show an understanding that the question asks for a relationship between the two quantities (6.RP.1); there was also an attempt to account for the statement “3 times as much”, though an error was made.

**Mathematical Practices:** This student is able to make sense of the problem (Practice 1) and model the situation with a diagram (Practice 4). The student abstracts the information from the problem and re-contextualizes it in terms of the context (Practice 2). The student shows the habit of labeling columns in tables (Practice 6). Although the student’s diagram can be seen as a reasonable justification for the answer to part a) (Practice 3), the student does not provide a written explanation.

**Next Instructional Steps:** Ask the student to express the ratio between sugar and egg using symbols. Ask the student what this type of ratio is called (unit rate). Ask him/her to show you how the table can help them show the relationship between baking powder and salt.

NYC Grade 6 Assessment 1  
Sugar Cookies Task  
Annotated Student Work

**Annotation of Student Work With a Score of 1**

- a) 1 cup of sugar for  
each 2 eggs because  
 $2 \div 2 = 14 \div 2 = 8$
- b)  $1\frac{1}{2} + 1\frac{1}{2} + 1\frac{1}{2} = 4\frac{1}{2}$   
 $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$

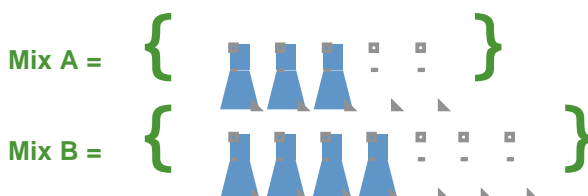
**Content Standards:** The student received a score of 1 because, although s/he correctly forms a ratio of cups of sugar to eggs and expresses it as a rate “1 cup of sugar for each 2 eggs”, s/he does not represent the unit rate (6.RP.2). The student uses informal language to express the rate but does not use standard ratio notation (6.RP.1). Although the student shows an understanding of what “3 times as much” means in part b), s/he does not seem to understand this to be a relationship between two quantities (6.RP.1).

**Mathematical Practices:** This student interprets the context and finds an entry point (Practice 1). The student also abstracts and manipulates the quantitative information from the problem and interprets the answer in terms of the context (Practice 2). The student provides a mathematical justification (division) for his/her procedure (Practice 3). The student understands the relationship between the division operation and the context (Practice 4) and labels the quantities appropriately in part a), though s/he fails to label them in part b) (Practice 6).

**Next Instructional Steps:** Ask the student to draw a diagram and use it to figure out how much sugar there will be for one egg. Ask the student how that can be expressed symbolically as a ratio. For part b), have the student draw diagrams to show different examples of “three times as much baking powder as salt” (without referring to the quantities in the table). See if the student can express these examples as ratios, and ask where the “three times as much” can be seen in the ratio. Ask the student whether the relationship between baking powder and salt given in the table fits this criterion.

## Assessment 1: Question 5

5. Fashion designers are trying to decide on just the right shade of blue for a new line of jeans. They have several bottles of fabric color, some with blue color and some with white color. They plan to mix these together to get the desired color.



- a) Will both mixes produce the same color jeans? Justify your reasoning.
- b) A designer uses the table below to think about her own special mix, Mix C. How many liters of blue color will she need to make a total of 40 liters? Explain your reasoning.

Liters of Blue Color	Liters of White Color	Total Liters
5	3	8
10	6	16

CCSS (Content) Addressed by this Task:

- 6.RP.1 Understand the concept of ratio and use ratio language to describe a ratio relationship between two quantities.
- 6.RP.2 Understand the concept of a unit rate  $a/b$  associated with a ratio  $a:b$  with  $b \neq 0$ , and use rate language in the context of a ratio relationship.
- 6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g. by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
- 6.RP.3a Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. (possible)
- 6.RP.3b Solve unit rate problems including those involving unit pricing and constant speed.
- 6.RP.3c Find a percent of a quantity as a rate per 100; solve problems involving finding the whole, given a part and the percent.

NYC Grade 6 Assessment 1  
Mixture Task  
Annotated Student Work

CCSS for Mathematical Practice Addressed by this Task:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with Mathematics (possible)
6. Attend to precision
7. Look for and make use of structure

NYC Grade 6 Assessment 1  
Mixture Task  
Annotated Student Work

Annotation of Student Work With a Score of 3

5 a)  $2:3 \times 4 = 8:12$

$3:4 \times 3 = 9:12$

Mix B has more white than Mix A

b)

Liters of Blue color	Liters of White color	Total Liters
5	3	8
10	6	16
15	9	24
20	12	32
25	15	40

25

When you use a graf if you continue the pattern the mix will remain the same except the total will get bigger


**Content Standards:** The student received a score of 3 because s/he represents the context using ratios (6.RP.1), correctly uses equivalent ratios with a common denominator to compare the ratio of blue to white dye in the two mixes and arrives at the correct conclusion (6.RP.3; 6.RP.3a). The student also extends the table of equivalent ratios, explaining his/her reasoning that “if you continue the pattern (sic) the mix will remain the same except the total will get bigger” (6.RP.3a).


**Mathematical Practices:** This student analyzes the problem and uses an appropriate strategy to answer part a) (Practice 1), abstracting the relevant quantities, working mathematically with those quantities and relating the results back to the context (Practice 2). The student constructs a viable argument, going beyond what was requested for part b) (Practice 3). The student does not label the quantities in the initial ratios (Practice 6); however, the explanation “mix B has more white than (sic) mix A” suggests that s/he keeps track of the meaning of the quantities in the ratios 8:12 and 9:12. The student can give a more complete explanation of the reasoning in part a) by stating that the amount of blue dye was the same (Practice 3). The student also fails to accurately express the “scaling up” of the ratios when s/he says simply “2:3 x 4” and “3:4 x 3” (Practice 6), although it is clear that she multiplied both numerator and denominator to arrive at the solution. The student notes and uses the structures of proportional reasoning, and can clearly use more than one strategy associated with those structures (Practice 7).

NYC Grade 6 Assessment 1  
Mixture Task  
Annotated Student Work

**Next Instructional Steps:** Ask the student to explain what s/he did to scale up the ratios and how the factors were chosen. Press the student to represent what s/he actually did mathematically and to explain what happens when you multiply a ratio by  $\frac{4}{4}$  or  $\frac{3}{3}$ . Ask the student to explain what s/he means by “Mix B has more white than Mix A”. Play devil’s advocate: “in the original picture, Mix B also has more white than Mix A. Why can’t you just answer the question from what is shown there?”

**Annotation of Student Work With a Score of 2**

Mix A = {  }  $\frac{3}{2}$

Mix B = {  }  $\frac{4}{3}$

a) No, because Mix A =  $\frac{3}{2}$  and Mix B =  $\frac{4}{3}$ .

b) I divided 40 and 8 = 5 and  $5 \times 5 = 25$ , because

Liters of Blue Color	Liters of White Color	Total Liters
5	3	8
10	6	16
25	15	40

**Content Standards:** The student received a score of 2 because the ratio of blue to white dye is correctly represented (6.RP.1). The student uses a process of scaling up in part b) to produce equivalent ratios to determine the amount of blue dye that would be in 40 liters (6.RP.3, 6.RP.3a) and explains how the factor of change was determined. However, in part a) the quantities are not labeled in the ratios, and the student shows no evidence of understanding that the task requires him/her to determine whether the ratios are equivalent (6.RP.3a).

**Mathematical Practices:** This student's work shows an effort to justify the responses through the consistent use of "because" (Practice 3). This student analyzes the problem and uses an appropriate strategy to answer part b) (Practice 1). The student does not label the quantities in the initial ratios (Practice 6). While the student recognizes the need to think proportionally in part b), s/he does not discern the need to do so in part a) (Practice 7).

**Next Instructional Steps:** Ask the student to explain how s/he knows that the two mixes do not produce the same color. If the response is "3/2 is not the same as 4/3" ask him/her how s/he knows this. Ask the student what they would have said if Mix B had been 6 parts blue to 4 parts white or 6/4". If the student argues that these are equivalent, ask him/her how to convince their neighbor that 3/2 and 4/3 are not equivalent.

NYC Grade 6 Assessment 1  
Mixture Task  
Annotated Student Work

**Annotation of Student Work With a Score of 1**

a) Yes. in both there is only one less in the whites: to the blues

b)

Liters of Blue Color	Liters of White Color	Total Liters
5	3	8
10	6	16
15	9	24
20	12	32
25	15	40

**Content Standards:** The student received a score of 1 because, although s/he correctly extends the table to find the missing value in part b) (6.RP.3a), the answer (25 liters) is not given and there is no explanation. In addition, the student uses an additive, rather than multiplicative, approach to compare the ratios in part a). The student also fails to provide the ratios that are being compared (6.RP.1) and fails to provide an explanation for part b).

**Mathematical Practices:** This student attempts to make sense of the problem (Practice 1) and produce a coherent explanation of his/her reasoning (Practice 3).

**Next Instructional Steps:** The student will need many opportunities to work with problems that involve scaling up (as in part b) using tables and diagrams where s/he will recognize that the total “mix” remains the same as the quantity grows. The student will need to explore patterns in the table, and to figure out why the mix still remains the same as it grows. As the student explores patterns in such tables, s/he can be asked to identify patterns that exist – both within columns and between columns.





# GRADE 6 MATH: RATIOS AND PROPORTIONAL RELATIONSHIPS

## INSTRUCTIONAL SUPPORTS

The instructional supports on the following pages include a unit outline with formative assessments and suggested learning activities. Teachers may use this unit outline as it is described, integrate parts of it into a currently existing curriculum unit, or use it as a model or checklist for a currently existing unit on a different topic.

In addition to the unit outline, these instructional materials include:

Three sequences of high-level instructional tasks (an arc), including detailed lesson guides for select tasks, which address the identified Common Core Standards for Mathematical Content and Common Core Learning Standards for Mathematical Practice. Each of the lessons includes a high-level instructional task designed to support students in preparation for the final assessment.

The lessons guides provide teachers with the mathematical goals of the lesson, as well as possible solution paths, errors and misconceptions, and pedagogical moves (e.g., promoting classroom discussion) to elevate the level of engagement and rigor of the tasks themselves. Teachers may choose to use them to support their planning and instruction.

## Unit Outline – 6<sup>th</sup> grade Math

**INTRODUCTION:** Teachers may (a) use this unit as it is described below; (b) integrate parts of it into a currently existing curriculum unit; or (c) use it as a model or checklist for a currently existing unit on a different topic or (d) Teach the unit as it is while including additional supplementary materials to support student understanding.

### Grade 6 Math: Ratios and Proportional Relationships

#### UNIT OVERVIEW

This 4-5 week unit focuses on developing an understanding of ratio concepts and using ratio reasoning to solve problems. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Students learn to use ratio and rate language to describe relationships; expand the scope of problems for which they can use multiplication and division; connect ratios and fractions; and solve a wide variety of problems involving ratios and rates.

#### COMMON CORE LEARNING STANDARDS:

Understand ratio concepts and use ratio reasoning to solve problems.

