KINDERGARTEN MATH: BOOKS ON SHELVES

UNIT OVERVIEW

Books on Shelves is the culminating task in a multi-week unit focused on operations and algebraic thinking. Students demonstrate mastery by completing the Books on Shelves task in one class period.

TASK DETAILS

Task Name: Books on Shelves

Grade: K

Subject: Math

Depth of Knowledge: 2

Task Description: Students are asked to demonstrate the different possible combinations when placing 6 books on 2 shelves.

Standards Assessed:
K.OA.3 Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).

Standards for Mathematical Practice:
MP.1 Make sense of problems and persevere in solving them.
MP.2 Reason abstractly and quantitatively.
MP.3 Construct viable arguments and critique the reasoning of others.
MP.6 Attend to precision.

NYC Department of Education
Dennis M. Walcott, Chancellor

TABLE OF CONTENTS

The task and instructional supports in the following pages are designed to help educators understand and implement tasks that are embedded in Common Core-aligned curricula. While the focus for the 2011-2012 Instructional Expectations is on engaging students in Common Core-aligned culminating tasks, it is imperative that the tasks are embedded in units of study that are also aligned to the new standards. Rather than asking teachers to introduce a task into the semester without context, this work is intended to encourage analysis of student and teacher work to understand what alignment looks like. We have learned through the 2010-2011 Common Core pilots that beginning with rigorous assessments drives significant shifts in curriculum and pedagogy. Universal Design for Learning (UDL) support is included to ensure multiple entry points for all learners, including students with disabilities and English language learners.

Acknowledgements: The unit outline was developed by Kerry Cunningham (CFN 208) with input from Curriculum Designers Alignment Review Team. The tasks were developed by the schools in the 2010-2011 NYC DOE Elementary School Performance Based Assessment Pilot, in collaboration Exemplars, Inc. and the Center for Assessment.

KINDERGARTEN MATH: BOOKS ON SHELVES PERFORMANCE TASK

NYC Department of Education
Dennis M. Walcott, Chancellor

Name_____

Books on Shelves

Miguel has two shelves. Miguel has six books. Miguel wants to put books on the two shelves. How many different ways can Miguel put books on the two shelves? Show and tell how you know.

KINDERGARTEN MATH: BOOKS ON SHELVES

UNIVERSAL DESIGN FOR LEARNING (UDL) PRINCIPLES

Books on Shelves – Math Grade K
Common Core Learning Standards/
Universal Design for Learning

The goal of using Common Core Learning Standards (CCLS) is to provide the highest academic standards to all of our students. Universal Design for Learning (UDL) is a set of principles that provides teachers with a structure to develop their instruction to meet the needs of a diversity of learners. UDL is a research-based framework that suggests each student learns in a unique manner. A one-size-fits-all approach is not effective to meet the diverse range of learners in our schools. By creating options for how instruction is presented, how students express their ideas, and how teachers can engage students in their learning, instruction can be customized and adjusted to meet individual student needs. In this manner, we can support our students to succeed in the CCLS.

Below are some ideas of how this Common Core Task is aligned with the three principles of UDL; providing options in representation, action/expression, and engagement. As UDL calls for multiple options, the possible list is endless. Please use this as a starting point. Think about your own group of students and assess whether these are options you can use.

REPRESENTATION: *The "what" of learning.* How does the task present information and content in different ways? How do students gather facts and categorize what they see, hear, and read? How are they identifying letters, words, or an author's style?

In this task, teachers can…

- ✓ **Highlight or emphasize key elements in text, graphics, and diagrams** by providing access to concrete or virtual manipulatives, such as pictures or models of book on shelves.

ACTION/EXPRESSION: *The "how" of learning.* How does the task differentiate the ways that students can express what they know? How do they plan and perform tasks? How do students organize and express their ideas?

In this task, teachers can…

- ✓ **Provide graphic organizers and templates for data collection and organizing information** to help students organize their thoughts and establish relationships between ideas. Graphic organizers can be paper/pencil or found on-line..

ENGAGEMENT: *The "why" of learning.* How does the task stimulate interest and motivation for learning? How do students get engaged? How are they challenged, excited, or interested?

In this task, teachers can…

- ✓ **Provide feedback that is frequent, timely, and specific** by helping students see what they did well and why, see their errors and learn how to correct them by providing explicit and informative feedback when assessing student work, making feedback a part of the learning process.

Visit http://schools.nyc.gov/Academics/CommonCoreLibrary/default.htm to learn more information about UDL.

KINDERGARTEN MATH:BOOKS ON SHELVES RUBRIC

The following section contains two rubrics that were used to score student work: a content rubric and a process rubric. The content rubric describes student performance according to the content standards in the CCLS. The process rubric describes student performance according to the National Council of Teachers of Mathematics (NCTM) process standards. Students' were given a score based on their achievement on the CCLS content rubric and the process rubric. Given that the process rubric is not in the language of the Common Core's Mathematical Practices, we have also included a document that NCTM has posted regarding the relationship between the NCTM process standards and the Standards for Mathematical Practice.

CCSS Mathematics Content Standards & Standards in Practice

Students apply mathematical reasoning, knowledge, and skills in problems-solving situations and support their solutions using mathematical language and appropriate representations (data).

Grades K-1 Progress-Monitoring Focus: Addition & subtraction; Comparing & ordering numbers

Gr. K-1 **Math CCSS Criteria/Clusters**	**Novice**	**Apprentice**	**Practitioner**	**Expert**
Counting & Cardinality (K only)	May recognize number symbols and names, but lacks counting sequence A numerical answer may be correct, but is not supported by student work (e.g., solves problem without applying properties of operations; just copies numbers) OR stated answer is incorrect or lacks relevance	Some parts of problem correct and those parts supported by student work Represents and solves simple addition and subtraction problems using counting, models, visuals, manipulatives, number lines, sounds, etc. May apply commutative property	***Represents number of objects for written numerals, 0-20*** **K.CC-3** ***Counts to find out how many; Determines greater than/less than of groups of objects (up to 20 if ordered; up to 10 if random)*** **K.CC- 4, 5** ***Compares numbers between 1 and 10 using written numerals*** **K.CC-6, 7**	Represents, compares, and solves problems using numbers greater than 20 Applies associative or commutative properties to solve problems

Operations & Algebraic Thinking **And** **Number & Operations in Base Ten**	Uses place value to show 10 or less	Represents and solves simple addition and subtraction problems using counting, models, visuals, manipulatives, number lines, sounds, etc. May apply commutative property	***Represents, compares, & solves addition & subtraction using strategies – counting, objects, drawings, etc. …*** ***-to 10*** **K.OA-1, 2** ***-or to 20; with multiple addends; using equal sign and equations*** **1. OA-1, 2, 6, 7, 8** ***Composes/ decomposes and compares numbers using tens and ones*** **K.OA-3; K.NBT-1** **1.NBT- 2, 3** ***Applies associative & commutative properties*** **1.OA-3** *Minor computation flaws do not affect outcome of a correct solution*	Represents, compares, and solves problems using numbers up to 100 (K) 120 (gr1) All parts of problem correct, precise, and supported by student work Applies associative or commutative properties to solve problems in more than one way Uses place value to expand numbers (gr 1)
Measurement & Data	Still demonstrates limited number sense (e.g., difficulty counting, estimating; representing quantities; recognizing measurement attributes,) Tells time to the hour using a digital clock	Recognizes and uses 1 measureable attribute to compare or classify Tells time to hour, & half hour (gr 1) using a digital clock	***Describes measurable attributes; compares measures of objects*** **K.MD-1, 2** ***Measures to whole unit; compares 2 or 3 objects; classifies objects using more than 1 attribute*** **1.MD-1, 2** ***Measures & compares lengths indirectly*** **1.MD-1** ***Tells time to hour & half hour using digital and analog clocks*** **1.MD-3**	Uses a variety of strategies to estimate, measure, and compare Measures and compares lengths of more than 2 objects and determines the difference in lengths

Exemplars® Standards-Based Math Rubric*

	Problem Solving	Reasoning and Proof	Communication	Connections	Representation
Novice	No strategy is chosen, or a strategy is chosen that will not lead to a solution. Little or no evidence of engagement in the task present.	Arguments are made with no mathematical basis. No correct reasoning nor justification for reasoning is present.	No awareness of audience or purpose is communicated. or Little or no communication of an approach is evident or Everyday, familiar language is used to communicate ideas.	No connections are made.	No attempt is made to construct mathematical representations.
Apprentice	A partially correct strategy is chosen, or a correct strategy for only solving part of the task is chosen. Evidence of drawing on some previous knowledge is present, showing some relevant engagement in the task.	Arguments are made with some mathematical basis. Some correct reasoning or justification for reasoning is present with trial and error, or unsystematic trying of several cases.	Some awareness of audience or purpose is communicated, and may take place in the form of paraphrasing of the task. or Some communication of an approach is evident through verbal/written accounts and explanations, use of diagrams or objects, writing, and using mathematical symbols. or Some formal math language is used, and examples are provided to communicate ideas.	Some attempt to relate the task to other subjects or to own interests and experiences is made.	An attempt is made to construct mathematical representations to record and communicate problem solving.

*Based on revised NCTM standards.

Exemplars® Standards-Based Math Rubric (cont.)*

	Problem Solving	Reasoning and Proof	communication	connections	Representation
Practitioner	A correct strategy is chosen based on mathematical situation in the task. Planning or monitoring of strategy is evident. Evidence of solidifying prior knowledge and applying it to the problem solving situation is present. Note: The practitioner must achieve a correct answer.	Arguments are constructed with adequate mathematical basis. A systematic approach and/or justification of correct reasoning is present. This may lead to... • clarification of the task. • exploration of mathematical phenomenon. • noting patterns, structures and regularities.	A sense of audience or purpose is communicated. and/or Communication of an approach is evident through a methodical, organized, coherent sequenced and labeled response. Formal math language is used throughout the solution to share and clarify ideas.	Mathematical connections or observations are recognized.	Appropriate and accurate mathematical representations are constructed and refined to solve problems or portray solutions.
Expert Work at this level is exceeding grade-level expectations	An efficient strategy is chosen and progress towards a solution is evaluated. Adjustments in strategy, if necessary, are made along the way, and / or alternative strategies are considered. Evidence of analyzing the situation in mathematical terms, and extending prior knowledge is present. Note: The expert must achieve a correct answer.	Deductive arguments are used to justify decisions and may result in formal proofs. Evidence is used to justify and support decisions made and conclusions reached. This may lead to... • testing and accepting or rejecting of a hypothesis or conjecture. • explanation of phenomenon. • generalizing and extending the solution to other cases.	A sense of audience and purpose is communicated. and/or Communication at the Practitioner level is achieved, and communication of argument is supported by mathematical properties. Precise math language and symbolic notation are used to consolidate math thinking and to communicate ideas.	Mathematical connections or observations are used to extend the solution.	Abstract or symbolic mathematical representations are constructed to analyze relationships, extend thinking, and clarify or interpret phenomenon.

