GRADE **3** MATH: COOKIE DOUGH

UNIT OVERVIEW

This packet contains a curriculum-embedded CCLS aligned task and instructional supports. The task is embedded in a 4-5 week unit on interpreting and linking representations, modeling situations, solving non-routine problems and justifying arguments of multiplication and division.

TASK DETAILS

Task Name: Cookie Dough

Grade: 3

Subject: Mathematics

Task Description: The tasks in the unit access the full range of *Depth of Knowledge,* including Recalling and Recognizing, Using Procedures, Explaining, Concluding and Making Connections, Extensions and Justifying.

Standards:
 3.OA.1 Interpret products of whole numbers, e.g., interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each.
3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects and partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each.
3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers.
3.OA.5 Apply properties or operations as strategies to multiply and divide.
3.OA.6 Understand division as an unknown-factor problem.
3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 x 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.
3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
3.OA.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.

Standards for Mathematical Practice:
MP.1 Make sense of problems and persevere in solving them.
MP.2 Reason abstractly and quantitatively.
MP.3 Construct viable arguments and critique the reasoning of others.
MP.4 Model using mathematics.

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The task and instructional supports in the following pages are designed to help educators understand and implement tasks that are embedded in Common Core-aligned curricula. While the focus for the 2011-2012 Instructional Expectations is on engaging students in Common Core-aligned culminating tasks, it is imperative that the tasks are embedded in units of study that are also aligned to the new standards. Rather than asking teachers to introduce a task into the semester without context, this work is intended to encourage analysis of student and teacher work to understand what alignment looks like. We have learned through the 2010-2011 Common Core pilots that beginning with rigorous assessments drive significant shifts in curriculum and pedagogy. Universal Design for Learning (UDL) support is included to ensure multiple entry points for all learners, including students with disabilities and English language learners.

GRADE 3 MATH: COOKIE DOUGH PERFORMANCE TASK

Performance Task
Cookie Dough

Clear Creek School is fundraising. They are selling Cookie Dough in tubs.

Chocolate Chip Cookie Dough $5 a tub	Peanut Butter Cookie Dough $4 a tub	Oatmeal Cookie Dough $3 a tub

1. Jill sold 2 tubs of Oatmeal Cookie Dough.

    How much money did she raise? $ _____

    Show how you figured it out.

2. Joe sold 4 tubs of Peanut Butter Cookie Dough and 4 tubs of Chocolate Chip Cookie Dough.

    How much money did he raise in all? $ _____

    Show how you figured it out.

3. Jade sold *only* Peanut Butter Cookie Dough.   She raised $32.

   How many tubs did she sell?   _____ tubs
   Show how you figured it out.

4. Jermaine's mother loves oatmeal cookies.   She has $20 to spend.

   What is the greatest number of tubs of Oatmeal Cookie Dough she can

   buy?   _____ tubs

   Explain how you figured it out.

   _____

   _____

5. Justin's arithmetic work looks like this: 3 x $5 + 7 x $4 + 12 x $3

   Write a word problem for Justin's arithmetic AND give the answer.

   _____

   _____

   _____

GRADE 3 MATH: COOKIE DOUGH
RUBRIC

The rubric section contains a scoring guide and performance level descriptions for the Cookie Dough task.

Scoring Guide: The scoring guide is designed specifically to each small performance task. The points highlight each specific piece of student thinking and explanation required of the task and help teachers see common misconceptions (which errors or incorrect explanations) keep happening across several papers. The scoring guide can then be used to refer back to the performance level descriptions.

Performance Level Descriptions: Performance level descriptions help teachers think about the overall qualities of work for each task by providing information about the expected level of performance for students. Performance level descriptions provide score ranges for each level, which are assessed using the scoring guide.

Grade 3 Math: Cookie Dough

Cookie Dough Scoring Guide

Cookie Dough	Rubric	
The core elements of performance required by this task are: • work with multiplication and division in a real context Based on these, credit for specific aspects of performance should be assigned as follows:	points	section points
1. Gives correct answer: $**6.00**	1	1
2. Gives correct answer: $**36.00** Shows work such as: 4 x 4 and 4 x 5	1 1	2
3. Gives correct answer: **8** Shows work such as: 32 ÷ 4	1 1	2
4. Gives correct answer**: 6** Gives a correct explanation such as: The most her mother can buy is 6 tubs because I counted by 3s and she can buy 6 but doesn't have enough for 7 or, I counted 3, 6, 9, 12, 15, 18, 21. She doesn't have 21.	1 1	2
Total Points		**7**

Grade 3 Math: Cookie Dough
Rubric

Performance Level Description and Cut Scores
Performance is reported at four levels: 1 through 4, with 4 as the highest.