- **6.RP.1** Understand the concept of ratio and use ratio language to describe a ratio relationship between two quantities. *For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”*
- **6.RP.2** Understand the concept of a unit rate  $a/b$  associated with a ratio  $a:b$  with  $b \neq 0$ , and use rate language in the context of a ratio relationship. *For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is  $3/4$  cup of flour for each cup of sugar.” “We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger.”*
- **6.RP.3** Use ratio and rate reasoning to solve real-world and mathematical problems, i.e., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
  - a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
  - b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*
  - c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means  $30/100$  times the quantity); solve problems involving finding the whole, given a part

## Unit Outline – 6<sup>th</sup> grade Math

<p>and the percent.</p> <p><b>d.</b> Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.</p> <ul style="list-style-type: none"> <li>➤ <b>MP.1</b> Make sense of problems and persevere in solving them.</li> <li>➤ <b>MP.2</b> Reason abstractly and quantitatively.</li> <li>➤ <b>MP.3</b> Construct viable arguments and critique the reasoning of others.</li> <li>➤ <b>MP.4</b> Model with mathematics. Use appropriate tools strategically.</li> <li>➤ <b>MP.5</b> Use appropriate tools strategically.</li> <li>➤ <b>MP.6</b> Attend to precision.</li> <li>➤ <b>MP.7</b> Look for and make use of structure.</li> <li>➤ <b>MP.8</b> Look for and express regularity in repeated reasoning.</li> </ul>	
<p><b>BIG IDEAS/ENDURING UNDERSTANDINGS:</b></p> <ul style="list-style-type: none"> <li>➤ A ratio is a multiplicative comparison of two quantities, or it is a joining of two quantities in a composed unit.</li> <li>➤ Reasoning with ratios involves attending to and coordinating two quantities.</li> <li>➤ If one quantity in a ratio is multiplied or divided by a particular factor, then the other quantity must be multiplied by the same factor to maintain the proportional relationship.</li> <li>➤ Knowledge of equivalent fractions and equivalent ratios can be used to model ratio, rate and proportional reasoning grounded in sense making.</li> <li>➤ This knowledge can be generalized into algorithms for solving ratio, rate and proportion problems.</li> </ul>	<p><b>ESSENTIAL QUESTIONS:</b></p> <ul style="list-style-type: none"> <li>➤ How are ratios and rates related to fractions?</li> <li>➤ How can I use multiplication and division to solve ratio and rate problems?</li> <li>➤ What is the difference between a ratio and a rate?</li> <li>➤ How can I use tables of equivalent ratios, tape diagrams, double number line diagrams, or equations to compare rates of change?</li> <li>➤ What important elements do I need to know about proportional reasoning and when should I apply it?</li> </ul>
<p><b>CONTENT:</b></p> <p><b>Prior Knowledge:</b></p> <ul style="list-style-type: none"> <li>➤ Fluency with multiplication and division of whole numbers</li> <li>➤ Ability to generate factors and multiples</li> <li>➤ Understanding of fractions and fraction</li> </ul>	<p><b>SKILLS AND PRACTICES:</b></p> <p>The unit takes a holistic approach to the skills listed below and emphasizes the interdependence of the skills.</p> <ul style="list-style-type: none"> <li>➤ <b>Apply</b> multiplicative reasoning to</li> </ul>

## Unit Outline – 6<sup>th</sup> grade Math

<p style="text-align: center;">equivalence</p> <hr style="border-top: 1px dashed black;"/> <p><b>Ratio and Rate:</b></p> <ul style="list-style-type: none"> <li>➤ Whole number multiplication and division</li> <li>➤ “Scaling up” and “scaling down”</li> <li>➤ Rate</li> <li>➤ Unit Rate</li> <li>➤ Speed</li> <li>➤ Fractions</li> <li>➤ Equivalent fractions</li> <li>➤ Percent</li> <li>➤ Problem solving</li> <li>➤ Reasoning</li> <li>➤ Tools: <ul style="list-style-type: none"> <li><i>Multiplication tables</i></li> <li><i>Tables of equivalent ratios</i></li> <li><i>Tape diagrams</i></li> <li><i>Double number line diagrams</i></li> <li><i>Equations</i></li> </ul> </li> </ul> <hr style="border-top: 1px dashed black;"/> <p><b>Proportion:</b></p> <ul style="list-style-type: none"> <li>➤ Equivalent ratios</li> <li>➤ Equivalent fractions</li> <li>➤ Product of the means and extremes</li> <li>➤ Problem Solving</li> <li>➤ Reasoning</li> </ul>	<ul style="list-style-type: none"> <li>➤ explain the concept of ratio</li> <li>➤ <b>Use</b> ratio language to describe a ratio relationship between two quantities</li> <li>➤ <b>Understand</b> and <b>apply</b> the concept of a unit rate <math>a/b</math> associated with a ratio <math>a:b</math> with <math>b \neq 0</math></li> <li>➤ <b>Use</b> rate language in the context of a ratio relationship.</li> <li>➤ <b>Distinguish</b> the difference between rate and ratio</li> <li>➤ <b>Apply</b> concepts of rate and ratio to solve real world problems, using tables of equivalent ratios, tape diagrams, double number line diagrams, or equations</li> <li>➤ <b>Make</b> tables of equivalent ratios relating quantities with whole-number measurements</li> <li>➤ <b>Create</b> equivalent fractions, given a fraction</li> <li>➤ <b>Solve</b> unit rate problems, including those involving unit pricing and constant speed</li> <li>➤ <b>Express</b> ratios in different forms</li> </ul> <hr style="border-top: 1px dashed black;"/> <ul style="list-style-type: none"> <li>➤ Express equivalent ratios as a proportion</li> <li>➤ Solve proportions using equivalent fractions</li> <li>➤ Verify the proportionality of two ratios using the Cross Products Property (<math>a/b = c/d</math> is true if <math>ad = bc</math>, <math>d \neq 0</math>)</li> <li>➤ Read, write, and identify percents of a whole (0% to 100%)</li> <li>➤ Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity);</li> <li>➤ Solve problems involving percent, rate, and base</li> <li>➤ Use tables to compare ratios.</li> <li>➤ Use ratio reasoning to convert measurement units;</li> <li>➤ Manipulate and transform measurement units appropriately when multiplying or</li> </ul>
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## Unit Outline – 6<sup>th</sup> grade Math

<hr style="border-top: 1px dashed black;"/> <p><b>Coordinate Geometry and Algebra:</b></p> <ul style="list-style-type: none"> <li>➤ Coordinate plane</li> <li>➤ Ordered pairs</li> <li>➤ Plotting points</li> </ul>	<p style="text-align: center;">dividing quantities.</p> <hr style="border-top: 1px dashed black;"/> <ul style="list-style-type: none"> <li>➤ Find missing values in tables of equivalent ratios, and plot the pairs of values on the coordinate plane.</li> </ul> <p>In planning to address these skills and their interdependence, it is critically important also to consider the Standards for the Mathematical Practices, which require that the student:</p> <ul style="list-style-type: none"> <li>➤ Make sense of problems and persevere in solving them</li> <li>➤ Reason abstractly and quantitatively</li> <li>➤ Construct viable arguments and critique the reasoning of others.</li> <li>➤ Model with mathematics.</li> <li>➤ Use appropriate tools strategically</li> <li>➤ Attend to precision.</li> <li>➤ Look for and make use of structure</li> <li>➤ Look for and express regularity in repeated reasoning.</li> </ul> <p>Together, these skills and practices should enable a student to achieve the objective of the unit.</p>
<p><b>VOCABULARY/KEY TERMS:</b></p> <p>Ratio, rate, equivalent ratios, unit rate, per, percent, form, proportion, proportional relationships, scale, composed unit, unit price, constant speed, rate of change, multiplicative relationship, coordinate plane, x-axis, y-axis, origin, ordered pairs,</p>	

## Unit Outline – 6<sup>th</sup> grade Math

### ASSESSMENT EVIDENCE AND ACTIVITIES:

#### INITIAL ASSESSMENT : THE FRACTION TASK

The *initial assessment* also allows for what is sometimes called a *touchstone task*. The Fraction Task is rich enough as it can be solved from a variety of approaches, thereby allowing students to make sense of it in natural ways. As the unit progresses, students should be able to move to more efficient or grade-level appropriate strategies. As the students learn new ideas or procedures, students and the teacher can reflect upon how these new ideas and procedures might apply to the Fractions Task. *Please reference the Fractions Task for full task details.*

#### FORMATIVE ASSESSMENT: CATERPILLARS TASK

The purpose of formative assessment is to surface misconceptions and, through the course of the lessons, to provide ways for students to resolve these misconceptions and deepen their understanding. By surfacing misconceptions, the teacher is then able to make mid-unit corrections to instruction. Thus, students' experiences help to improve learning, rather than waiting until the final assessment to uncover problems or gaps in learning. Throughout this unit, periodic collection and analysis of work should yield a wealth of information teachers can use formatively.

#### FINAL PERFORMANCE TASK: 6<sup>TH</sup> GRADE PERFORMANCE ASSESSMENT 1

At the end of the unit the teacher should give the class The 6<sup>th</sup> Grade Performance Assessment 1 to see how students have improved their thinking and mathematical skills over the course of the instructional unit. This set of tasks assesses students' skills in, and knowledge of the big ideas skills and strategies described in this unit. *See assessment for full details.*

### LEARNING PLAN & ACTIVITIES:

Each *Arc of Lessons* and the student tasks included therein can be used in sequence for individual and group exploration and discussion; and for ongoing formative assessment. Each sequence of lessons here (an arc) includes specific student tasks. The tasks were designed to be cognitively demanding and to provide options in representation, action/expression, and engagement. See *The Mathematical Task Analysis Guide* and *Universal Design for Learning Principles* in this packet. These tasks are arranged in a particular order to support the development of the big ideas of the unit.

#### Arc 1 (Multiplicative Comparisons)

- Fractions Task
- Cookies Task
- Snakes Task\*
- Towns Task\*

#### Arc 2 (Rates and Part to Part Relationships)

- Caterpillar Task
- Dripping Faucet
- Worms Task\*

## Unit Outline – 6<sup>th</sup> grade Math

- Lemonade Mix Task\*

### Arc 3 (Rates and Part to Part and Part to Whole Relationships)

- Distance Task
- Mixing Paint Task\*
- Wheat Flour Task

Tasks marked with an asterisk (\*) include a detailed lesson guide, which can be used for ideas on how to implement these tasks in ways that deepen student understanding while building procedural competence. All the tasks are intended to be used as formative assessment tools.