*Based on revised NCTM standards.

National Council of Teachers of Mathematics (NCTM) Process Standards and the Common Core State Standards for Mathematics

From ***NCTM Action on the Common Core State Standards for Mathematics***

by NCTM President J. Michael Shaughnessy

"The preeminent message in both the NCTM *Principles and Standards for School Mathematics* (2000) and CCSSM is the importance of nurturing mathematical thinking and reasoning processes in our students. No bulleted list of specific content standards will hold together as a coherent, meaningful whole, or make any significant contribution to our students' growth in mathematics, without interweaving mathematical "practices." Mathematics curricula must show students the power of reasoning and sense making as they explore mathematical structures, of communication as they construct viable arguments, and of multiple representations as they engage in mathematical modeling. The close connections between the NCTM Process Standards and the CCSSM Standards for Mathematical Practice are represented in the chart below.

The upcoming NCTM publication, *Making it Happen*, will provide a deeper analysis of the connections between the NCTM Process Standards and detail the potential of the existing NCTM resources to interpret and implement CCSSM."

NCTM Process Standards and the CCSS Mathematical Practices

NCTM Process Standards	CCSS Standards for Mathematical Practice
Problem Solving	1. Make sense of problems and persevere in solving them. 5. Use appropriate tools strategically.
Reasoning and Proof	2. Reason abstractly and quantitatively. 3. Critique the reasoning of others. 8. Look for and express regularity in repeated reasoning
Communication	3. Construct viable arguments
Connections	6. Attend to precision. 7. Look for and make use of structure
Representations	4. Model with mathematics.

KINDERGARTEN MATH: BOOKS ON SHELVES ANNOTATED STUDENT WORK

This section contains annotated student work at a range of score points, student summaries, and implications for instruction for each performance level. The annotated student work and student summaries demonstrate performance at different levels and show examples of student understandings and misunderstandings of the task that can be used with the implications for instruction to understand how to move students to the next performance level.

Kindergarten Math: Books on Shelves
Annotated Student Work

This student is an Expert according to both the Exemplars Rubric and the CCSS Content Standards Rubric, (both included in the supporting materials).

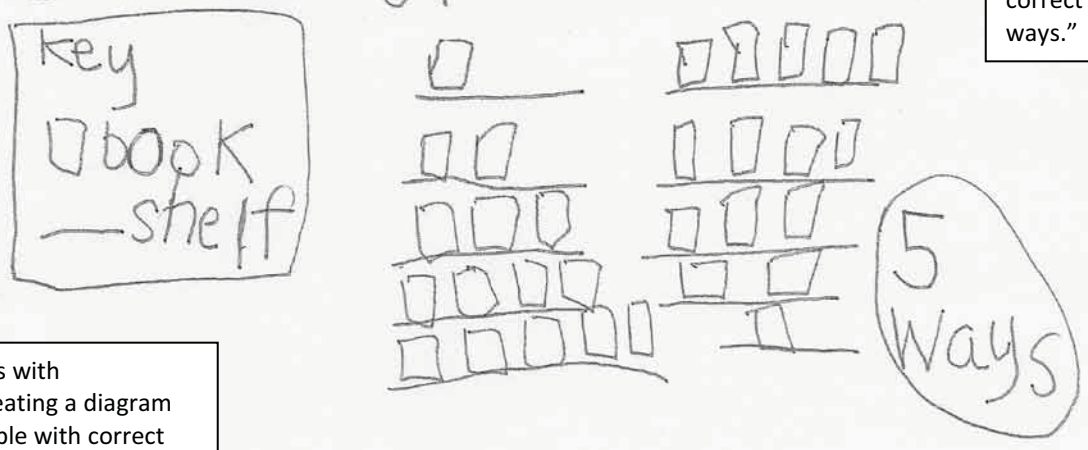
Expert

Name Student 1

Books on Shelves

Miguel has two shelves.
Miguel has six books.
Miguel wants to put books on the two shelves.
How many different ways can Miguel put books on the two shelves?
Show and tell how you know.

Do a diagram.



The student is able to make sense of the problem and persevere in finding a pathway to solving it. The student uses a diagram to show the five possible combinations for six books on two shelves. The student states a correct answer, "5 ways."

The student models with mathematics by creating a diagram and a key, and a table with correct labels and data. The student uses both representations to clarify and support the answer.

The student can decompose numbers less than or equal to 10 in pairs in more than one way, and record each decomposition by a drawing or equation (K.OA 3).

"This is a fair share of books for the two shelves. That means equal. 6 books means even number."

combinations

"This word is combinations. Miguel has 5 of 'em."

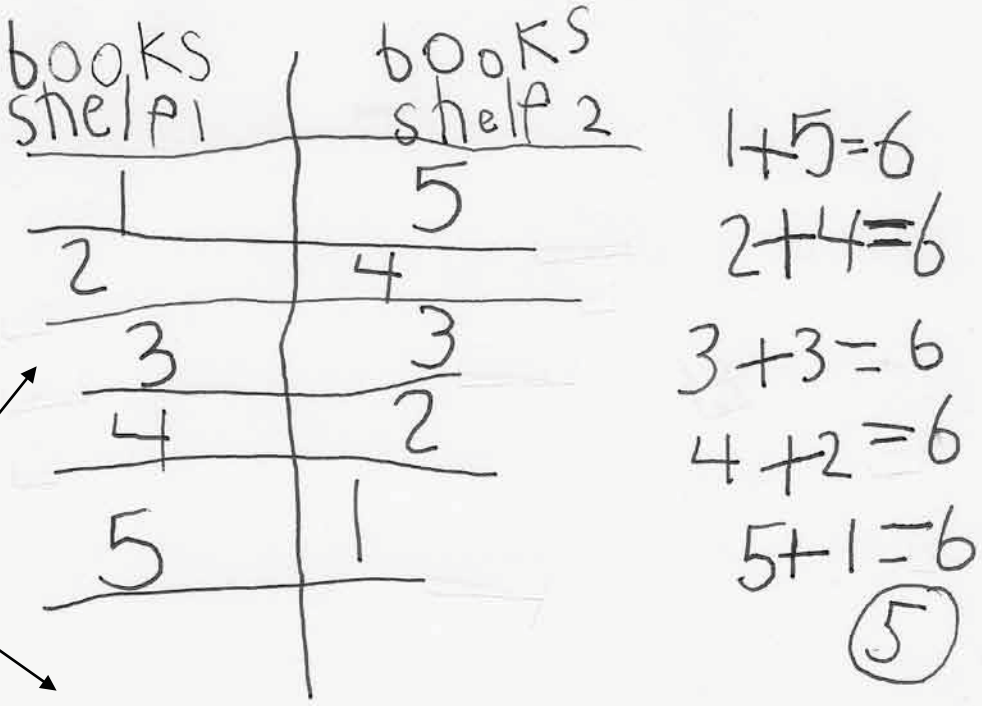
"If the books can go on only one shelf you can have two more ways - zero and six, six and zero. I think I can do a table."

The student applies the precise mathematical language- diagram, key, fair share, equal, even number, combinations, table, pattern, to help construct viable arguments.

The student constructs a new argument, "If the books can go on only one shelf you can have two more combinations- zero and six, six and zero."

page 1 of 2

Kindergarten Math: Books on Shelves
Annotated Student Work



"Look, the books go up by 1 here (pointed to shelf 1) and books go down by 1 here (pointed to shelf 2) that is a pattern.
I did 5 ways here and here (pointed to diagram and table). Tell Miguel it is 5 ways cuz I know it is." A2

The student justifies the reasonableness of her/his conclusions by verifying the answer using a new strategy of a table and states, "Tell Miguel it is five ways cuz I know it is." The student searches for regularity and discerns a pattern, "Look the books go up by 1 here and books go down by one here. That is a pattern."

page 2 of 2

Kindergarten Math: Books on Shelves
Annotated Student Work

Expert – Student 1 Summary

Achievement Level: Student 1 is an Expert according to both the Exemplars Process Rubric and the CCSS Content Standards Rubric, (both of which are included in the supporting materials). The table below provides a rationale for the student's performance level in each of the criteria identified in the Exemplars Process Rubric.

Criteria and Performance Level	Rationale
Problem Solving Expert	The student's strategy of using a diagram to show the total combinations for placing six books on two shelves works to solve the problem. The student shows five correct combinations and correctly states, "5 ways." The student uses an alternative strategy of a table to support her/his answer.
Reasoning and Proof Expert	The student shows correct reasoning of the underlying mathematical concepts of combinations to six, the commutative property, and understanding that 0 + 6 = 6 and 6 + 0 = 6 can not be considered as possible combinations for the problem. The student verifies her/his solution by solving the problem a second way.
Communication Expert	The student correctly uses the mathematical terms-diagram, key, fair share, equal, even number, combinations, table, pattern.
Connections Expert	The student makes the Practitioner mathematically relevant connections, "This is a fair share of books for the two shelves," "That means equal," "Six books means even number," "If the books can go on only one shelf you can have two more ways-zero and six, six and zero," and, "Look the books go up by one here and books go down by one here, that is a pattern." The student makes the Expert connection by using a table to solve the problem a second way and linking the two strategies together to verify that her/his solution is correct. The student states, I did five ways here and here. Tell Miguel it is five ways cuz I know it is."
Representation Expert	The student's diagrams are appropriate to the problem and accurate. The student's key and the scribing correctly define the books and shelves. The student's table is appropriate to the problem and accurate. Each column is correctly labeled and all entered data is correct. The student uses her/his table to clarify and support the answer.