Level 1: Demonstrates Minimal Success (0 – 1 point)
The student's response shows few of the elements of performance that the tasks demand as defined by the CCSS. The work shows a minimal attempt on the problem and struggles to make a coherent attack on the problem. Communication is limited and shows minimal reasoning. The student's response rarely uses definitions in his/her explanations. The student struggles to recognize patterns or the structure of the problem situation.

Level 2: Performance Below Standard (2 – 3 points)
The student's response shows some of the elements of performance that the tasks demand, and some signs of a coherent attack on the core of some of the problems, as defined by the CCSS. However, shortcomings are substantial, and evidence suggests the student would not be able to produce high-quality solutions without significant further instruction. The student might ignore or fail to address some of the constraints of the problem. The student may occasionally make sense of quantities in relationships within the problem, but their use of quantity is limited or not fully developed. The student response may not state assumptions, definitions, and previously established results. While the student makes an attack on the problem, it is incomplete. The student may recognize some patterns or structures, but has trouble generalizing or using them to solve the problem.

Level 3: Performance at Standard (4 – 5 points)
For most of the task, the student response shows the main elements of performance the tasks demand as defined by the CCSS. The response is organized as a coherent attack on the core of the problem. There are errors or omissions, some of which may be important, but of a kind that the student could well fix, with more time for checking, revision and some limited help. The student explains the problem and identifies constraints, they make sense of quantities and their relationship to the problem situations. S/he often uses abstractions to represent a problem symbolically or with other mathematical representations. The student response may use assumptions, definitions, and previously established results in constructing arguments. S/he may make conjectures and build a logical progression of statements to explore the truth of their conjectures. The student might discern patterns or structures and make connections between representations.

Level 4: Achieves Standards at a High Level (6 – 7 points)
The student's response meets the demands of nearly all of the tasks as defined by the CCSS, with few errors. With some more time for checking and revision, excellent solutions would seem likely. The student response shows understanding and use of stated assumptions, definitions and previously established results in the construction of arguments. The student is able to make conjectures and build a logical progression of statements to explore the truth of his/her own conjecture. The student response routinely interprets his/her mathematical results in the context of the situation and reflects on whether the results make sense. The communication is precise, using definitions clearly. The student looks closely to discern a pattern or structure. The body of the work looks at the overall situation of the problem and process, while attending to the details.

GRADE 3 MATH: COOKIE DOUGH ANNOTATED STUDENT WORK

This section contains annotated student work at a range of score points and implications for instruction for each performance level. The student work shows examples of student understandings and misunderstandings of the task, which can be used with the implications for instruction to understand how to move students to the next performance level.

Level 4: Achieves Standards at a High Level (Score Range 6 – 7)

The student's response meets the demands of nearly the entire task, with few errors. With more time for checking and revision, excellent solutions would seem likely. The student response shows understanding and use of stated assumptions, definitions and previously established results in constructing arguments. The student is able to make conjectures and build a logical progression of statements to explore the truth of their conjecture. The student response routinely interprets their mathematical results in the context of the situation and reflects on whether the results make sense. The communication is precise, using definitions clearly. The students look closely to discern a pattern or structure. The body of work looks at the overall situation of the problem and process, while attending to the details.

The student achieved standards at a high level (4) with a score of 7. The student demonstrates understanding of all parts of the task and provides sound explanations.

Cookie Dough

This problem gives you the chance to:
- choose and use number operations in context

Clear Creek School is fundraising. They are selling Cookie Dough in tubs.

Chocolate Chip Cookie Dough	Peanut Butter Cookie Dough	Oatmeal Cookie Dough
$5 a tub	$4 a tub	$3 a tub

1. Jill sold 2 tubs of Oatmeal Cookie Dough. How much did she raise?

$$3 \times 2 = 6$$

$6.00

The student reasoned quantitatively and modeled a mathematical situation using number sentences. 3.OA, MP2, MP4

2. Joe sold 4 tubs of Peanut Butter Cookie Dough and 4 tubs of Chocolate Chip Cookie Dough. How much money did he raise in all? Show how you figured it out.

$$4 \times 4 = \$16$$
$$4 \times 5 = 20$$

$$\begin{array}{r} 20 \\ + 16 \\ \hline 36 \end{array}$$

$36.00

The student reasons quantitatively and creates a pattern of growth by fours and threes to determine the answer to the number of tubs. In this manner the student understood the growing pattern at a constant rate. 3.OA, MP2, MP2

3. Jade sold only Peanut Butter Cookie Dough. She raised $32. How many tubs did she sell? Show how you figured it out.

$$4, 8, 12, 16, 20, 24, 28, 32$$
$$1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8$$

___8___ tubs

4. Jermaine's mother loves oatmeal cookies. She has $20. to spend. What is the greatest number of tubs of Oatmeal Cookie Dough she can buy?

$$3, 6, 9, 12, 15, 18,$$
$$1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$$

___6___ tubs

Explain how you figured it out.