*Please note:* None of these arcs directly address 6.RP.3.c: Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

### RESOURCES: RESOURCES:

#### SUPPORTING UNITS FOR STUDENTS

- *Activities for the Differentiated Classroom, Math Grades 6-8, Corwin (2008).*
- *Teaching Centered Mathematics Grades 5 - 8, Volume Three; Van de Walle, John & LouAnn H. Lovin; Pearson Education Inc. NY 2006*
- *Elementary and Middle School Mathematics: Teaching Developmentally; Van de Walle; Pearson Education Inc NY*
- The following Connected Math Project (CMP) units provide additional investigations and tasks to extend or enrich this unit:
  - *Bits and Pieces 1, 2, and 3 (Grade 6)*
  - *Prime Time (Grade 6)*




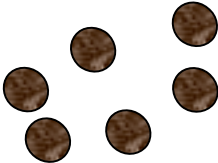
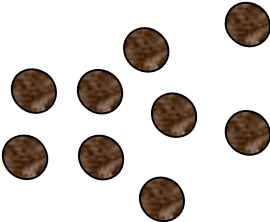
<http://connectedmath.msu.edu/mathcontent/contents.shtml>

- 6.RP.1 Understand the concept of ratio and use ratio language to describe a ratio relationship between two quantities. For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”
- 6.RP.2 Understand the concept of a unit rate  $a/b$  associated with a ratio  $a:b$  with  $b \neq 0$ , and use rate language in the context of a ratio relationship. For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is  $3/4$  cup of flour for each cup of sugar.” “We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger.”
- 6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, i.e., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
- Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
  - Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?
  - Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means  $30/100$  times the quantity); solve problems involving finding the whole, given a part and the percent.
  - Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

#### Essential Understandings of Ratios, Proportions & Proportional Reasoning (NCTM)

- Reasoning with ratios involves attending to and coordinating two quantities.
- A ratio is a multiplicative comparison of two quantities, or it is a joining of two quantities in a composed unit.
- Forming a ratio as a measure of a real-world attribute involves isolating that attribute from other attributes and understanding the effect of changing each quantity on the attribute of interest.
- A number of mathematical connections link ratios and fractions:
  - Ratios are often expressed in fraction notation, although ratios and fractions do not have identical meaning.
  - Ratios are often used to make “part-part” comparisons, but fractions are not.
  - Ratios and fractions can be thought of as overlapping sets.
  - Ratios can often be meaningfully reinterpreted as fractions.
- Ratios can be meaningfully reinterpreted as quotients.
- A proportion is a relationship of equality between two ratios. In a proportion, the ratio of two quantities remains constant as the corresponding values of the quantities change.
- Proportional reasoning is complex and involves understanding that:
  - Equivalent ratios can be created by iterating and/or partitioning a composed unit;
  - If one quantity in a ratio is multiplied or divided by a particular factor, then the other quantity must be multiplied or divided by the same factor to maintain the proportional relationship; and
  - The two types of ratios – composed units and multiplicative comparisons – are related.
- A rate is a set of infinitely many equivalent ratios.
- Several ways of reasoning, all grounded in sense making, can be generalized into algorithms for solving proportion problems.
- Superficial cues present in the context of a problem do not provide sufficient evidence of proportional relationships between quantities.




<p>Fraction Tasks</p> <p>6.RP.1                  Concept introduction.                  May use language.</p> <p>EU #2</p>	<p>Look at the picture below? Can you see <math>\frac{3}{5}</math>? What is the unit?</p>  <p>Can you see <math>\frac{2}{3}</math>? What is the unit? Can you see <math>\frac{5}{3}</math>?</p>  <p>What is the unit? Can you see <math>\frac{2}{3}</math> of <math>\frac{3}{5}</math>? What is the unit?</p>  <p style="text-align: right;">2002 NCTM Yearbook, pp. 103 – 104)</p>
<p>Cookie Task</p> <p>6.RP.1                  Concept.                  May use language.</p> <p>EU #1                  EU #2</p>	<p>Look at the number of cookies that Marcus has and the number of cookies that Nadia has.</p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">  <p>Marcus' cookies</p> </div> <div style="text-align: center;">  <p>Nadia's cookies</p> </div> </div> <ol style="list-style-type: none"> <li>1. How many times would you have to stack up Marcus' cookies to get a pile as high as Nadia's cookies?</li> <li>2. What fraction of a dozen cookies does Nadia have?</li> <li>3. There are 6 cookies in a package of cookies. What part of a package of cookies does Marcus have? Nadia have?</li> </ol> <p style="text-align: right;">Keeping the Problem Situation the Same, Varying the Types of Questions                  Adapted from Lamon, 1999</p>

<p>String Bean and Slim</p> <p>6.RP.1 Concept.</p> <p>6.RP.3 a or b., May use unit rate.</p> <p>EU #1 EU #2 EU #3 EU #4</p>	<p>Jo has two snakes, String Bean and Slim. Right now, String Bean is 3 feet long and Slim is 10 feet long. Jo knows that two years from now, String Bean will be about 8 feet long, while Slim's length will be about 15 feet.</p> <p>Over the next two years, will both snakes grow the same amount? Explain and justify your response.</p> <div style="text-align: center;"> <p>Now ...</p> <p>String Bean 3'      Slim 10'</p> <p>2 years from now ...</p> <p>String Bean 8'      Slim 15'</p> </div> <p>Arthur thinks each snake grew the same amount. Chenda agrees, but says she is thinking about the growth of the snakes in another way. In what other ways can we think about the growth of the snakes?</p> <p style="text-align: right;">Adapted from Lamon, J., (2008). <u>Teaching fractions and ratios for understanding</u>, pp.30-31. NY: Routledge.</p>
<p>Towns</p> <p>6.RP.1 Concept.</p> <p>6.RP.3 a or b., May use unit rate.</p> <p>EU #1 EU #2 EU #3 EU #4</p>	<p>In 1980 the populations of Towns A and B were 5000 and 6000, respectively. In 1990, the populations of Towns A and B were 8000 and 9000, respectively. Brian claims that from 1980 to 1990 the two towns' populations grew by the same amount. Use mathematics to explain how Brian might have justified his answer. Darlene claims that from 1980 to 1990 the population of Town A had grown more. Use mathematics to explain how Darlene might have justified her answer.</p> <p style="text-align: right;"><i>(released NAEP item #M069601)</i></p>

- 6.RP.1 Understand the concept of ratio and use ratio language to describe a ratio relationship between two quantities. For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”
- 6.RP.2 Understand the concept of a unit rate  $a/b$  associated with a ratio  $a:b$  with  $b \neq 0$ , and use rate language in the context of a ratio relationship. For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is  $3/4$  cup of flour for each cup of sugar.” “We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger.”
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- A rate is a set of infinitely many equivalent ratios.
- Several ways of reasoning, all grounded in sense making, can be generalized into algorithms for solving proportion problems.
- Superficial cues present in the context of a problem do not provide sufficient evidence of proportional relationships between quantities.

<p>Caterpillars and Leaves</p> <p>6.RP.1          Concept.          6.RP.2          May be used for unit rate.          6.RP.3          a or b.          May use unit rate.          EU #1</p>	<div style="text-align: center;"> <div style="border: 1px solid black; padding: 2px; display: inline-block;">CATERPILLARS AND LEAVES</div>  </div> <p style="text-align: center;">A fourth-grade class needs 5 leaves each day to feed its 2 caterpillars.          How many leaves would they need each day for 12 caterpillars?</p>
<p>Dripping Faucet</p> <p>6.RP.1          Concept,          Language          6.RP.2          6.RP.3          a or b, d          May use unit rate          EU #1, 2,          3, 4, 5, 8</p>	<p>A large bucket is used to collect water. Suppose that 4 ounces of water drip into the bucket every 2 minutes. How full will the bucket be at 4 minutes? 10 minutes? One hour?</p> <p>Suppose that 2 ounces of water drip every 5 minutes. How full will the bucket be at 4 minutes? 10 minutes? One hour?</p>
<p>Worms Task</p> <p>6.RP.1          Concept,          Language.</p> <p>6.RP.3          a or b.          May use unit rate.          EU #2, 5,          6, 8</p>	<p>Use number lines to answer the following question: Andy Worm travels 6 cm every 4 minutes. Betty Worm travels 15 cm every 10 minutes. Are Andy and Betty traveling at the same rate? If not, who is traveling "faster"? Explain how you know!</p>
<p>Lemonade Mix</p> <p>6.RP.1          Concept.          May use language.</p> <p>6.RP.3          a or b, d.          May use unit rate.          EU #1, 2,          3, 6, 7, 8</p>	<p>Solve this problem in at least three different ways: To make one glass of lemonade, use 3 tablespoons of lemonade mix and 6 oz. of water. How much lemonade mix do you need to make 2 quarts of lemonade?</p>

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  - a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
  - b. Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?
  - c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means  $30/100$  times the quantity); solve problems involving finding the whole, given a part and the percent.
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- 3. Forming a ratio as a measure of a real-world attribute involves isolating that attribute from other attributes and understanding the effect of changing each quantity on the attribute of interest.
- 4. A number of mathematical connections link ratios and fractions:
  - a. Ratios are often expressed in fraction notation, although ratios and fractions do not have identical meaning.
  - b. Ratios are often used to make “part-part” comparisons, but fractions are not.
  - c. Ratios and fractions can be thought of as overlapping sets.
  - d. Ratios can often be meaningfully reinterpreted as fractions.
- 5. Ratios can be meaningfully reinterpreted as quotients.
- 6. A proportion is a relationship of equality between two ratios. In a proportion, the ratio of two quantities remains constant as the corresponding values of the quantities change.
- 7. Proportional reasoning is complex and involves understanding that:
  - a. Equivalent ratios can be created by iterating and/or partitioning a composed unit;
  - b. If one quantity in a ratio is multiplied or divided by a particular factor, then the other quantity must be multiplied or divided by the same factor to maintain the proportional relationship; and
  - c. The two types of ratios – composed units and multiplicative comparisons – are related.
- 8. A rate is a set of infinitely many equivalent ratios.
- 9. Several ways of reasoning, all grounded in sense making, can be generalized into algorithms for solving proportion problems.
- 10. Superficial cues present in the context of a problem do not provide sufficient evidence of proportional relationships between quantities.

<p>Distance Tasks                  6.RP.1                  Concept, Rate, Language.                  6.RP.2 and 6.RP.3                  a or b.                  May use unit rate.                  EU #3, 4c, d, 7, 8, 9</p>	<p>Suppose a rabbit starts from home and, after 3 seconds, it is 12 cm from home. Generate several “distance from home” and elapsed time values for other parts of the rabbit’s journey so that the rabbit travels the same speed throughout its journey. Describe several different patterns that you see in the table. Write a rule or description that tells us how far from home the rabbit is for any number of seconds he travels.</p> <p>An automobile is traveling so that at the end of 5 hours it has gone 294 miles. Generate several “distance from home” and elapsed time values for other parts of the journey so that the automobile travels the same speed throughout its journey. Write a rule or description that tells us how far the automobile travels for any number of seconds it is on the road. An airplane will go 480 miles in 2 hours. Generate several “distance from home” and elapsed time values for other parts of the journey so that the airplane travels the same speed throughout its journey. Write a rule or description that tells us how far the airplane travels for any number of seconds it is in the air.</p>									
<p>Mixing Paint</p> <p>Tasks                  6.RP.1                  Concept, Language.                  6.RP.3                  a or b.                  May use unit rate.                  EU #4c, d, 6, 7, 8, 9</p>	<p>Mark was mixing blue paint and yellow paint in the ratio of 2:3 to make green paint. He wants to make 45 liters of green paint. He began to make a table to help him think about the problem, but is unsure of what to do next.</p> <table border="1" data-bbox="636 907 1161 1150" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>Liters of Blue Paint</th> <th>Liters of Yellow Paint</th> <th>Liters of Green Paint</th> </tr> </thead> <tbody> <tr> <td>2</td> <td>3</td> <td>5</td> </tr> <tr> <td>4</td> <td>6</td> <td>10</td> </tr> </tbody> </table> <p>a.) Explain how to continue to add values to the table.                  b.) Write an explanation to Mark about how he can use his table to find how many liters of blue paint and how many liters of yellow paint will he need to make 45 liters of green paint.</p>	Liters of Blue Paint	Liters of Yellow Paint	Liters of Green Paint	2	3	5	4	6	10
Liters of Blue Paint	Liters of Yellow Paint	Liters of Green Paint								
2	3	5								
4	6	10								
<p>Wheat Flour                  6.RP.1                  Concept.                  May use language.                  6.RP.2 and 6.RP.3                  a or b, d.                  May use unit rate.                  EU #4c, d, 5, 6, 7, 8, 9</p>	<p>Melissa bought 0.43 of a pound of wheat flour for which she paid \$0.86. How many pounds of flour could she buy for one dollar?</p> <p style="text-align: right;"><i>(based on Post et al., 1991)</i></p>									

## STRING BEAN AND SLIM TASK LESSON GUIDE

### LESSON OVERVIEW:

The String Bean and Slim Task asks students to determine whether or not two snakes grow the same amount. Students will have an opportunity to compare strategies that analyze the information using *absolute* differences (e.g., 5 feet) vs. *relative* differences (e.g., 5 feet relative to 8 feet vs. 5 feet relative to 15 feet). In addition, the open-ended nature of the task will allow students to utilize a variety of strategies and representations to help them make sense of these relative differences. A whole-class Share, Discuss and Analyze Phase provides an opportunity for these strategies to be highlighted, analyzed, and connections among strategies to be made. Students will recognize that determining relative differences allows them to coordinate two quantities within the context at the same time.