Instructional Implications: Books on Shelves K

Achievement Level: Expert

Note: Student work identified at this level is exceeding grade-level expectations

The following is a list of instructional implications that you may want to consider for students performing at the Expert level. In addition, you may want to consult the suggestions for the Practitioner level:

- Find combinations for numbers greater than ten
- Investigate and prove generalization-total combinations is one less than sum being found
- Investigate and prove generalization-if zero can be considered the total combinations is one more than sum
- Solve problem more than one way to verify that answer is correct
- Relate problem to a similar one completed and discuss how they are mathematically similar
- Discover and discuss why an even number has a fair share/equal combination but an odd number does not

Kindergarten Math: Books on Shelves
Annotated Student Work

This student is a Practitioner according to both the Exemplars Rubric and the CCSS Content Standards Rubric, (both included in the supporting materials).

Practitioner

Name Student 1

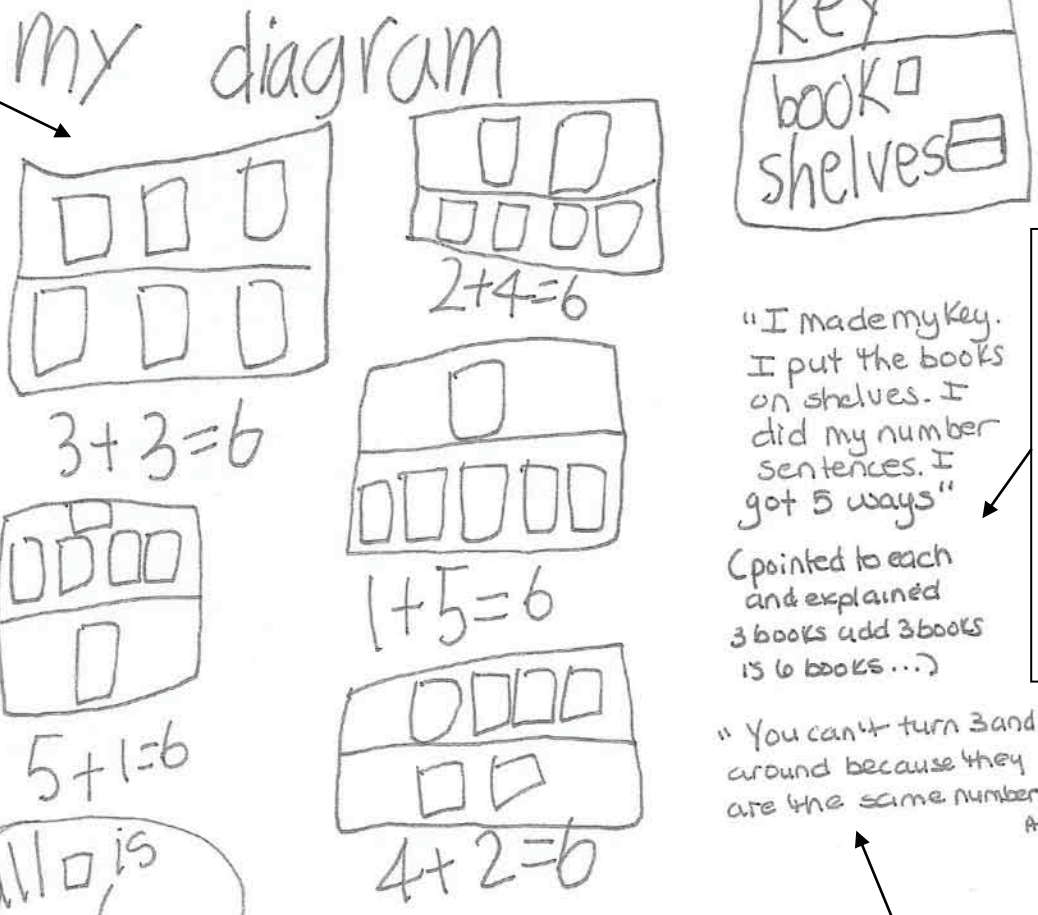
K

Books on Shelves

Miguel has two shelves.
Miguel has six books.
Miguel wants to put books on the two shelves.
How many different ways can Miguel put books on the two shelves?
Show and tell how you know.

The student applies the precise mathematical language-diagram, key, number sentences, equal, to help construct her/his viable argument.

The student can decompose numbers less than or equal to 10 in pairs in more than one way, and record each decomposition by a drawing or equation (K.OA 3).



The student is able to make sense of the problem and persevere in finding a pathway to solving it. The student shows the five possible combinations for six books on two shelves and correctly states, "I got five ways."

"I made my key. I put the books on shelves. I did my number sentences. I got 5 ways"

(pointed to each and explained 3 books add 3 books is 6 books...)

The student models mathematics by creating a diagram that is appropriate and accurate. A key and scribing correctly defines the books and shelves.

"You can't turn 3 and 3 around because they are the same number." A-2.

The student reasons quantitatively by making the connection, "You can't turn three and three around because they are the same number."

page 1 of 1

Kindergarten Math: Books on Shelves
Annotated Student Work

Practitioner – Student 1 Summary

Achievement Level: Student 1 is a Practitioner according to both the Exemplars Process Rubric and the CCSS Content Standards Rubric, (both of which are included in the supporting materials). The table below provides a rationale for the student's performance level in each of the criteria identified in the Exemplars Process Rubric.

Criteria and Performance Level	Rationale
Problem Solving Practitioner	The student's strategy of using a diagram and number sentences to show the total combinations for placing six books on two shelves works to solve the problem. The student shows five correct combinations and correctly states, "I got five ways."
Reasoning and Proof Practitioner	The student shows correct reasoning of the underlying mathematical concepts of combinations to six, the commutative property, and understanding that 0 + 6 = 6 and 6 + 0 = 6 can not be considered as possible combinations for the problem.
Communication Practitioner	The student correctly uses the mathematical terms-diagram, key, number sentences, equal.
Connections Practitioner	The student makes the mathematically relevant connection, "You can't turn three and three around because they are the same number." The student's statements, "all □ is + 6," and, "The two sides has to equal six," are not considered connections as they represent the reasoning a student must understand to solve the problem.
Representation Practitioner	The student's diagram is appropriate to the problem and accurate. The student's key and the scribing correctly defines the books and shelves.

Kindergarten Math: Books on Shelves
Annotated Student Work

Exemplars Rubric: Practitioner
CCSS Content Rubric: Expert

This student is a Practitioner according to the Exemplars Rubric and an Expert according to the CCSS Content Standards Rubric, (both included in the supporting materials).

The student is able to make sense of the problem and persevere in solving it. The student shows the five possible combinations for six books on two shelves, and correctly states, "Miguel has five ways to do the books."

Name Student 2

K

Books on Shelves

Miguel has two shelves.
Miguel has six books.
Miguel wants to put books on the two shelves.
How many different ways can Miguel put books on the two shelves?
Show and tell how you know.

My table

Key
shelf = 1 2
book □

miguels books A

1	2
□	□□□□□
□□	□□□□
□□□	□□□
□□□□	□□
□□□□□	□

5

The student can decompose numbers less than or equal to 10 in pairs in more than one way, and record each decomposition by a drawing or equation (K.OA 3).

The student applies the precise mathematical language-table, key, to help construct her/his viable argument.

Table 2

1	2
1	5
2	4
3	3
4	2
5	1

"I did a table I made a key for shelf 1 and 2 and 6 books I went 1 book, 2 book 3 book, 4 book, 5 book so I don't miss any ways. We learned that plan. Miguel has 5 ways to do the books. 1 plus 5, 2 plus 4, 3 plus 3 4 plus 2, 5 plus 1 (pointed to table) They all end at 6 books."

"I did it with the numbers It is faster this way."

"3 books and 3 books is the same on shelf 1 and shelf 2"

AZ

page 1 of 1

The student reasons abstractly and quantitatively by making the connection, "3 books and 3 books is the same on shelf one and shelf two and searches for regularity by stating, "I went 1 book, 2 book, 3 book, 4 book, 5 book, so I don't miss any ways.

The student models with mathematics by using a table with a key to define the shelves and books. The student reflects and improves the model/table by keeping the key for shelves but uses numbers in place of the books and states, "I did it with numbers. It is faster this way."

Kindergarten Math: Books on Shelves
Annotated Student Work

Practitioner/Expert – Student 2 Summary

Achievement Level: Student 2 is a Practitioner according to the Exemplars Process Rubric and an Expert according to the CCSS Content Standards Rubric, (both of which are included in the supporting materials). The table below provides a rationale for the student's performance level in each of the criteria identified in the Exemplars Process Rubric.

Criteria and Performance Level	Rationale
Problem Solving Practitioner	The student's strategy of using a table to show the total combinations for placing six books on two shelves works to solve the problem. The student shows five correct combinations and states, "5," which is also correct. The scribing also correctly states, "Miguel has five ways to do the books. One plus five, two plus four, three plus three, four plus two, five plus one."
Reasoning and Proof Practitioner	The student shows correct reasoning of the underlying mathematical concepts of combinations to six, the commutative property, and understanding that 0 + 6 = 6 and 6 + 0 = 6 can not be considered as possible combinations for the problem.
Communication Practitioner	The student correctly uses the mathematical terms-table, key.
Connections Practitioner	The student makes the mathematically relevant connection of using a different form of a table to show the five correct combinations. The student states, "I did it with numbers. It is faster this way." The student makes the mathematically relevant observation, "Three books and three books is the same on shelf one and shelf 2."
Representation Practitioner	The student's first table is appropriate to the problem and accurate. The key, column labels, and scribing correctly identify the books and shelves.

Kindergarten Math: Books on Shelves
Annotated Student Work

This student is a Practitioner according to both the Exemplars Rubric and the CCSS Content Standards Rubric, (both included in the supporting materials).