This task highlights the difference between additive change and multiplicative change (or *absolute* growth vs. *relative* growth). From an additive perspective, since 5 feet is added to the length of both snakes (*absolute* growth which is independent of, and unrelated to, anything else), the snakes will grow the same amount. However, from a comparative or *relative* perspective, the snakes will not grow the same amount. Several types of ratios can be formed in order to reason about the growth of each snake relative to its original or final length and compare their growth.

The task is appropriate for several grade levels; however the possible solutions described in the Lesson Guide may not be constructed at all grade levels.

### COMMON CORE STATE STANDARDS:

- **6.RP.1** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.
- **6.RP.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
  - c) Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity).
- **7.RP.3** Use proportional relationships to solve multistep ratio and percent problems. *Example: percent increase.*

### DRIVING QUESTION:

- What information does comparing by ratio give us that comparing by subtraction does not (absolute difference versus relative difference)?

### NCTM ESSENTIAL UNDERSTANDINGS<sup>1</sup>:

1. Reasoning with ratios involves attending to and coordinating two quantities.
2. A ratio is a multiplicative comparison of two quantities, or it is a joining of two quantities in a composed unit.
3. Ratios can be meaningfully reinterpreted as quotients.
7. Proportional reasoning is complex and involves understanding that: if one quantity in a ratio is multiplied or divided by a particular factor, then the other quantity must be multiplied or divided by the same factor to maintain the proportional relationship.

### SKILLS DEVELOPED:

Students will be able to interpret the meaning of different types of ratios that can be formed given a particular situation and use the ratios to make comparisons and reason about the situation.

### MATERIALS:

String Bean and Slim task sheet, calculators.  
Either a document reader, overhead transparencies, or chart paper.

### GROUPING:

Students will begin their work individually, but will then work in pairs, trios, or groups of four to discuss the task and arrive at a common solution.

<sup>1</sup> NCTM (2010) Developing Essential Understandings of Ratios, Proportions & Proportional Reasoning: Grades 6 - 8.  
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**SET-UP****Instructions to Students:**

Ask a student to read the task while others follow along. Ask students: “What do we know?” “What do we want to find out?” Tell students that there are many ways to reason about the question.

Remind students that they will be expected to: justify their solutions; explain their thinking and reasoning to others; make sense of other students’ explanations; ask questions of the teacher or other students when they do not understand; and use correct mathematical language and symbols.

**EXPLORE PHASE: Supporting Students’ Exploration of the Mathematical Ideas**

**Private Think Time:** Allow students to work individually for 3 – 5 minutes without intervening, though you might want to circulate quickly to get an idea of the strategies that they are using.

**Small-Group Work:** After 3 – 5 minutes, ask students to work with their partner or in their small groups. As students are working, circulate around the room. Be persistent in:

- asking questions related to the mathematical ideas, problem-solving strategies, and connections between representations.
- asking students to explain their thinking and reasoning.
- asking students to explain in their own words, and build onto, what other students have said.

As you circulate, identify solution paths that you will have groups share during the Share, Discuss, Analyze Phase, and decide on the sequence that you would like for them to be shared. Give groups a “heads up” that you will be asking them to come to the front of the room. If a document reader is not available, give selected groups an OVH transparency or chart paper to write their solution on.

**Possible Solution Paths**

If a group is unable to start:

Press students to consider whether the original lengths, or final lengths, also matter.

1. *They both grew the same amount because they both grew five feet.*

**Possible Assessing and Advancing Questions**

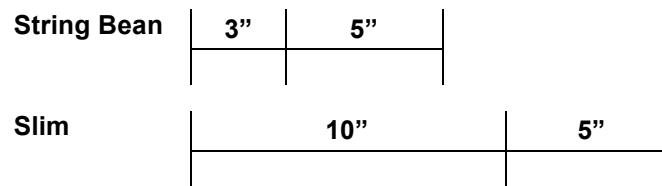
Assessing Question

- *Tell me about your work. Explain your thinking.*

Advancing Questions

- *Can you think about other ways to look at their growth, as Chenda did? What are some strategies for examining problems that we have used in the past?*
- *Can you draw a picture to help you think of other ways like Chenda did?*

**Informal comparisons of the growth to the initial or final lengths looking at the numbers or by a drawing or diagram.**



*String Bean and Slim both grew 5 feet but 5 feet is a bigger part of String Bean’s length (3) than Slim’s length (10) so String Bean grew more*

Assessing Questions

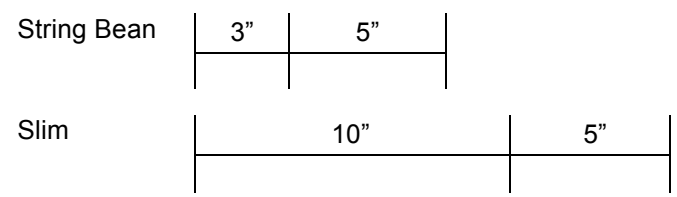
- *Tell me about your work.*
- *What does it tell you when you compare 5 feet with 8 feet; and 5 feet with 15 feet?*
- *How are you comparing 5 feet with 8 feet; and 5 feet with 15 feet?*

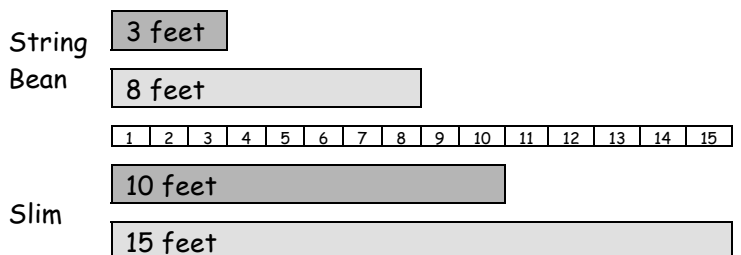
Advancing Questions

- *What do you mean when you say that 5 feet is a bigger part of 8 feet than 15 feet? How do you know that it’s a bigger part? Be prepared to show me how you made your decisions.*
- *Can you finish your statement? You said, “String Bean*



<p>compared to her original length.</p>	<p>and Slim both grew 5 feet, but 5 feet is a bigger part of String Bean's length (8) than Slim's length (15) so String Bean grew more." Say more, BECAUSE.....can you be prepared to share your reasoning?</p>	
<p>Using ratios to compare amount of growth to the initial or final growths, and comparing them numerically, either as ratios or percents. There are three different ratios that can be formed:</p>	<p>Assessing Questions</p> <ul style="list-style-type: none"> <li>• Tell me about your work. Explain your thinking.</li> <li>• What are the quantities you are using to form your ratio? How does that help you think about which snake grew more?</li> <li>• How do you know that <math>\frac{5}{3}</math> (or <math>\frac{5}{8}</math>) is bigger than <math>\frac{5}{10}</math> (or <math>\frac{5}{15}</math>)? (One possible answer: there are the same number of parts but the parts are larger.)</li> <li>• What do you mean, "She grew a bigger amount of her size?" Why does her original size make a difference?</li> </ul> <p>Advancing Questions</p> <ul style="list-style-type: none"> <li>• It looks like you are comparing her growth to her original size (growth to final size, original size to final size). You are looking at her 'relative' growth. Would your response change if you compared another relative growth, her growth to her final size (original size to final size, growth to original size)? Why or why not?</li> </ul>	
<p><b>amount of growth</b> original</p>		<ul style="list-style-type: none"> <li>• String Bean grew 5 feet which was <math>\frac{5}{3}</math> of her initial length; Slim grew <math>\frac{5}{10}</math> of his initial length. String Bean's growth relative to starting length is greater than Slim's.</li> </ul>
<p><b>amount of growth</b> final</p>		<ul style="list-style-type: none"> <li>• String Bean grew 5 feet which was <math>\frac{5}{8}</math> of her final length, Slim grew 5 feet which was <math>\frac{5}{15}</math> of his final length. String Bean's growth relative to her final length was greater than Slim's, so String Bean grew more.</li> </ul>
<p><b>original length</b> final</p>		<ul style="list-style-type: none"> <li>• String Bean was only <math>\frac{3}{8}</math> of her final length at the beginning but Slim was already <math>\frac{10}{15}</math> of his final length. So, String Bean had to grow more in the 2 years to reach 100%.</li> </ul>
<p>Analyzing the problem in terms of factor of change – similar to the previous solution, but looking at the ratio as an operator:</p> <ul style="list-style-type: none"> <li>• String Bean's length increased <math>166\frac{2}{3}\%</math>, while Slim's length increased only 50%.</li> </ul>	<p>Assessing Questions</p> <ul style="list-style-type: none"> <li>• Tell me about your work. Explain your thinking.</li> <li>• What are the quantities you are using to form your percent? How does that help you think about growth of the snakes?</li> <li>• Why does the snakes' original size make a difference?</li> </ul> <p>Advancing Questions</p> <ul style="list-style-type: none"> <li>• It looks like you are comparing the growth of a snake to the original size (growth to final size, original size to final size). You are looking at 'relative' growth. Would your response change if you compared another relative growth, growth to final size (original size to final size, growth to original size)? Why or why not?</li> <li>• So, if String Bean had started out at 8 feet, and grew at this rate, your <math>166\frac{2}{3}\%</math>, how long would she be in 2 years? How long would Slim be if he started out at 16 feet and grew at this rate?</li> </ul>	