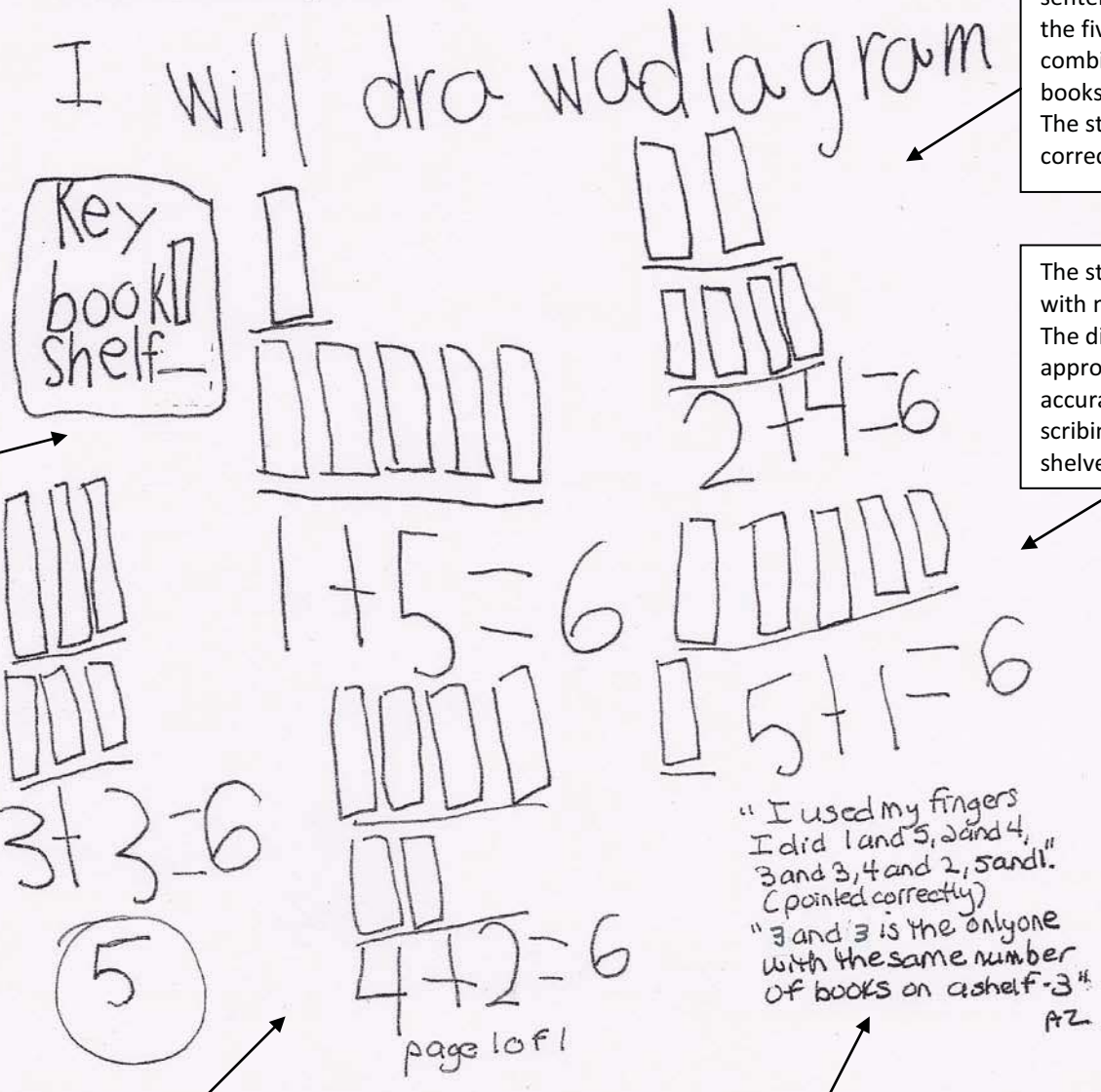
Practitioner

Name Student 3C

K

Books on Shelves

Miguel has two shelves.
Miguel has six books.
Miguel wants to put books on the two shelves.
How many different ways can Miguel put books on the two shelves?
Show and tell how you know.



The student is able to make sense of the problem and persevere in finding a pathway to solving it. The student uses a diagram and number sentences to show the five possible combinations for six books on two shelves. The student states a correct answer, "5."

The student applies the precise mathematical language-diagram, key, to help construct her/his viable argument.

The student models with mathematics. The diagram is appropriate and accurate. A key and scribing define the shelves and books.

The student can decompose numbers less than or equal to 10 in pairs in more than one way, and record each decomposition by a drawing or equation (K.OA 3).

The student reasons abstractly and quantitatively by making the connection, "three and three is the only one with the same number books on a shelf-three."

Kindergarten Math: Books on Shelves
Annotated Student Work

Practitioner – Student 3 Summary

Achievement Level: Student 3 is a Practitioner according to both the Exemplars Process Rubric and the CCSS Content Standards Rubric, (both of which are included in the supporting materials). The table below provides a rationale for the student's performance level in each of the criteria identified in the Exemplars Process Rubric.

Criteria and Performance Level	Rationale
Problem Solving Practitioner	The student's strategy of using a diagram and number sentences to show the total combinations for placing six books on two shelves works to solve the problem. The student shows five correct combinations and correctly states, "5." The student also correctly states, "I used my fingers. I did one and five, two and four, three and three, four and two, five and one."
Reasoning and Proof Practitioner	The student shows correct reasoning of the underlying mathematical concepts of combinations to six, the commutative property, and understanding that 0 + 6 = 6 and 6 + 0 = 6 can not be considered as possible combinations for the problem.
Communication Practitioner	The student correctly uses the mathematical terms-diagram, key
Connections Practitioner	The student makes the mathematically relevant connection, "six and six is the only one with the same number of books on a shelf-three."
Representation Practitioner	The student's diagram is appropriate to the problem and accurate. The student's key and the scribing correctly label the books and shelves.

Instructional Implications: Books on Shelves K

Student Achievement Level: Practitioner

The following is a list of instructional implications that you may want to consider for students performing at the Practitioner level. In addition, you may want to consult the suggestions for the Novice and Apprentice levels:

- Include more writing of number sentences to support the possible combinations for any number up to ten in a student's diagram, table, etc.
- Discuss and solve problems where zero would be used in a combination and when it would not be used
- Introduce the language term-commutative property
- Encourage student to make more than one mathematically relevant connection about her/his work
- Investigate where a combination is a fair share
- Introduce another strategy to solve the same problem-model, diagram/key, organized list, table

Kindergarten Math: Books on Shelves
Annotated Student Work

This student is an Apprentice according to both the Exemplars Rubric and the CCSS Content Standards Rubric, (both included in the supporting materials).

Apprentice

Name Student 1 K

Books on Shelves

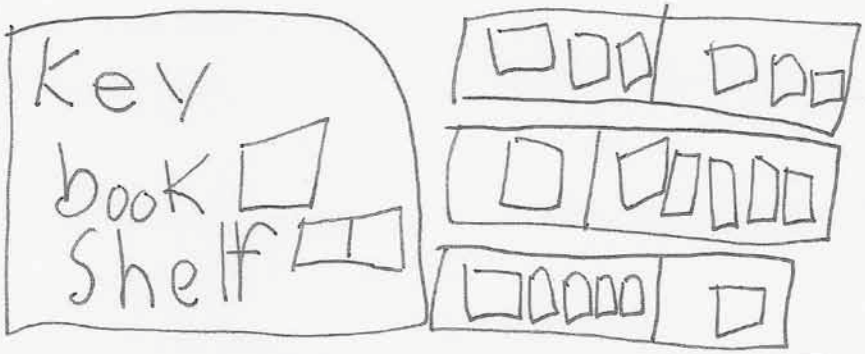
Miguel has two shelves.
Miguel has six books.
Miguel wants to put books on the two shelves.
How many different ways can Miguel put books on the two shelves?
Show and tell how you know.

The student is able to make sense of the problem and persevere in finding a solution pathway. The student's answer, "5," is correct.

make a diagram.

5

The student applies the precise mathematical language- diagram, key, to help construct her/his viable argument.



The student can decompose numbers less than or equal to 10 in pairs in more than one way, and record each decomposition by a drawing or equation (K.OA 3).

"I used my brain to find out the answer. All of them had 6. Any time I made 6 I kept it "A2

The student models with mathematics. The diagram is appropriate and accurate. A key defines the books and shelves.

The student does not reflect on her/his solution to make connections to regularities, patterns, structures, or trends.

there is 6 book

page 1 of 1

Kindergarten Math: Books on Shelves
Annotated Student Work

Apprentice - Student 1 Summary

Achievement Level: Student 1 is an Apprentice according to both the Exemplars Process Rubric and the CCSS Content Standards Rubric, (both of which are included in the supporting materials). The table below provides a rationale for the student's performance level in each of the criteria identified in the Exemplars Process Rubric.

Criteria and Performance Level	Rationale
Problem Solving Practitioner	The student's strategy of using a diagram to show the total combinations for placing six books on two shelves works to solve the problem. The student shows five correct combinations and correctly states, "5."
Reasoning and Proof Practitioner	The student shows correct reasoning of the underlying mathematical concepts of combinations to six, the commutative property, and understanding that 0 + 6 = 6 and 6 + 0 = 6 cannot be considered as possible combinations for the problem.
Communication Practitioner	The student correctly uses the mathematical terms-diagram, key.
Connections Novice	The student does not make a mathematically relevant connection about her/his solution.
Representation Practitioner	The student's diagram is appropriate to the problem and accurate. The student's key correctly defines the books and shelves.

Kindergarten Math: Books on Shelves
Annotated Student Work

This student is an Apprentice according to both the Exemplars Rubric and the CCSS Content Standards Rubric, (both included in the supporting materials).

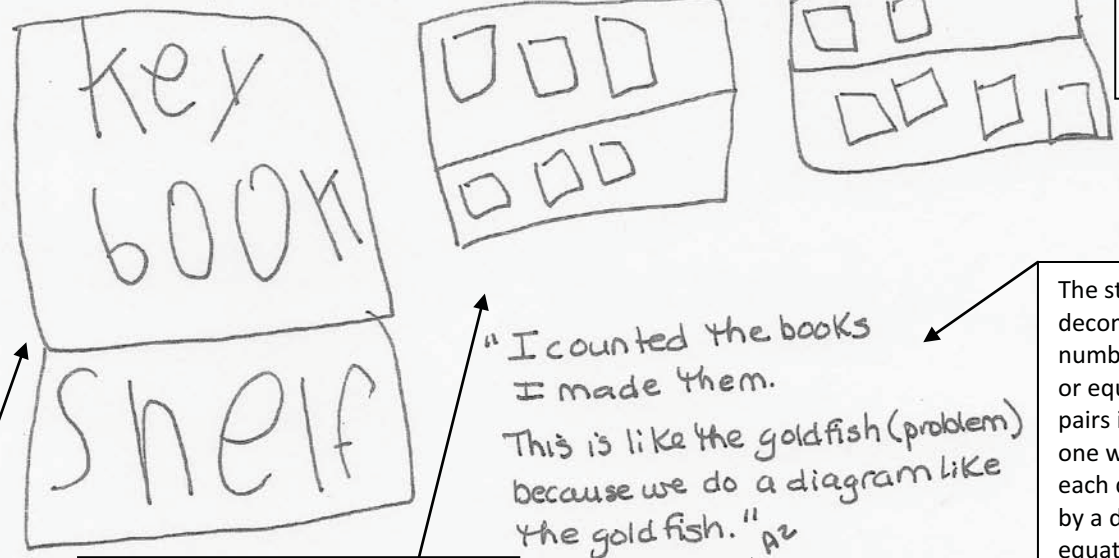
Apprentice

Name Student 2 K

Books on Shelves

Miguel has two shelves.
Miguel has six books.
Miguel wants to put books on the two shelves.
How many different ways can Miguel put books on the two shelves?
Show and tell how you know.

I will draw a diagram



"I counted the books I made them.
This is like the goldfish (problem) because we do a diagram like the gold fish." A2

The student is able to make sense of the problem but does not persevere in solving it. The student only finds two possible combinations.

The student applies the precise mathematical language- diagram, to help construct her/his viable argument. The term, key, is not credited because the key is not accurate.

The student models with mathematics making a diagram that is appropriate but not accurate. The key is not complete and the student's scribing only defines the books.

The student can decompose numbers less than or equal to 10 in pairs in more than one way, and record each decomposition by a drawing or equation (K.OA 3), but only does so for two possible combinations for six.

The student does not reflect on her/his solution to make connections to regularities, patterns, structures, or trends.

page 1 of 1

Kindergarten Math: Books on Shelves
Annotated Student Work

Apprentice – Student 2 Summary

Achievement Level: Student 2 is an Apprentice according to both the Exemplars Process Rubric and the CCSS Content Standards Rubric, (both of which are included in the supporting materials). The table below provides a rationale for the student's performance level in each of the criteria identified in the Exemplars Process Rubric.

Criteria and Performance Level	Rationale
Problem Solving Apprentice	The student's strategy of using a diagram to show the total combinations for placing six books on two shelves would work to solve the problem. The student shows two correct combinations which is a partially correct answer.
Reasoning and Proof Apprentice	The student shows correct reasoning of the underlying mathematical concept of combinations to six but only completes two combinations. The student is missing 1 + 5 = 6, 4 + 2 = 6, and, 5 + 1 = 6.
Communication Apprentice	The student correctly uses the mathematical term-diagram. The student does not earn credit for the term-key because the student does not complete her/his key to demonstrate understanding of the term.
Connections Apprentice	The student attempts to make a mathematically relevant connection about her/his solution. The student starts to compare the Books on Shelves problem to the Goldfish problem but does not make a mathematical link about the common underlying mathematics of the two problems. The student needs to explain how the diagrams are similar and why.
Representation Apprentice	The student's diagram is appropriate to the problem but is not accurate. The student does not define the □ 's and does not include all five possible combinations.

Kindergarten Math: Books on Shelves
Annotated Student Work

This student is an Apprentice according to both the Exemplars Rubric and the CCSS Content Standards Rubric, (both included in the supporting materials).

Apprentice

Name Student 3 K

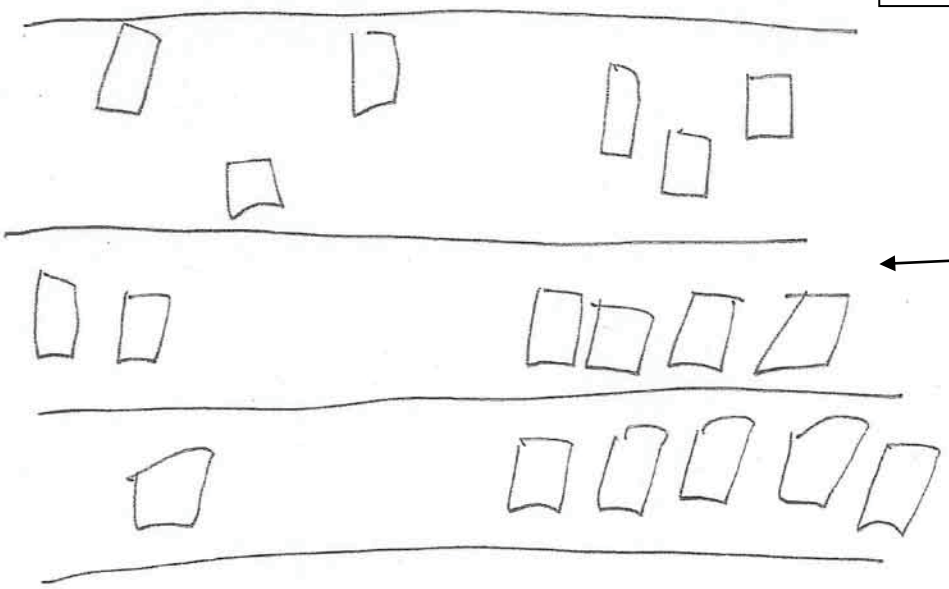
Books on Shelves

Miguel has two shelves.
Miguel has six books.
Miguel wants to put books on the two shelves.
How many different ways can Miguel put books on the two shelves?
Show and tell how you know.

The student applies the precise mathematical language- diagram to help construct her/his viable argument.

The student is able to make sense of the problem but does not persevere in solving it. The student only finds three possible combinations for six books on two shelves which leads to a partial answer.

My diagram



The student's diagram is appropriate but not accurate. Two combinations are not indicated and a key or scribing is not included to define the shelves and books.

3 ways

The student does not reflect on her/his solution to make connections to regularities, patterns, structures, or trends.

page 1 of 1

The student can decompose numbers less than or equal to 10 in pairs in more than one way, and record each decomposition by a drawing or equation (K.OA 3), but only shows three possible combinations.

Kindergarten Math: Books on Shelves
Annotated Student Work

Apprentice – Student 3 Summary

Achievement Level: Student 3 is an Apprentice according to both the Exemplars Process Rubric and the CCSS Content Standards Rubric, (both of which are included in the supporting materials). The table below provides a rationale for the student's performance level in each of the criteria identified in the Exemplars Process Rubric.

Criteria and Performance Level	Rationale
Problem Solving Apprentice	The student's strategy of using a diagram to show combinations for placing six books on two shelves would work to solve the problem. The student only shows three correct combinations. The student's answer, "3 ways," is not correct.
Reasoning and Proof Apprentice	The student shows correct reasoning of the underlying mathematical concepts of combinations to six, and the understanding that 0 + 6 = 6 and 6 + 0 = 6 cannot be considered as possible combinations for the problem. The student does not apply the commutative property in finding the combinations for 4 + 2 = 6 and 5 + 1 = 6
Communication Apprentice	The student correctly uses the mathematical term-diagram.
Connections Novice	The student does not make a mathematically relevant connection about her/his solution.
Representation Apprentice	The student's diagram is appropriate to the problem but is not accurate. The student does not define the books and shelves in a key or in any scribing. The diagram is missing the combinations for 4 and 2 and 5 and 1.

Instructional Implications: Books on Shelves K

Student Achievement Levels: Novice and Apprentice

The following is a list of instructional implications that you may want to consider for students performing at the Novice and Apprentice levels:

- continue to subitize with student to "see" sixness, or any number up to ten using hands and dot cards
- use manipulatives to investigate "fact families"/combinations of six or other numbers up to ten to discover how applying the commutative property in an organized manner can support a correct answer.
- use a ten frame to find combinations of six or other numbers up to ten
- use games such as cup and counters to find combinations of six or other numbers to up ten
- use graph paper and two color crayons to show combinations to six or other numbers up to ten, cut apart to show how the two "staircases" match for commutative property
- use number sentences to represent combinations/commutative property
- review mathematical language-model, number sentence, diagram, key, per, total, equal, add, fair share, combination, more than, less than
- review how to make a model with manipulatives or a diagram with a key
- have centers available for investigation and practice
- Provide leading questions to begin reflection on the solution in order to see regularities, structures, patterns, trends, etc.
- Solve similar problems using four, five, seven, eight, nine, or ten

Kindergarten Math: Books on Shelves
Annotated Student Work

This student is a Novice according to both the Exemplars Rubric and the CCSS Content Standards Rubric, (both included in the supporting materials).

Novice

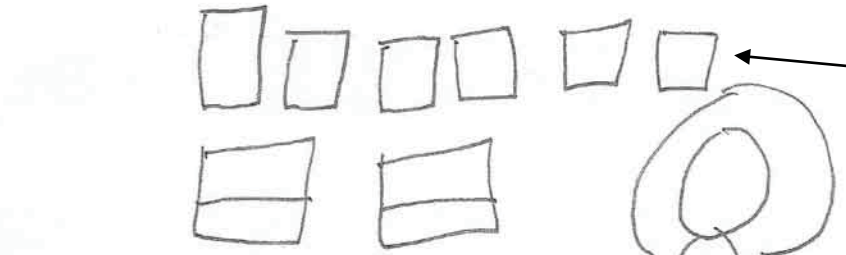
Name Student 1 K

Books on Shelves

Miguel has two shelves.
Miguel has six books.
Miguel wants to put books on the two shelves.
How many different ways can Miguel put books on the two shelves?
Show and tell how you know.

I will draw a diagram
Miguel has two shelves

The student does not discern that the problem requires decomposing six books to find five possible combinations. It appears that the student adds the two shelves and six books for a total of "8" which is an incorrect answer.



The student does not decompose numbers less than or equal to 10 into pairs in more than one way, and record each decomposition by a drawing or equation (K.OA 3).

"I did 6'
1,2,3,4,5,6
I did 2
1,2
I got 8
1,2,3,4,5,6,7,8"
A-2

The student applies the precise mathematical language-diagram and key, to help construct her/his argument.

The student attempts to model with mathematics but the diagram is not accurate.

page 1 of 1

Kindergarten Math: Books on Shelves
Annotated Student Work

Novice - Student 1 Summary

Achievement Level: Student 1 is a Novice according to both the Exemplars Process Rubric and the CCSS Content Standards Rubric, (both of which are included in the supporting materials). The table below provides a rationale for the student's performance level in each of the criteria identified in the Exemplars Process Rubric.

Criteria and Performance Level	Rationale
Problem Solving Novice	The student's strategy of using a diagram to show six books and what appears to be two shelves, adding the two numbers for a total of eight would not work to solve the problem. The student's answer, "8," is not correct.
Reasoning and Proof Novice	The student shows no correct reasoning of the underlying mathematical concepts of combinations, applying zero to combinations, and the commutative property. It seems that the student is simply adding the two numbers stated in the problem for a sum of eight.
Communication Practitioner	The student correctly uses the mathematical terms-diagram and key to support her/his thinking.
Connections Novice	The student does not make a mathematically relevant connection about her/his solution.
Representation Apprentice	The student attempts a diagram of books and shelves but the diagram is not accurate. The student is diagramming the six books and two shelves stated in the problem and adding the two numbers together.