Possible Errors and Misconceptions	Possible Questions to Address Errors and Misconceptions
<p>Comparing numbers without attending to their meaning, thus not knowing how to interpret the result, e.g., <math>3/8</math> or <math>37.5\%</math> is smaller than <math>10/15</math> or <math>66\ 2/3\%</math>. So, <i>Slim grew more.</i></p>	<p>Assessing Questions</p> <ul style="list-style-type: none"> <li>• Explain what you have done.</li> <li>• What do the numbers mean in the context?</li> <li>• What does <math>3/8</math> tell us in terms of the context? What does <math>10/15</math> tell us in terms of the context? (If necessary: What does the 3 mean? The 8?)</li> </ul> <p>Advancing Questions</p> <ul style="list-style-type: none"> <li>• What is the relationship between the context and the numbers you wrote? Between the lengths of the snakes and the numbers you wrote?</li> <li>• What does it mean, in terms of the problem, for <math>3/8</math> to be smaller than <math>10/15</math>?</li> <li>• You say <math>37.5\%</math> (or <math>66\ 2/3\%</math>) – what is the whole? What is the part? What does the percent mean in terms of the problem?</li> </ul>
<b>SHARE DISCUSS ANALYZE PHASE: Deepening and Connecting Mathematical Understanding</b>	
<p><b>General Considerations:</b></p> <ul style="list-style-type: none"> <li>• Orchestrate the class discussion so that it builds on, extends, and connects the thinking and reasoning of students</li> <li>• Sequence the solution paths so that you will be able to press students to make comparisons and connections across, and between, the various solution paths.</li> </ul>	
Possible Sequence of Solution Paths	Possible Questions and Possible Student Responses
<p>Have a group share an <b>explanation or diagram that compares the amount of growth to either the initial, or final, length.</b></p> <div style="margin-left: 40px;">  </div> <ul style="list-style-type: none"> <li>• <i>String Bean and Slim both grew 5 feet but 5 feet is a bigger part of String Bean's length (3) than Slim's length (10) so String Bean grew more.</i></li> </ul>	<p>Explain your group's solution.</p> <ul style="list-style-type: none"> <li>• <i>We saw that they both grew 5 feet, but we thought that String Bean grew in a different way from Slim because she grew more of her length than Slim did. She started out 3 feet and grew to almost three times her whole length, but Slim just grew half of his length.</i></li> </ul> <p>How is this solution strategy different than Arthur's?</p> <ul style="list-style-type: none"> <li>• <i>We thought it was important to compare how much they grew to how long they were in the beginning, but Arthur just looked at how long they grew.</i></li> </ul> <p>OK, this is a kind of 'relative change' (<i>writing on chart paper</i>) when we compare the change in something to, in this case, its original size. Since we are talking about the growth of the snakes, it is the relative growth. How does it change things when we look at where they started?</p> <ul style="list-style-type: none"> <li>• <i>It's like, if I had \$5 dollars and then I had \$10 I would have twice as much, but if I had \$1000 and then I had \$1005, I would hardly notice the difference. So, how much you start with makes a difference in how much of a difference the change makes.</i></li> </ul>

	<p>How did the diagram help you think about it?</p> <ul style="list-style-type: none"> <li>The diagram helped us to show how even though the 5-foot lengths were the same, you can see that String Bean got more than twice as long, but Slim didn't.</li> </ul>
<p>Have a group share a <b>strategy that uses mathematical processes to compare the amount of growth with either the beginning or ending length.</b> If both comparisons were made, it would be important to discuss them both.</p>	<p>How did your group solve the problem?</p> <ul style="list-style-type: none"> <li>We saw that String Bean grew 5 feet, but she started at 3 feet, so she grew <math>\frac{5}{8}</math> of her original length. Slim grew <math>\frac{5}{15}</math> of his original length. We know that <math>\frac{5}{8}</math> is bigger than <math>\frac{5}{15}</math>, so we know String Bean grew more.</li> </ul> <p>How does this method compare to the solution we just saw?</p> <ul style="list-style-type: none"> <li>It's almost the same because we were comparing how much they grew with how long they were to start with, but we used ratios and then compared the ratios.</li> </ul> <p>So, you also looked at relative change. I saw a group that also used ratios. They made the ratios <math>\frac{3}{8}</math> (37.5%) and <math>\frac{10}{15}</math> (66 <math>\frac{2}{3}</math>%) and then said that <math>\frac{3}{8}</math> was less. So, they said that Slim grew more. Do you agree or disagree?</p> <ul style="list-style-type: none"> <li>I see what they did, but I disagree. They compared their "now" length with their "then" length. So now, Slim is a bigger ratio of his "then" length than String Bean is. So, he grew less 'cause he started more.</li> </ul> <p>I'm not convinced. Could a diagram help you to convince me that this is wrong?</p> <ul style="list-style-type: none"> <li>Let me try. Slim started at 10 feet and ended at 15 feet so, if we compare the start to the end, String Bean starts at a smaller part than Slim. See here, String Bean starts out at about a half of where she ends up, but Slim starts out at way over half. So, String Bean's ratio is smaller, because she grows more.</li> </ul>  <p>String Bean: 3 feet to 8 feet</p> <p>Slim: 10 feet to 15 feet</p>
<p>Have a group that thought of the <b>ratio as percent increase share their solution.</b></p>	<p>How did your group solve the problem?</p> <ul style="list-style-type: none"> <li>We did the same thing as the other group but we turned our ratios into percents. We saw that String Bean grew 166 <math>\frac{2}{3}</math>% of her length while Slim only grew 50%. So, String Bean grew more.</li> </ul> <p>How does thinking of percent increase help us to reason about the problem?</p> <ul style="list-style-type: none"> <li>Percent increase lets us think about how the amount they grew compared to the whole for both of them, even though the wholes are different. So, we can just compare the percents, without having to think about how the fractions compare. The percent does that for us.</li> </ul>

	<p>If String Bean's growth is proportional to Slim's, how much will String Bean grow in 2 years?"</p> <ul style="list-style-type: none"> <li>• <i>String Bean would grow 50% of 3 or 1.5 feet, so she'd only be 4.5 feet long.</i></li> </ul> <p>Or, if Slim's growth is proportional to String Bean's how much will Slim grow in 2 years?</p> <ul style="list-style-type: none"> <li>• <i>Slim would grow 166 2/3% of 10 or 16 2/3 feet, so Slim would be 26 2/3 feet long. This makes it easy to see that String Bean grew more than Slim.</i></li> </ul>
<p>Compare the <b>four possible ratios</b>.</p>	<p>Different groups used different ratio comparisons – <math>5/3</math> vs <math>5/10</math>; <math>5/8</math> vs <math>3/15</math>; and <math>3/8</math> vs <math>10/15</math>. How can you decide what each comparison tells you?</p> <ul style="list-style-type: none"> <li>• <i>You have to always think about what the numbers mean. That helps you.</i></li> </ul> <p>Sometimes we saw that the biggest ratio went with the most growth, and sometimes the smallest ratio went with the most growth. How can this be?</p> <ul style="list-style-type: none"> <li>• <i>It all depends on what you're comparing. Since 5 was the amount they grew, when we compared the 5 with the original or final length, the biggest ratio was the biggest growth. But when you compared their original length to their final length, it was different. The closer the original was to the final length, the less they grew, so it is opposite.</i></li> </ul>
<p>Ask students to consider the <b>difference between additive reasoning and multiplicative reasoning</b>.</p>	<p>So, what do you think? Was Chenda right when she said there are other ways to think about how the snakes have grown?</p> <ul style="list-style-type: none"> <li>• <i>Yes. Even though they both grew 5 feet, so they grew the same amount, it tells you a lot more when you compare how much they grew to how much they started out. String Bean more than doubled while Slim just changed a little bit of his whole length (say, if Slim had started out at 20 feet). So, it matters.</i></li> </ul>

<p><b>CLOSURE</b></p>
<p><b>Quick Write:</b></p> <ul style="list-style-type: none"> <li>• When something is changing, or growing larger, what information can we get from forming a ratio to describe the change?</li> </ul>
<p><b>Possible Assessment:</b></p> <ul style="list-style-type: none"> <li>• Provide some additional contexts where they need to compare quantities. Ask them to explain their thinking in writing.</li> </ul>
<p><b>Homework:</b></p> <ul style="list-style-type: none"> <li>• Find items from the current curriculum that will allow them to apply these ideas and understandings.</li> </ul>

## TOWNS TASK SIXTH GRADE LESSON GUIDE

### LESSON OVERVIEW:

The Towns task asks students to compare strategies that analyze the information using absolute differences (e.g., a growth of 3000 people) vs. relative differences (e.g., 5000 people relative to 8000 people vs. 6000 to relative to 9000 people). In addition, the open-ended nature of the task will allow students to utilize a variety of strategies and representations to help them make sense of relative differences. A whole-class Share, Discuss and Analyze Phase provides an opportunity for these strategies to be highlighted, and a comparison between additive and relative approaches to be made. Building on what they learned from the String Bean and Slim Task, students will recognize that although an additive approach is one way to analyze the situation, determining relative differences allows them to coordinate two quantities within the context at the same time.

This task highlights the difference between additive change and multiplicative change (or absolute growth vs. relative growth). From an additive perspective, since 3000 people is added to the population of both towns (absolute growth which is independent of, and unrelated to, anything else), the towns will grow the same amount. However, from a comparative or relative perspective, the populations will grow the same amount. Several types of ratios can be formed in order to reason about the growth of each population relative to their initial or final population compare to their growth.

The task is appropriate for several grade levels; however the possible solutions described in the Lesson Guide may not be constructed at all grade levels.

### COMMON CORE STATE STANDARDS:

- **6.RP.1** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.
- **6.RP.2** Understand the concept of a unit rate  $a/b$  associated with a ratio  $a:b$  with  $b \neq 0$ , and use rate language in the context of a ratio relationship.
- **6.RP.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
  - a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
  - b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*
  - c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
  - d. Use ratio reasoning to convert measurement units to manipulate and transform units appropriately when multiplying or dividing quantities.

<p><b>DRIVING QUESTION:</b></p> <ul style="list-style-type: none"> <li>• What information does comparing by ratio give us that comparing by subtraction does not (absolute difference between versus relative difference)?</li> </ul>	<p><b>NCTM ESSENTIAL UNDERSTANDINGS<sup>1</sup>:</b></p> <ol style="list-style-type: none"> <li>1. Reasoning with ratios involves attending to and coordinating two quantities.</li> <li>2. A ratio is a multiplicative comparison of two quantities, or it is joining of two quantities in a composed unit.</li> <li>3. Ratios can be meaningfully reinterpreted as quotients.</li> <li>4. Proportional reasoning is complex and involves understanding that: if one quantity in a ratio is multiplied or divided by a particular factor, then the other quantity must be multiplied or divided by the same factor to maintain the proportional relationship.</li> </ol>	<p><b>SKILLS DEVELOPED:</b></p> <p>Students will be able to interpret the meaning of different types of ratios that can be formed given a particular situation and use the ratios to make comparisons and reason about the situation.</p>
<p><b>MATERIALS:</b> The Towns task sheet, calculators, either a document reader, overhead transparencies, or chart paper and markers</p>	<p><b>GROUPING:</b> Students will begin their work individually, but will then work in pairs, trios, or groups of four to discuss the task and arrive at a common solution.</p>	

<p><b>LALAINA</b></p>
<p><b>SET-UP</b></p>
<p><b>Instructions to Students:</b></p> <p>Ask a student to read the task while others follow along. Ask students: “What do we know?” “What do we want to find out?” Tell students that there are many ways to reason about the question.</p> <p>Remind students that they will be expected to: justify their solutions; explain their thinking and reasoning to others; make sense of other students’ explanations; ask questions of the teacher or other students when they do not understand; and use correct mathematical language and symbols.</p>
<p><b>EXPLORE PHASE: Supporting Students’ Exploration of the Mathematical Ideas</b></p>
<p><b>Private Think Time:</b> Allow students to work individually for 3-5 minutes without intervening, though you might want to circulate quickly to get an idea of the strategies that they are using.</p> <p><b>Small-Group Work:</b> After 3-5 minutes, ask students to work with their partner or in their small groups. As students are working, circulate around the room. Be persistent in:</p> <ul style="list-style-type: none"> <li>• asking questions related to the mathematical ideas, problem-solving strategies, and connections between representations.</li> <li>• asking students to explain their thinking and reasoning.</li> <li>• asking students to explain in their own words, and build onto, what other students have said.</li> </ul>

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As you circulate, identify solution paths that you will have groups share during the Share, Discuss, Analyze Phase, and decide on the sequence that you would like for them to be shared. Give groups a “heads up” that you will be asking them to come to the front of the room. If a document reader is not available, give selected groups an OVH transparency or chart paper to write their solution.