Kindergarten Math: Books on Shelves
Annotated Student Work

This student is a Novice according to both the Exemplars Rubric and the CCSS Content Standards Rubric, (both included in the supporting materials).

Novice

Name Student 2

K

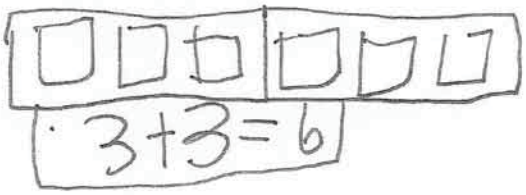
Books on Shelves

Miguel has two shelves.
Miguel has six books.
Miguel wants to put books on the two shelves.
How many different ways can Miguel put books on the two shelves?
Show and tell how you know.

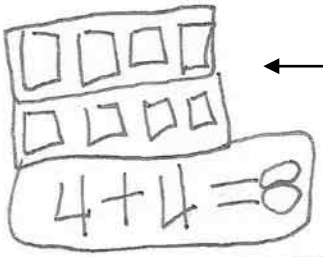
I will draw a diagram

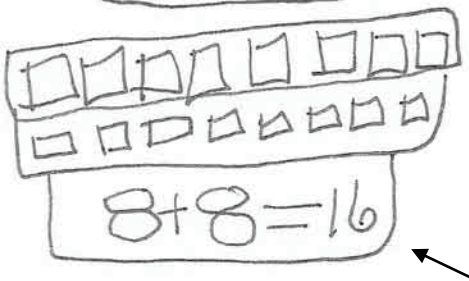
The student applies the precise mathematical language-diagram, key, to help construct her/his argument.

The student does not discern that the problem requires decomposing six books to find five possible combinations. The student states a correct combination for six but does not apply the concept of combinations of six to her/his solution.



The student does not decompose numbers less than or equal to 10 in pairs in more than one way, and record each decomposition by a drawing or equation (K.OA 3).





The student does not reflect on her/his solution to make connections to regularities, patterns, structures, or trends in the problem.

The student attempts to model with mathematics but the diagram is not accurate. The student is representing books and shelves to find the double of three, four, and eight.

page 1 of 1

Kindergarten Math: Books on Shelves
Annotated Student Work

Novice - Student 2 Summary

Achievement Level: Student 2 is a Novice according to both the Exemplars Process Rubric and the CCSS Content Standards Rubric, (both of which are included in the supporting materials). The table below provides a rationale for the student's performance level in each of the criteria identified in the Exemplars Process Rubric.

Criteria and Performance Level	Rationale
Problem Solving Novice	The student's strategy of using a diagram to show three books and three books equaling six books, four books and four books equalling eight books, and eight books and eight books equalling sixteen books would not work to solve the problem. Although the student states a correct combination for six, the student is not applying the concept of combinations to her/his solution.
Reasoning and Proof Novice	The student shows no correct reasoning of the underlying mathematical concept of combinations to six. It appears that the student is finding "doubles" using three, four, and eight.
Communication Practitioner	The student correctly uses the mathematical terms-diagram and key to support her/his thinking.
Connections Novice	The student does not make a mathematically relevant connection about her/his solution.
Representation Apprentice	The student attempts a diagram of books and shelves but the diagram is not accurate. The student is using books and shelves to find the double of three, four, and eight instead of diagramming possible combinations of six.

KINDERGARTEN MATH: BOOKS ON SHELVES INSTRUCTIONAL SUPPORTS

The instructional supports on the following pages include a unit outline with formative assessments and suggested learning activities. Teachers may use this unit outline as it is described, integrate parts of it into a currently existing curriculum unit, or use it as a model or checklist for a currently existing unit on a different topic.

Unit Outline –Kindergarten Math

INTRODUCTION: This unit outline provides an example of how teachers may integrate performance tasks into a unit. *Teachers may (a) use this unit outline as it is described below; (b) integrate parts of it into a currently existing curriculum unit; or (c) use it as a model or checklist for a currently existing unit on a different topic.*

Kindergarten Mathematics: Operations and Algebraic Thinking Unit

UNIT TOPIC AND LENGTH:

- This unit focuses on initial addition concepts with objects, drawings, dramatization, verbal explanations or expressions and equations. Students will work on decomposing numbers up to 10, using and recording their work with objects, drawings and or equations. At this time students should understand cardinal counting, but not necessarily conserve number.
- For developmental reasons this unit should happen in the second half of the year and can last several weeks. *(In Kindergarten, routines and games that support the mathematics in this unit should be happening all year and are not limited to one unit.)*

COMMON CORE CONTENT STANDARDS:

- K.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings[1], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.
- K.OA.3 Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).
- K.CC.4 Understand the relationship between numbers and quantities; connect counting to cardinality.
- MP.1 Make sense of problems and persevere in solving them.
- MP.3 Construct viable arguments and critique the reasoning of others.
- MP.4 Model with mathematics
- MP. 6 Attend to precision

BIG IDEAS/ENDURING UNDERSTANDINGS:

- Mathematicians can organize, represent, and compare the same number using different groupings (numbers or objects).
- Mathematicians can explain how numbers are organized, represented, and compared.

ESSENTIAL QUESTIONS:

- How do we show that numbers work together?
- How can we show and explain our thinking?

[1] Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)

Unit Outline –Kindergarten Math

CONTENT:

Numbers and quantities up to 10

- Order of numbers
- One to one correspondence
- Count
- Written numbers up to 10
- Quantities up to 10

Addition and subtraction up to 10 with objects

Number composition and decomposition

- Put together/take apart number strategies. Numbers less than and up to 10, not including zero, can be composed and put back together in more than one way.

SKILLS:

- **Recognize and name** numbers up to 10
- **Count** up to 10 orally
- **Match** a written number to objects
- **Sequence** numbers 1-up to 10
- **Write** numbers 1 – up to 10
- **Count** a number of objects
- **Demonstrate** that numbers have a quantity using objects

- **Add** objects to a set to show a number
- **Take away** objects in a set to show a number
- **Manipulate** objects to show a number sentence
- **Demonstrate** at least 2 different combinations of objects for one number
- **Explain** how to add or subtract objects to show a different number

KEY TERMS/ VOCABULARY:

- Add, subtract, explain

ASSESSMENT EVIDENCE AND ACTIVITIES:

INITIAL ASSESSMENT : GOLDFISH

Students are given a narrative situation in which they are asked to decompose the number 4 in as many ways as they can. Students represent the different combinations using pictures, numbers or any manipulative allowing them to show pairs of numbers that make four. Some Kindergarteners may need manipulatives such as stickers, unifix cubes, etc rather than actual drawings or symbolic numerical representation to show the decomposition pairs. The teacher should be carefully observing students as they work on this task noting the strategies they use. *See the task Goldfish and the associated planning sheet for full details.*

FORMATIVE ASSESSMENT: PRETTY TULIPS

Students are given another narrative situation in which they are asked to decompose the number five in as many ways as they can. Five is a benchmark number in our base ten system and it's important that students have fluency with the number. The students should be able to represent their solutions with manipulatives, pictures, five frames, numbers and or equations. The teacher should be carefully observing students as they work on this task noting the

strategies they use. *See the task Pretty Tulips and the associated planning sheet for full details.*

FINAL PERFORMANCE TASK: BOOKS ON SHELVES

Students are given a narrative situation in which they are asked to decompose the number six into as many ways as they can. Some Kindergarteners may need manipulatives such as stickers, unifix cubes etc rather than actual drawings or symbolic numerical representation to show the decomposition pairs. The teacher should be carefully observing students as they work on this task noting the strategies they use. *See the task Books on Shelves and the associated planning sheet for full details.*

LEARNING PLAN & ACTIVITIES:

- Routines and activities around counting and creating sets for a given number are important.
- Five and Ten frame counting and initial adding activities are listed below in a developmental sequence. Corresponding activity sheets are included in the unit. **It should be noted that most kindergarten instructional programs include games and routines that address these standards. The list below is a sampling of routines and games you may add to your repertoire. ***

  Quick Images: This routine can be done with dice faces, dot cards, five frames and ten frames. Initially children can use manipulatives to show the quantity projected on the screen. As the student's skills development around subitizing you can project more complex image amounts from the ten frames for a shorter amount of time. Challenge the students to say a quantity that is to more or two less than what is on the screen.

  Breaking cubes: The teacher should model this activity first before children to play with their partners.

  Unifix Towers

  Make Five on the Five Frame

  Shake Five and Spill

  Sums of Five

  Five Squares

  Part-Whole Mats

  Make Ten on the Ten Frame

  Making Apple Ten Packs

  Bunk Bed Problem

  On and Off

  Counters in a Cup

- Closely observe students while they are engaged in the activities and make note of the strategies they are using. Are they beginning to subitize or are they counting all? Are they finding several solutions or are they moving on after one or two?
- Make sure that you give students the opportunity to talk about their experience with their classmates.

Let children explain how they found their answers to each other.

- Students should be asked if later activities and assessment problems remind them of other activities they have done. Do they recognize that although the numbers and contexts are different, the mathematics is the same?

*Activities and games are included in this packet

RESOURCES:

Children's Related Literature Sampling:

- *Rooster's off to See the World* by Eric Carle
- *The Very Hungry Caterpillar* by Eric Carle
- *Ten Black Dots* by Donald Crews
- *Benny's Pennies* by Pat Brisson
- *Fish Eyes: A Book You Can Count On* by Lois Ehlert
- *Ten Little Rabbits* by Virginia Grossman
- *Ten, Nine, Eight* by Molly Bang
- *Mouse Count* by Ellen Stoll Walsh
- *Ten Flashing Fireflies* by Philemon Sturges
- *1 Hunter* by Pat Hutchins
- *12 ways to get to 11* by Eve Merriam

Professional Literature Resources and Research Articles:

Learning and Teaching Early Mathematics by Douglas Clements and Julie Surgma
Teaching Student Centered Mathematics Volume 1 Grades K-2 by Van de Walle and Lovin
Coming to Know Number by Wheatley and Reynolds
Young Mathematicians at Work: Constructing Number Sense, Addition and Subtraction by Fosnot and Dolk
Number Talks by Sherry Parrish

Research Articles

Number Relationships in Preschool by Myoungwhon Jung
An Algebraic-Habits-of-Mind Perspective on Elementary School by Goldenberg, Mark and Cuocco

Websites directly related to the unit, great for games activities and ideas:

For teachers:

- http://www.K-5mathteachingresources.com
- http://commoncoretools.files.wordpress.com/2011/05/ccss_progression_cc_oa_k5_2011_05_302.pdf

For students:

- http://illuminations.nctm.org/LessonDetail.aspx?ID=L54
- http://illuminations.nctm.org/LessonDetail.aspx?ID=L26
- http://www.fi.uu.nl/rekenweb/en/welcome.xml?groep=2

P/S	R/P	Com	Con	Rep	Ach/Level

Name_____

Goldfish

Jarod has four goldfish. Jarod has a square bowl and a round bowl. Jarod wants to put the four goldfish in the two bowls. How many different ways can Jarod put the four goldfish in two bowls? Show and tell how you know.

Preliminary Planning Sheet for a Mathematics Portfolio Piece/Task

(K)

Title of Task: Goldfish

Content Strand(s) Addressed: Operations + Algebraic Thinking

State Standard(s) Addressed:

Program Link:

Common Core Standard(s): K.OA.3

Underlying Mathematical Concepts

- Combinations of 4
- square / round
- number sense to 4
- counting on / addition (commutative property)

Problem Solving Strategies/Representation

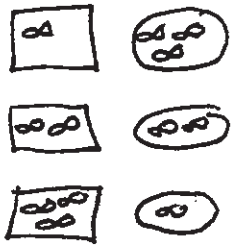
- diagram / key
- model (manipulatives)
- tallychart
- table

Mathematical Language

diagram, key, model, tallychart, table, odd, even, combination, per, commutative property, square, more/less/than, equal, fairshare, pattern, equation

Possible Solution(s)

Answer: 3 ways / combinations



Key
- square bowl
- round bowl
- gold fish

Goldfish	

1 + 3 = 4
2 + 2 = 4
3 + 1 = 4

Goldfish	
fishin square bowl	fishin round bowl
1	3
2	2
3	1

1 + 3 = 4
2 + 2 = 4
3 + 1 = 4

or use graph paper and colors

Connections

- 2 goldfish per bowl is a fair share
- Always even fish or odd gold-fish in a combination
- 1 less combination than total gold-fish - prove this phenomenon
- Recreate problem by adding more goldfish to see if "1 less rule" continues
- Relate to a similar problem and state math
- Verify by solving more than one way

Related Tasks

See Resource Binder

- Can't use zero - fish have to be in both bowls

P/S	R/P	Com	Con	Rep	Ach/Level

Name_____

Pretty Tulips

Hector has five tulips. Hector has two vases. Hector wants to put the tulips in the two vases. Hector needs your help. How many different ways can Hector put five tulips in two vases? Show and tell how you know.

Games & Activities

Breaking Cubes

Materials: 5-10 snap or unifix cubes in a stick

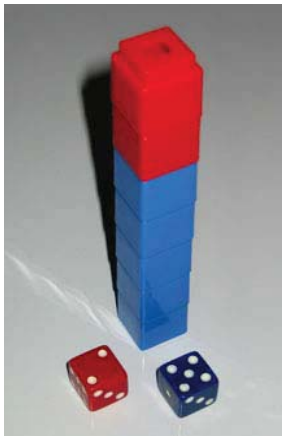
Players: 1-3

Object: To figure out how many cubes are hidden behind Your partner's back.

How to Play:

1. Make a stick of cubes 5-10 cubes long and one color.
2. Behind your back break apart your cube stick into two parts and show only one part.
3. Your partner will guess how many sticks you have hidden behind your back.
4. Show what you have hidden.
5. Let you partner have a turn and repeat steps 1-3

Unifix Towers



Materials dice, unifix cubes, paper, pencils

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1. Roll two dice and build a unifix tower to match the total.

2. Keep rolling and building until you have made 5 towers.

3. Put your towers in order from smallest to largest.

4. Draw a picture or write about your towers.

Make Five on the Five Frame

***Materials*:** two color counters (red and yellow), blank five frame

---

I placed five counters on my five frame. Some were red and some were yellow. What might my five frame have looked like?

Use pictures, numbers, or words to show as many different solutions as you can.

How many different solutions did you find?

Sums of Five

Materials: dot or numeral cards 0-5, Sums of Five gameboard, 10 counters for each player of different colors (e.g. one stick of 10 orange Unifix cubes and one stick of 10 blue Unifix cubes)

---

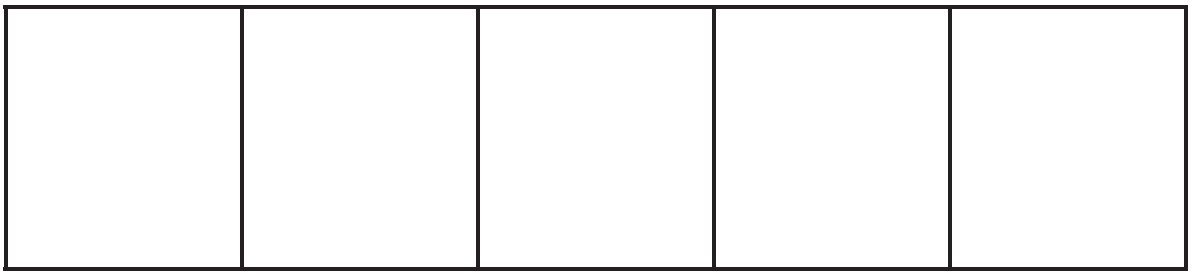
1. Work with a partner. Player A: Turn over a card and place that many counters on the first five-frame on the gameboard. Complete the math talk sentence

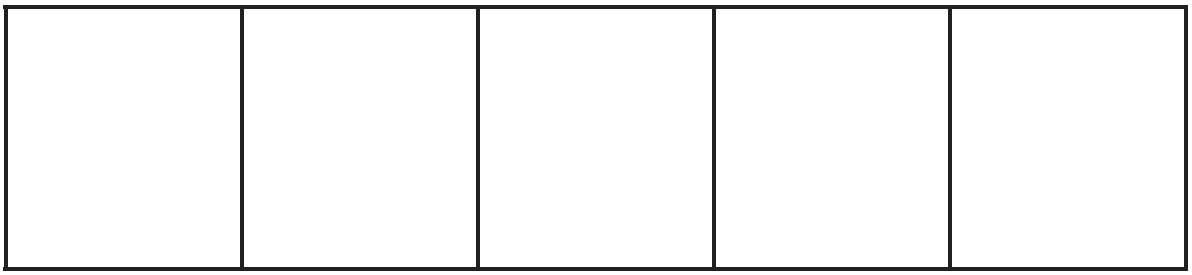
2. Player B: Turn over a card and place that many counters on the second five-frame on the gameboard. Complete the math talk sentences.

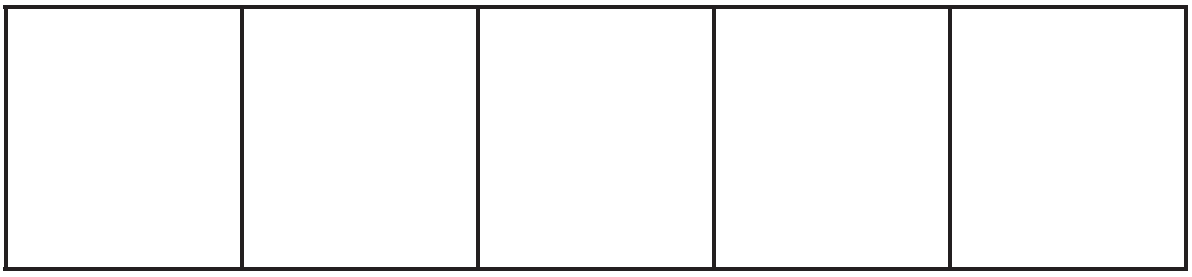
3. Player A: Turn over a card and place that many counters on the third five-frame on the gameboard. Complete the math talk sentences.

4. Continue to turn over cards to try and fill a five-frame. Each frame must be filled with only two cards (e.g. 4 and 1 or 2 and 3). If a player turns over a card that can not be used to complete a frame she or he misses a turn. Whoever completes a frame scores a point.

Sums of Five







Five Squares

Materials: container of square pattern blocks, numeral cards (0-5)

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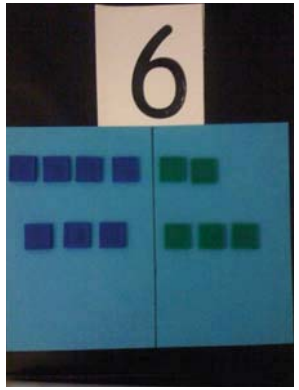
1. Count out five square pattern blocks.
2. Put your five squares into two sets.
3. Show a different way to put the five squares into two sets.
4. How many different ways can you find to do this?
5. Record your work using pictures, numbers, or words.

Part-Whole Mats



Materials: Part-Whole Mats, numeral cards, counters

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1. Choose a numeral card and place it above your Part-Whole Mat.

2. How many different pairs of numbers can you find to equal the number on your Part-Whole Mat?

3. Use pictures, numbers or words to record your work.

Part-Whole Mats

Mathematical Ideas: Part-whole relationships, Using counting strategies and number facts to solve problems, Commutativity. Communicating Mathematically

Math Vocabulary: different, equals, altogether, pairs

Materials: part-whole mats, numeral cards, counters, blank paper, pencils

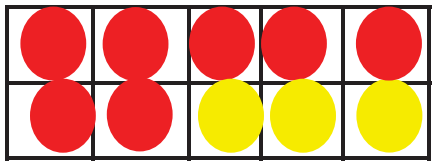
Starting Point: Choose a numeral card. Place the number you have chosen above your Part-Whole mat. How many different pairs of numbers can you find to equal the number on your Part-Whole Mat? Use pictures, numbers or words to record your work.

Possible questions to develop and extend students' thinking:

- How many different pairs of numbers have you found that equal (10)?
- Do you know any other pairs of numbers that equal (10)?
- Tell me about your recording.
- Can you order your number sentences? What do you notice when you put your number sentences in order?
- You recorded that (7) plus (3) equals (10) and (3) plus (7) equals (10). Why do (7) plus (3) and (3) plus (7) both equal (10)?

  * Adjust numbers in ( ) to match student's recording.