Possible Solution Paths	Possible Assessing and Advancing Questions				
<p>If a group is unable to start:</p> <p><i>TOWN A – POPULATION CHART</i></p> <p><i>Town A population in 1980 – 5,000</i></p> <p><i>Town A population in 1990 - 8,000</i></p> <p><i>TOWN B – POPULATION CHART</i></p> <p><i>Town B population in 1980 – 6,000</i></p> <p><i>Town B population in 1990 – 9,000</i></p>	<p>Assessing Questions</p> <ul style="list-style-type: none"> <li>• What do you think that Brian was focusing on when he said the two towns grew the same?</li> <li>• What do you think that Darlene was focusing on when she said that town A grew more?</li> </ul> <p>If students are still stuck:</p> <ul style="list-style-type: none"> <li>• What was the population of people for Town A in 1980?</li> <li>• What was the population of people for Town A in 1990?</li> <li>• What do you notice about the amount the population grew compared to the original population of the town?</li> </ul> <p>Advancing Questions</p> <ul style="list-style-type: none"> <li>• How does this task relate to other tasks we have done? [If additional prodding is needed: “When we solved the String Bean and Slim Task, we saw there was more than one way to think of growth. Try those strategies here. I’ll be back.”]</li> </ul>				
<p>Brian - possible strategy: Additive approach using subtraction</p> <table border="0" style="width: 100%;"> <tr> <td style="width: 50%;">1980 - Town A 5,000</td> <td style="width: 50%;">1990 Town A 8,000</td> </tr> <tr> <td style="padding-left: 40px;">Town B 6,000</td> <td style="padding-left: 40px;">1990 Town B 9,000</td> </tr> </table> <p>8,000 (1990) - 5,000 (1980) = 3,000 Difference in population for Town A</p> <p>9,000 (1990) - 6,000 (1980) = 3000 Difference in population for Town B</p> <p>Same number of people in the population growth from 1980 to 1990.</p>	1980 - Town A 5,000	1990 Town A 8,000	Town B 6,000	1990 Town B 9,000	<p>Assessing Questions</p> <ul style="list-style-type: none"> <li>• What was the population change from 1980 to 1990 for Town A?</li> <li>• What was the population change from 1980 to 1990 for Town B?</li> </ul> <p>Advancing Questions</p> <ul style="list-style-type: none"> <li>• Demonstrate your solutions using tables, charts, diagrams or calculations.</li> </ul>
1980 - Town A 5,000	1990 Town A 8,000				
Town B 6,000	1990 Town B 9,000				

Brian – possible strategy: Additive approach using the missing addend, and equations.

$$\begin{array}{r} 5,000 + X = 8,000 \\ \underline{-5,000} \quad \underline{-5,000} \end{array}$$

$X = 3,000$  Town A population growth of people from 1980 to 1990

$$\begin{array}{r} 6,000 + X = 9,000 \\ \underline{-6,000} \quad \underline{-6,000} \end{array}$$

$X = 3,000$  Town B population growth of people from 1980 to 1990

Darlene – possible strategy: expressing starting population as % of final population

Town A's population growth  $5/8 = .625 =$  Town A starts out at 62.5 % of its final population.

Town B's Population growth  $6/9 = .66 =$  Town B starts out at 66% of its final population.

To get to 100%

Town A will grow 37.5 % and

Town B will grow 33.33 %

Therefore, Town A grows at a faster rate.

#### Assessing Questions

- What does your equation represent in terms of the problem?

#### Advancing Questions

- Which method of finding the difference would you prefer and why? Support your answer.

#### Assessing Questions

- What do Darlene's ratios represent in terms of the problem?
- When Darlene compares the ratios to 100%, what does this mean in terms of the problem?
- What do you notice about the ratio for Town A compared to the ratio of Town B? What does that tell us?

#### Advancing Questions

- What is Darlene actually comparing when she is determining the population growth?
- How is Darlene's method different from Brian's method? Explain your answer.
- Why does Darlene's method give a different answer than Brian's method? Explain your answer.



Possible Errors and Misconceptions	Possible Questions to Address Errors and Misconceptions
<p>The meaning of certain words might lead to misconception such as the word “respectively”. Many students might have difficulty understanding that it relates to the populations of town A and town B.</p> <p>Comparing numbers without attending to their meaning, thus not knowing how to interpret the result.</p> <p>For example, the growth of the population of town A (62.5 %) is smaller than the population of town B (66%).</p>	<p>Assessing Questions</p> <ul style="list-style-type: none"> <li>• Explain what you have done.</li> <li>• How did you arrive at your conclusion?</li> <li>• Explain the meaning of the numbers in context.</li> <li>• What does <math>\frac{5}{8}</math> tell us about the context?</li> <li>• What does the 5 mean? The 8?</li> </ul> <p>Advancing Questions</p> <ul style="list-style-type: none"> <li>• Can you draw a diagram or number line to represent Town A and Town B, and how they each grew?</li> <li>• Is there any way that your diagram/number line might show that Town A grew more than Town B?</li> <li>• Can you apply that reasoning to Town A and Town B?</li> </ul>
<b>SHARE DISCUSS ANALYZE PHASE: Deepening and Connecting Mathematical Understanding</b>	
<p><b>General Considerations:</b></p> <ul style="list-style-type: none"> <li>• Orchestrate the whole-class discussion so that it builds on, extends, and connects the thinking and reasoning of students.</li> <li>• Sequence the solution paths so that you will be able to press students to make comparisons and connections across, and between, the various solution paths.</li> <li>• The focus of the discussion during the SDA phase should be comparing the absolute and relative approach.</li> </ul>	

Possible Sequence of Solution Paths	Possible Questions and <i>Possible Student Responses</i>
	<ul style="list-style-type: none"> <li>• Why did Brian and Darlene reach different conclusions?</li> <li>• What does Darlene's answer tell us that Brian's answer doesn't tell us?</li> <li>• What are some times in everyday life when we make <i>absolute</i> comparisons?</li> <li>• What are some times in everyday life when we make <i>relative</i> comparisons?</li> </ul>
<p style="text-align: center;">             Town A 1980 – 5000      Town B 1980 – 6000              1990 – 8000              1990 – 9000  <span style="margin-left: 100px;">3000</span>                      <span style="margin-left: 100px;">3000</span> </p> <p><i>If you find the difference in the starting figures and the ending totals, you will find that they are equal. Therefore, both towns grew by the same rate — 3000.</i></p>	<p>Explain your group's solution.</p> <ul style="list-style-type: none"> <li>• We saw that in 10 years, both towns had an increase in population of 3000 people. This gave the impression that the population growth was the same for both cities.</li> <li>• <i>Brian's solution supports this since the increase in population for both cities is equivalent. However, Brian does not take into consideration the starting figures, which differ by 1000.</i></li> <li>• If two towns have an increase in population that is equal, but their starting figures differ, can their growth rate be the same?</li> </ul>
<p>Town A <math>5000/8000 = 5/8 = .625 = 62.5\%</math>              Town B <math>6000/9000 = 6/9 = .66 = 66\%</math></p> <p>To get to 100% Town A will grow at a rate of 37.5% and              Town B will grow at a rate of 33.3%</p> <p><i>This method shows how you can use percents to compare growth rates.</i></p>	<ul style="list-style-type: none"> <li>• Another group compares the two totals by first creating ratios in the form of fractions and simplifying them to lowest terms. The fractions are then converted first to decimals, and then into percents where a comparison is then made. The outcome is that Town A has grown at a faster rate.</li> </ul> <p><i>This solution shows different ways that the growth rate of the cities can be expressed. They chose to use percents, which shows that Town A did indeed grow more. Actually, both towns did have a growth of 3000 people, so both groups would be able to provide support for their answers.</i></p>
<p>Town A <math>5000/8000 = 5/8</math>              Town B <math>6000/9000 = 6/9</math></p> <p style="text-align: center;"><math>5/8 &lt; 6/9</math></p> <p><i>Town B grew more because if you create equivalent fractions using a common denominator of 72, the numerator for 5/8 will be 45, and for 6/9 it will be 48. Since 48 is more than 45, Town B has grown more.</i></p>	<ul style="list-style-type: none"> <li>• Another group also created ratios in the form of fractions, but left them as fractions and compared their values by creating equivalent fractions with the same denominators.</li> </ul> <p><i>If you compare the fractions created with ratios comparing the individual growth of the cities, what values are you actually comparing?</i></p> <p style="text-align: center;"><i>We thought that since fractions can be compared, that we could compare the fractions and determine from whichever was the greater fraction, which city had the most growth.</i></p> <p><i>This group did not realize that the values they were comparing were not the rate of growth, but the actual totals of the population over the 10-year period.</i></p>

## CLOSURE

### Quick Write:

- Take a moment to write in your math journal and compare Brian's and Darlene's approaches – give similarities and differences.
- (Extending) explain how Darlene's approach provides more information about the towns being compared than Brian's approach.

### Possible Assessment:

- Collect student work on the problem, collect group work on the problem, collect Quick Write, review video of class share if taped.

### Homework:

- Assign a similar problem ...

Last year at MS345 the attendance at the school dance was 100 out of the 200 students at the school. This year, 150 students attended the MS345 dance.

Last year at MS789 the attendance at the school dance was 200 students out of the 400 students at the school. This year, 250 students attended the MS789 dance.

Joelle, a student at MS345 is having an argument with her friend Samantha. Samantha attends MS789. Joelle says her school has more school spirit than MS789. What mathematical reasoning might she use to make this argument?

Ask students to review notes from class since the start of the ARC of lessons. Prepare to have one page of your notebook shared as part of a gallery walk. Students should be prepared to share why this page of notes is a good resource in their notebook, and what concept it helped them to better understand and hand out.