Make Ten on the Ten Frame



***Materials*:** two color counters (red and yellow), blank ten frame

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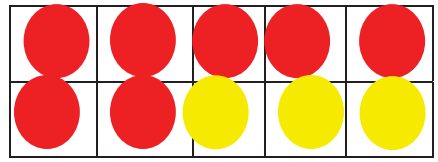
I placed ten counters on my ten frame. Some were red and some were yellow. What might my ten frame have looked like?

Draw a picture and write a number model for each solution that you find.

How many different solutions did you find?

Making Apple Ten Packs



***Materials*:** red and yellow counters, ten frames

The greengrocer wanted to make a pack of ten apples.

How many different ways could the greengrocer make a ten pack with some red and some yellow apples?

Use pictures, numbers, or words to show your thinking.

Bunk Bed Problem

Materials: counters, numeral cards 5-12, pencils, blank paper

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1. Turn over a numeral card and use it to complete this number story.

   ___ children sat on a bunk bed. Some sat on the top bunk and some sat on the bottom bunk. How many sat on the top bunk? How many sat on the bottom bunk?

2. Record as many different solutions to the problem as you can using pictures, numbers or words.

On and Off

Materials: Counters (5-10)
On and Off game grid
Sheet of paper

Players: 1-3
Object: Toss counters over a sheet of paper.
Record how many land on and off the paper.

How to Play

1. Decide how many counters you will toss each time. Write this total number on the game grid.
2. Lay the sheet of paper on a flat surface.
3. Hold the counters in one hand and toss them over the paper.
4. On the game grid, write how many landed on the paper and off the paper.
5. Repeat steps 3 and 4 until you have filled one game grid.

You can assign a total number and ask children to tally or represent the counters with a dot.

On and Off Game Grid

On	Off

Counters in a Cup

Materials: Counters (5-10)
Counters in a Cup game grid
Paper cup

Players: 2
Object: Figure out how many of a set of counters are hidden.

How to Play

1. Decide how many counters you will use each time. Write this total number on the game grid.
2. Player A hides a secret number of counters under the cup and leaves the rest out.
3. Player B figures out how many are hidden and says the number. Lift the cup to check.
4. On the game grid, write the number hidden in the cup and the number left out.
5. Players switch roles. Hide a different number of counters. (It's ok to hide the same number of counters more than once in a game.)
6. Repeat steps 2-5 until you have filled the game grid. (Hide the counters eight times.)

You can assign a total number and ask children to tally or represent the counters with a dot.

On and Off Game Grid

On	Off

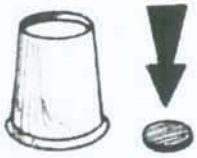
Name

Date

Student Sheet 3

Counters in a Cup Game Grid

Total number: _____

In	Out
	

PRE-K MATH: HOW MANY LITTLE SEEDS?

SUPPORTS FOR ENGLISH LANGUAGE LEARNERS

GRADE PRE-K MATH: HOW MANY LITTLE SEEDS?

Supports for ELLs

Title: Performance-Based Assessment 1: Planting Operations and Algebraic Thinking—Addition/Subtraction	**Grade:** Pre-K

Linguistic Access:

In this performance-based assessment, a distinction between the vocabulary and the language functions is needed to provide entry points to the math content. Both need to be clarified to ensure comprehension and to avoid misunderstanding. This can be done by introducing the most essential vocabulary/language functions with the concrete models in order for English Language Learners to grasp the meaning of the terms. The following vocabulary/language functions are suggested:

Vocabulary Words:

Tier I (Non academic language): line, more, less, enough, fewer

Tier II (General academic language): all together, number

Tier III (Math technical language/concepts that must be carefully developed): addition, subtraction, total, add, sum

Language Functions: combine, take away, explain

Content Access:

For this performance assessment, a clear understanding of addition and subtraction is required. Many opportunities need to be provided whereby students physically bring sets together and/or combine them into one new set.

Students need to physically experience the separation of one set into parts to find out what is left (the remainder set). Students need to know that this is only one model for subtraction, at an early age.

Scaffolds:

- Students need to have previous experiences with number lines and picture graphs. For example, students can build their own number line with the teacher's assistance.
- Additionally, 6 matching cards can be made (each one showing one of the faces on a die) for students to use with the pictures of objects (1 to 6), the numerals (1 to 6), and the words (for 1 to 6) to help develop the concept and vocabulary. Furthermore, this will provide necessary preparation for the math game.

PRE-K MATH:
HOW MANY LITTLE SEEDS?

SUPPORTS FOR STUDENTS WITH DISABILITIES

PRE-K Math: How Many Little Seeds?

Instructional Supports for Students with Disabilities using UDL Guidelines

Background Information

Learners differ in the ways that they perceive and comprehend information and may require a different process to acquire the same content. In addition, learners may differ markedly in the ways they engage, maintain attention to task and demonstrate what they know and have learned. Hence, the goal of a UDL curriculum is the interrelated components which comprise the goals, methods, materials and assessment. In this way, all students would then be able to generalize their mathematical understanding for real world application.

Day 1: PREPARING STUDENTS

Provide options for perception- Offer ways of customizing the display of information

- Offer students individual desk top number lines.
- Illustrate and concretize concepts using prepared clothes pins containing arrows to represent counting down (subtracting) seeds from the Number Line.

Provide options for physical action- Vary the methods for response and navigation

- *Offer students seeds of varying sizes (large vs. small) and rubber placemat or tray to facilitate manipulation of seeds.*

Provide options for comprehension- Guide information processing, visualization and manipulation

- Display the Ten Little Flower Seeds song at eye level. Choral read the lines of the song one at a time.
- Sketch, illustrate or affix flannel board prompts to provide explicit models for learning the verse.
- Offer students the opportunity to echo, sing along and/or repeat the same verse using pair share or individualize support.
- Stop, orient and support comprehension and clarify vocabulary using illustration, clip art technology and or flannel board materials
- Record the song and offer students the opportunity to listen on headphones for reinforcement
- Ask students to replay the song in their mind and sing in large group, small group or individually.
- After removing one seed at a time, offer students the opportunity to count how many seeds are left, counting down from ten on their individual desk-top number lines

Provide options for executive functioning- Enhance capacity for monitoring progress

- Provide prompts, reminders and ground rules/management plans that reduce the frequency of off-task behaviors in response to struggles or low stamina, as appropriate

- Offer children models and mentors that support the range of attention, cognitive, sensory and language strengths and challenges

Provide options for self-regulation- Facilitate personal coping skills and strategies

- Provide options for self regulation when students assemble for group activities, e.g., requests/permission to assemble on the rug, in chairs around a table, take a short "stretch" and/or excuse yourself for other appropriate actions.
- Provide appropriate procedures for transitions when students assemble for group activities, e.g., excuse yourself for personal needs, to work with related service providers or to "rejoin" the group to decrease distractions.

Provide options for comprehension- Highlight patterns, critical features, big ideas, and relationships

- Provide feedback and models for incorporating positive strategies for success, e.g., group leaders to facilitate at the flannel board for the group.

Day 2: How many Little Seeds? A Mathematics Game

Provide options for recruiting interest- Optimize relevance, value and authenticity

- Establish clear expectations for group work. Assign work group roles. Post class-created rubric where all students can view.
- Establish clear protocols for class discussions***:*** whole group; small groups; think-pair-share; and turn and talk.

Provide multiple means of action and expression- Provide options for physical action

- Illustrate through multiple media- Offer students the opportunity to view a "gardener placing seeds into the ground" and "birds taking seeds out of the ground." Generalize the concept, role play, demonstrate and practice the concept of using the flower pot and the flannel board number line and seeds. Think about using prompts or costumes, e.g., straw hat to portray the gardener and a feather headband to portray the bird in role.

- Enhance capacity for monitoring progress. Establish rituals and routines that prompt learners to identify the type of feedback, advice, and/or assistance as they practice taking the role of the Gardener, Bird and Record Keeper at the flannel board Number Line.
- Offer students the opportunity to create and perform a short skit.
- Increase mastery-oriented feedback**.** While students are observing/listening to classmates practice the steps needed to play the Mathematics Game, periodically select students to retell, sketch or elaborate to monitor comprehension.

Provide options for perception- Offer alternatives for auditory information

- If available, utilize an FM system to decrease distractions from extraneous/ambient noise.
- Offer children preferred seating, as appropriate.
- Repeat auditory prompts and encourage children to echo your questions, e.g., "How many seeds are in the pot now?"
- Provide alternatives to word call questions. Pointing to the Flannel Board Number Line, ask, "Do you see two seeds?" and "Can you come up and point to the number 5?" Or "Can you show me the total using your fingers?"

Provide options for comprehension-Activate or supply background knowledge

- Anchor instruction by linking to and activating relevant prior knowledge.
- Allow one or two students to model for the class.
- Ask children to predict, "What do you think would happen if there were five seeds in the pot and the gardener ***added*** two more seeds?" Ensure the students have a solid grasp of the vocabulary used for Operations and Algebraic Thinking.
- Prepare word cards for the following key words: gardener, bird, total, addition and subtraction. Place Velcro strips on the back of the word cards to connect actions in the mathematics game to the vocabulary words.

Formative Assessment Questions

As students play the game, use flexible grouping to assemble students. Allow individual attention as appropriate. Offer students the opportunity to work in pairs and small groups as appropriate. Provide multiple entry points for all students as the teacher questions and documents how students respond to the questions while playing the game.

Provide options for sustaining effort and persistence- Foster collaboration and community

- A teacher should be pre-teach and be present at the table to create cooperative learning groups with clear goals, roles and responsibilities. Provide prompts that guide learners in when and how to ask peers and/or teacher(s) for assistance. Encourage and support opportunities for peer tutors and construct communities of learners engaged in the Math Game for an appropriate length of time prior to assessment.

Provide options for sustaining and persistence- Increase mastery-oriented feedback.

- As students are prompted to respond to the assessment questions, the teacher should explain that responses will be written in the student's exact words. Offer preferential seating and ensure that all students assessed have optimal conditions for response. Limit extraneous noise. Request parental permission AND practice using video equipment prior to the assessment, if a camera will be used to capture the student in process of playing the game for assessment. Encourage student response through question repetition as needed. Prompt students to refocus on task, as appropriate.

- Document the student responses on the Teacher Template applying the Rubric. Ensure that appropriate concrete objects (Flower Pot, Desk Top Number Line and large sized seeds) are in proper repair for all the students.