## MIXING JUICE TASK LESSON GUIDE

### LESSON OVERVIEW

In the *Mixing Juice* task, students encounter an open-ended problem where they are asked to compare the “orangeyness” of four drink mixes. Students will likely approach the task using a range of different strategies, making comparisons among the mixes with ratios, percents, and fractions. Students will investigate how ratios can be formed and scaled up to find equivalent ratios. In addition, students will use proportional reasoning to decide how to use the different mixes to make juice for 240 people.

The strategies for comparing the mixes will be compared and connected during the whole-group discussion. Students should be able to see how each form ratios, percents, and fractions, and provides information needed to derive one of the other forms.

Before working on the *Mixing Juice* task, the Warm-Up task *Comparing by Using Ratios* will focus students’ attention on different ways to form ratios and different notations for ratios. Allow 15 to 20 minutes for students to engage in and discuss the Warm-Up task. The *Mixing Juice* task is a challenging problem for students and at least 1.5 class periods should be allowed for students to explore and have a whole-class discussion on the task.

### COMMON CORE STATE STANDARDS

- **6.RP.1** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.
- **6.RP.2** Understand the concept of a unit rate  $a/b$  associated with a ratio  $a:b$  with  $b \neq 0$ , and use rate language in the context of a ratio relationship.
- **6.RP.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
  - c) Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity).
- **7.RP.1** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units.
- **7.RP.3** Use proportional relationships to solve multistep ratio and percent problems. *Example: percent increase.*

<p><b>ESSENTIAL QUESTIONS:</b></p> <ul style="list-style-type: none"> <li>• What are different types of ratios and how are ratios used to make comparisons?</li> <li>• What strategies can be used to compare ratios?</li> <li>• How are ratios related to fractions?</li> </ul>	<p><b>NCTM ESSENTIAL UNDERSTANDINGS<sup>1</sup>:</b></p> <ol style="list-style-type: none"> <li>1. Reasoning with ratios involves attending to and coordinating two quantities.</li> <li>2. A ratio is a multiplicative comparison of two quantities, or it is a joining of two quantities in a composed unit.</li> <li>3. Forming a ratio as a measure of a real-world attribute involves isolating that attribute from other attributes and understanding the effect of changing each quantity on the attribute of interest.</li> <li>4. A number of mathematical connections link ratios and fractions: <ul style="list-style-type: none"> <li>• Ratios are often expressed in fraction notation, although ratios and fractions do not have identical meaning.</li> <li>• Ratios are often used to make “part-part” comparisons, but fractions are not.</li> <li>• Ratios can often be meaningfully reinterpreted as fractions.</li> </ul> </li> <li>5. Ratios can be meaningfully reinterpreted as quotients.</li> <li>7. Proportional reasoning is complex and involves understanding that: <ul style="list-style-type: none"> <li>• Equivalent ratios can be created by iterating and/or partitioning a composed unit.</li> <li>• If one quantity in a ratio is multiplied or divided by a particular factor, then the other quantity must be multiplied or divided by the same factor to maintain the proportional relationship.</li> </ul> </li> </ol>	<p><b>SKILLS DEVELOPED:</b></p> <ul style="list-style-type: none"> <li>• Use different representations to form ratios and make comparisons with ratios.</li> <li>• Use visual and numerical strategies to compare ratios.</li> <li>• Form equivalent ratios and use equivalent ratios to solve problems.</li> </ul>
<p><b>MATERIALS:</b> Warm-Up task <i>Comparing by Using Ratios</i> and <i>Mixing Juice</i> task sheet, Calculators, Chart paper.</p>	<p><b>GROUPING:</b> Students will begin their work individually, but will then work in groups of three or four to discuss the task and arrive at a common solution.</p>	

<sup>1</sup> NCTM (2010) *Developing Essential Understandings of Ratios, Proportions & Proportional Reasoning: Grades 6 -8.*

## Warm-Up Task (*Comparing by Using Ratios*)

### SET-UP

Tell students that a useful way to compare numbers is to form ratios. With students working in groups of three, give them the *Comparing by Using Ratios* task sheet and ask them to take turns reading the ratio statements to each other. Tell them they should form and interpret the ratios and think about different ways ratios can be written. They should also look for similarities and differences in the ratio statements.

### EXPLORE PHASE

#### Monitoring Student Work:

Circulate while students are reading and interpreting the ratios. Focus students' attention on the ratio comparisons. Listen to and make note of students' debating and deciding the types of ratios in Statements A-G. Tell students you will want them to share their reasons during the whole-class discussion.

#### Possible Solution Paths

Each ratio is a part-to-part ratio, a part-to-whole ratio, or a ratio comparing different kinds of measures or counts (also called a rate). Statement D compares a part to a whole. Statements C and F compare two different kinds of measures; this type of ratio is called a *rate*. The remaining statements compare parts to parts. Note that statement E can be interpreted as part-to-part or part-to-whole. Make note of students who are arguing either interpretation and highlight this during the Share Discuss Analyze Phase of the Warm-Up.

Ratios are often written in the form 5:6 or 5 to 6 to help students separate the ideas of ratios from fraction arithmetic.

- *How are these statements similar?*
- *How are they different?*

### SHARE DISCUSS ANALYZE PHASE

#### General Considerations:

Start by prompting students to focus on ratio statements that they think are similar, and ask them to explain why. You might start by asking them, "How are statements A and B similar?" Once they have identified part-part, you might then ask them if D is also part-part, thus distinguishing part-whole and part-part. You might then have them decide whether G fits one of these classes. Next, ask whether C and F fit into either of these groups. Name the groups as they are formed. You could then have a discussion of E – saying that you don't know which it is. Through this discussion you should separate the statements into the three types, and introduce the terminology – part-part, part-whole, and rate. If you allow 7-10 min for small-group discussion, that will leave only 10-13 minutes for the Share, Discuss, Analyze Phase.

## SET-UP (Mixing Juice Task)

Make sure students understand the context.

Suggested Questions:

- *How many of you have made juice by adding water to a mix before?*
- *What was involved in making it?*

You may want to bring in a can of frozen orange juice (thawed) and, with your class, make juice following the instructions on the can. You can discuss the fact that you have one container of concentrated juice and to this you add three containers of water (or whatever it says on the container of concentrate). Point out that the recipes given in the problem are different from the one on the can. At camp, the juice concentrate comes in a very large container without mixing proportions given.

You might let students begin to explore the juice recipes C in small-group, and then reassemble in whole-group to discuss the groups' initial ideas about the different mixes. This approach gives groups a chance to consider several representations and comparison strategies. You might discuss parts A, B and C in whole-group and then challenge groups to solve part D. Make sure that students have solved D for at least two mixes before calling them together to discuss it C in whole-group.

Remind students that they will be expected to: justify their solutions; explain their thinking and reasoning to others; make sense of other students' explanations; ask questions of the teacher or other students when they do not understand; and use correct mathematical language, and symbols.

## EXPLORE PHASE: Supporting Students' Exploration of the Mathematical Ideas

**Private Think Time:** Allow students to work individually for 3 – 5 minutes without intervening, though you might want to circulate quickly to get an idea of the strategies that they are using.

**Small-Group Work:** After 3 – 5 minutes, ask students to work with their partner or in their small groups. As students are working, circulate around the room. Be persistent in:

- asking questions related to the mathematical ideas, problem-solving strategies, and connections between representations.
- asking students to explain their thinking and reasoning.
- asking students to explain in their own words, and build onto, what other students have said.

As you circulate, identify solution paths that you will have groups share during the Share, Discuss, Analyze Phase, and decide on the sequence that you would like for them to be shared. Give groups a “heads up” that you will be asking them to put their solutions on chart paper to share during the whole-class discussion. Having the various strategies on chart paper will allow you to arrange the work in the room in a way that supports analyzing and making connections between and among them.

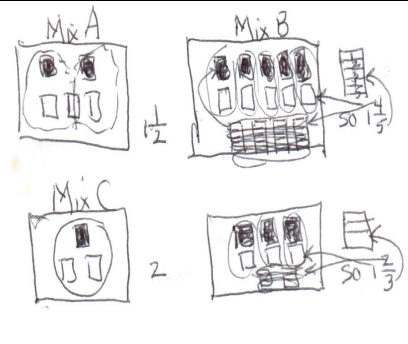
Some students will start with naïve strategies such as simply finding the difference between the number of cans of concentrate and ignore the water. Challenge this idea by asking: *Can I keep adding cans of water without making the juice less orangey?*

Questions A and B will allow misconceptions (additive strategies) as well as alternate multiplicative approaches for comparing ratios to emerge.

Question C is designed to raise the issue that the phrase “of Mix B” signals that this is a part-whole statement, thus  $\frac{5}{9}$  is not correct. Though students will discuss it during the Explore Phase, you will focus on questions A, B and D as you circulate during the Explore Phase.

**Possible Solution Paths for Parts A and B**

**Possible Assessing and Advancing Questions**



Draw pictures to show how much water there is for each cup of concentrate in each mix. The goal is to partition the water squares so that each cup of concentrate gets the same amount of water. In this way, you can see that Mix C has the most water for each cup of concentrate (least orangey) and Mix A has the least amount of water (most orangey).

**Assessing Questions**

- Tell me about your diagram. What does it show?
- How does this help to decide which mix is most or least orangey?
- What kind of ratios does this visual represent?

**Advancing Questions**

- If you wanted to write numerical ratios to represent what you have in this visual strategy, what would they look like? How would they help you decide which is most or least orangey?

Mix C	Mix B	Mix D	Mix A
$\frac{1}{3}$	$\frac{5}{14}$	$\frac{3}{8}$	$\frac{2}{5}$

Use part-to-whole ratios written in fraction form to express the relationships of concentrate to total liquid in a batch.

Using prior knowledge about fractions, students may represent the fractions as decimals or percents. There are a variety of strategies that can then be used to order the fractions, i.e., benchmark comparisons or common denominators.

**Assessing Questions**

- Tell me about your work.
- Tell me, for example, what  $\frac{3}{8}$  means in the context of the problem. What does the 3 represent? The 8? Is there any way you can let me know that in your explanation?

**Advancing Questions (if no comparisons are being made)**

- How are you going to compare the mixes? Which is most orangey and how do you know?
- What are some ways that you have used in the past to make comparisons? See if you can think about some of these and then try to make comparisons, in one or two ways.

**Advancing Questions (if comparisons are being made)**

- How does knowing that  $\frac{1}{3}$  is the ratio of concentrate to mix help you to answer Question D?

A.  $\frac{1\frac{1}{2}}{1}$ ;  $\frac{1\frac{4}{5}}{1}$ ;  $\frac{2}{1}$ ;  $\frac{1\frac{2}{3}}{1}$

B.  $\frac{2}{1}$ ;  $\frac{5}{1}$ ;  $\frac{1}{1}$ ;  $\frac{3}{1}$

A. Figure out how much water goes with each cup of concentrate. Notice that with these ratios we focus on most and least water.

B. Figure out how much concentrate goes with each cup of water. Notice that with these ratios we focus on most and least concentrate.

**Assessing Questions**


- Tell me about your work. What did you do and why?
- What kind of ratios have you created? What do they represent? Is there any way you can let me know that in your explanation?

**Advancing Questions**

- How are you using your ratios to decide which is most orangey?
- How are you going to compare the mixes? Which is most orangey and how do you know?
- What strategy are you using to order your ratios? What does the order mean in the context of the problem?



Mix A $\frac{2}{30} : \frac{3}{45}$ Mix B $\frac{5}{30} : \frac{9}{54}$	Use part-to-part ratios and make the number of cups of concentrate the same. Notice that with these ratios we focus on most and least water.	Assessing Questions <ul style="list-style-type: none"> <li>• <i>Tell me about your work.</i></li> <li>• <i>What kind of ratios have you created? What do they represent? Is there any way you can let me know that in your explanation?</i></li> <li>• <i>How did you decide that 30 cups of concentrate would be a helpful amount?</i></li> <li>• <i>How did you make the new ratios? What strategy did you use?</i></li> <li>• <i>How does making the amount of concentrate the same help you to reason about the problem?</i></li> </ul>
Mix C $\frac{1}{30} : \frac{2}{60}$ Mix D $\frac{3}{30} : \frac{5}{50}$		

Possible Errors and Misconceptions for Parts A and B	Possible Questions to Address Errors and Misconceptions
<p>Comparing the amount of water using absolute differences approach:</p>  <p>Mix A: cold water, concentrate</p> <p>Mix B: cold water, concentrate</p> <p>Mix C: water, conc</p> <p>Mix D: water, conc</p> <p>A. The juice that will taste most orangey is Mix C because it does not have as much water as mixes A, B, and D.</p> <p>B. The juice that will taste least orangey is Mix B because it has more water than mixes A, C, and D.</p>	<p>Assessing Question</p> <ul style="list-style-type: none"> <li>• <i>Tell me about your work. Explain your thinking.</i></li> </ul> <p>Advancing Questions</p> <ul style="list-style-type: none"> <li>• <i>What would 2 batches of Mix C look like? How would the “orangeyness” of this new one compare to Mix A? Why?</i></li> <li>• <i>For every one cup of water in Mix A how many cups of concentrate would I have?</i></li> </ul>
Possible Solution Paths for Part D	Possible Assessing and Advancing Questions
<p>Find the number of batches needed to make 120 cups of juice from each recipe, then multiply to find the amount of water and concentrate. For example, one batch of Mix A makes 5 cups of juice and since 120 cups are needed, <math>120 \div 5</math> yields 24 batches. So, <math>2 \times 24</math> or 48 cups of concentrate and <math>3 \times 24</math> or 72 cups of cold water are needed. 48 cups of concentrate plus 72 cups of water yields 120 cups of juice for the 240 campers.</p>	<p>Assessing Questions</p> <ul style="list-style-type: none"> <li>• <i>Why did you decide to work with 120 cups of juice?</i></li> <li>• <i>How did you use 120 to help you solve the problem?</i></li> <li>• <i>What does the 24 mean in the context of the problem and why did you multiply the 2 and 3 by it?</i></li> </ul> <p>Advancing Questions</p> <ul style="list-style-type: none"> <li>• <i>How can you check to see that the amounts you calculated are correct?</i></li> <li>• <i>So the ratio of cups of juice in the big batch to cups of juice in the recipe is 120:5 (pointing to <math>120 \div 5</math> on the paper). What is the ratio of cups of concentrate in the big batch to cups of concentrate in the recipe? What about the ratio of cups of water in the big batch to cups of water in the recipe? Do you think that will happen with the other mixes, too?</i></li> <li>• <i>Write those ratios in equation form and study the way they look. Is there a way to decide by looking at the statement that two ratios are equal?</i></li> </ul>

Make a rate or ratio table to scale up:

**Mix D** (for example)

Concentrate in cups	Water in cups	Total in cups
3	5	8
6	10	16
9	15	24
12	20	32
15	25	40
45	75	120

Students may continue to add 3:5:8 to each row until they reach 45:75:120, or they may notice that they can multiply a row by a scale factor to get to their result more quickly as is shown in the last 2 rows of this table.

Assessing Questions

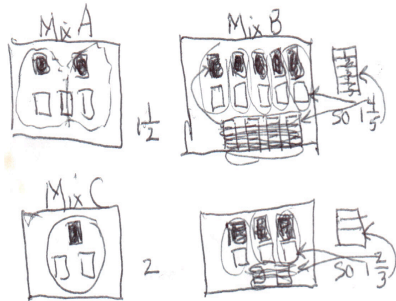
- *Why did you decide to organize your work in a table? How is that helpful?*
- *How are you getting from one row to the next in your table?*
- *How did you know when to stop making new rows?*
- *What patterns do you see in the table?*

Advancing Question

- *Can you show what you did in the last two rows so we don't have to guess? Use either a numerical expression or a written explanation. Then say WHY you knew you could do what you did.*

Possible Errors and Misconceptions for Part D		Possible Questions to Address Errors and Misconceptions																						
<p>If students are stuck on the question of making a recipe for 240 people, ask them to consider Mix A to start.</p>	<ul style="list-style-type: none"> <li>• <i>How much total juice does one batch of Mix A make? How can we figure out how many people one batch of this juice will serve?</i></li> <li>• <i>What if each serving is one cup? What if each serving is <math>\frac{1}{2}</math> cup?</i></li> <li>• <i>If you were going to serve juice to 50 people, how many cups of juice would you have to make if each person gets <math>\frac{1}{2}</math> cup of juice? How many batches of juice would this be?</i></li> <li>• <i>What are different strategies you might use to answer this question?</i> (Students might divide 50 people by 10 servings per batch to determine that 5 batches are needed. Alternatively, some students may reason that if 1 batch makes 10 servings, then 2 batches make 20 servings, 3 batches makes 30 servings, etc. Students may make a table from which to reason.</li> </ul>	<table border="1"> <thead> <tr> <th colspan="6">Servings of Juice</th> </tr> <tr> <th>Number of Batches</th> <th>1</th> <th>2</th> <th>3</th> <th>4</th> <th>5</th> </tr> </thead> <tbody> <tr> <td>Servings</td> <td>10</td> <td>20</td> <td>30</td> <td>40</td> <td>50</td> </tr> </tbody> </table>					Servings of Juice						Number of Batches	1	2	3	4	5	Servings	10	20	30	40	50
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<p><b>General Considerations:</b></p> <ul style="list-style-type: none"> <li>• Orchestrate the class discussion so that it builds on, extends, and connects the thinking and reasoning of students.</li> <li>• Sequence the solution paths so that you will be able to press students to make comparisons and connections across, and between, the various solution paths.</li> </ul> <p>It is recommended that you have groups share and discuss Parts A and B, then discuss C and D. For Parts A and B, the point you want students to think about is: <i>What does it mean to be most orangey tasting? To be least orangey tasting? Why do we have to consider BOTH the amount of concentrate AND the amount of water?</i></p>																								
<b>Possible Sequence of Solution Paths</b>		<b>Possible Questions and Possible Student Responses</b>																						
<p>Begin with the visual strategy under Possible Misconceptions and Errors for Parts A and B.</p>		<p>While the reasoning that was used to decide which mix is most orangey is faulty, the visual can be used to make connections to some of the other strategies. You might decide to have the group share their drawing and come back to them after they have heard some of the other strategies described to see if they will reconsider their reasoning and their answer about which is most orangey.</p>																						
<p>Have a group who used a part-to-whole ratio share their strategy. If there are a variety of strategies that were used to reason about the part-to-whole ratios, start with reasoning from the ratio itself, then move to those that converted the ratio to a decimal, then move to percents, and finally connect to the common denominator approach.</p>		<p><b>How did your group decide to compare the mixes?</b></p> <ul style="list-style-type: none"> <li>• <i>We made our comparisons using part-to-whole ratios. We compared the part of the mix that is concentrate to the whole mix. Then we made these fractions into decimals and looked for the largest because that would tell us which mix was most orangey.</i></li> </ul> <p><b>How is your strategy related to the diagrams we saw in the first strategy?</b></p> <ul style="list-style-type: none"> <li>• <i>The numerator in our fraction is the number of pieces that are shaded and these represent the concentrate. The denominator in our fraction is the total number of pieces in the diagram for a particular mix.</i></li> </ul> <p><b>How do your decimals relate to the diagrams?</b></p> <ul style="list-style-type: none"> <li>• <i>The decimal (percent) would represent the portion of the whole diagram that is shaded. In this case, the whole mix would have a value of 1 or 100%.</i></li> </ul>																						

Use a visual that shows how much water there is for each cup of concentrate in each mix. This is a unit rate approach.



**Tell us what your drawing means.**

- The solid squares represent cups of concentrate and the empty squares represent cups of water. We divided the water squares so that each cup of concentrate gets the same amount of water.

**How does your drawing help you decide which mix is most orangey?**

- We can see that Mix A has the least water,  $1 \frac{1}{2}$  cups, for each cup of concentrate, so it is the most orangey.

**How does your drawing help you decide which mix is least orangey?**

- We can see that Mix C has the most water, 2 cups, for each cup of concentrate, so it is the least orangey.

**How does your strategy compare to the ones we saw earlier?**

- We were using part-to-part comparisons and they were using part-to-whole comparisons.
- For them to say which mix was most orangey they had to look for the mix that had the most concentrate. For us to say which mix was most orangey, we looked for the mix that has the least water.

Have a group that used a unit rate approach with unit ratios expressed numerically share next.

**Tell us about the ratios your group made. How did you calculate them?**

- We compared cups of water to cups of concentrate. We wanted to figure out how much water goes with each cup of concentrate, so we divided the number of cups of water by the number of cups of concentrate.

**How does this compare to the last groups' strategy?**

- They divided up the area of the squares and we used ratios, unit rates, but they both convey similar

information. Like for Mix B, our ratio was  $\frac{1 \frac{4}{5}}{1}$  which is water to concentrate and that's what their picture shows.

Have a group that used part-to-part ratios and made the number of cups of concentrate (or water) the same.

*(This strategy is helpful to scaffold student thinking for Part D.)*

**Tell us about your approach. What kind of ratios did you use to compare the mixes?**

- We used part-to-part ratios. We thought if we had 4 big pots and used each pot to make many batches of each mix we could compare them. We wanted each pot to have the same amount of concentrate so we could think about how much water is in each pot.

**How will the multiple batches of juice in the pots compare to the original mix?**

- The juice in each pot will taste the same as the original mix because we kept the ratio of concentrate to water the same.

**How does your strategy compare to the other strategies we have seen?**

- Our strategy is probably most like the part-to-whole ratios where they found common denominators. Even though we used part-to-part ratios to reason, their strategy was similar because, like us, they ended up with a lot more juice than was in the original mix.

## CLOSURE

### Quickwrite:

- Why is a ratio a useful way to make comparisons?
- Which of the following will taste the most orangey? 2 cups of concentrate and 3 cups of water; 4 cups of concentrate and 6 cups of water; or 10 cups of concentrate and 15 cups of water? Explain your reasoning.

### Possible Assessment:

- Provide some additional contexts where they need to compare quantities. Ask them to explain their thinking in writing.

### Homework:

- Find items from the current curriculum that will allow them to apply these ideas and understandings.

## References

Lappan, Fey, Fitzgerald, Friel, Phillips (2009). *Teacher's Guide: Connected Mathematics 2. Comparing and Scaling: Ratio, Proportion, and Percent*, Pearson.