I skipped counted by 3's when I got to 18 I was going to skip more but it would be 21 dollars

The student was able to explain their process, which is the first step in justifying their findings. 3.OA.8&9, MP3

Third Grade: Cookie Dough
Annotated Student Work

This student's work is not only correct, but the explanations are very clear and complete. This student received a score of 7 and performed at Level 4.

Cookie Dough
This problem gives you the chance to:
- choose and use number operations in context

Clear Creek School is fundraising. They are selling Cookie Dough in tubs.

Chocolate Chip Cookie Dough	Peanut Butter Cookie Dough	Oatmeal Cookie Dough
$5 a tub	$4 a tub	$3 a tub

1. Jill sold 2 tubs of Oatmeal Cookie Dough. How much did she raise?

$$1 \text{ oatmeal} = \$3 \times 2 = \$6$$

$ __6__

In part 1 and 2, the student shows correct equations that models the situations, and labels the units that are part of reasoning quantitatively.
3.OA
MP2, MP4

2. Joe sold 4 tubs of Peanut Butter Cookie Dough and 4 tubs of Chocolate Chip Cookie Dough. How much money did he raise in all? Show how you figured it out.

$16 $ 20
         + $ 16
         $ 36   - This is my work of how I figured it out.

$ __36__

3. Jade sold only Peanut Butter Cookie Dough. She raised $32. How many tubs did she sell? Show how you figured it out.

$$(\frac{8}{?} \times 4) = 32$$ or another way...

$$\underset{}{32 \div 4 = 8} \text{ tubs}$$ $\frac{8}{}$ tubs

In part 3, the student links the operations of multiplication and division.
3.OA.5&6
MP2

4. Jermaine's mother loves oatmeal cookies. She has $20. to spend. What is the greatest number of tubs of Oatmeal Cookie Dough she can buy?

__6__ tubs

Explain how you figured it out.

First, I thought in my mind hmmm.. how many times can 3 go into 20? Then I counted 3,6,9,12,15,18, But then I noticed 21 was too much to spend. Then I looked how many numbers did I write when counting by 3's? Then I counted 3=1 6=2 9=3 12=4 15=5 18=6. I can't go on anymore because than it'll be too much money. So the answer is six tubs.

I erased 21 because it was too much.

The student writes a complete explanation for part 4, explains his/her thinking process, and associates the tub with the dollar amount at each interval. In addition, the student explains his/her justification to why 21 is too much. At third grade level, this is an example of a convincing argument. MP3

Third Grade: Cookie Dough
Annotated Student Work
Level 3: Performance at Standard (Score Range 4 – 5)

For most of the task, the student's response shows the main elements of performance that the tasks demand, and is organized as a coherent attack on the core of the problem. There are errors or omissions, some of which may be important, but of a kind that the student could well fix with more time for checking and revision, and some limited help. The student explains the problem and identifies constraints. The student makes sense of the quantities and their relationship in the problem situations. S/he often uses abstractions to represent a problem symbolically or with other mathematical representations. The student response may use assumptions, definitions, and previously established results in constructing arguments. S/he may make conjectures and build a logical progression of statements to explore the truth of their conjectures. The student might discern patterns or structures and make connections between representations.

This student met standard (Level 3) and scored 4. This student showed understanding, but made errors in both calculating and interpreting their own work.

The answer to part 3 is correct, yet the erased work does not indicate how the student arrived at the answer.

In part two, the student interpreted the situation, reasoned through the multi-steps, and modeled the number sentences correctly, but when multiplying 4 times 5 made a calculation error. The rest of the reasoning is sound.
3.OA.1 to 3, MP2, MP4

In part 4, the student used a chart associating 3 (dollars) to circles (tubs). The student correctly determined when to stop the pattern and stated that 21 went passed 20, but then the student did not distinguish correctly between the amount of tubs and dollars. The reasoning and problem solving was sound. MP1, MP2

Clear Creek School is fundraising. They are selling Cookie Dough in tubs.

Chocolate Chip Cookie Dough $5 a tub	Peanut Butter Cookie Dough $4 a tub	Oatmeal Cookie Dough $3 a tub

1. Jill sold 2 tubs of Oatmeal Cookie Dough. How much did she raise?

$ __6__

2. Joe sold 4 tubs of Peanut Butter Cookie Dough and 4 tubs of Chocolate Chip Cookie Dough. How much money did he raise in all?
Show how you figured it out.

$ __31__

$P = 16$ $C = 25$

$$\begin{array}{r} 16 \\ +25 \\ \hline \$31 \end{array}$$

3. Jade sold only Peanut Butter Cookie Dough. She raised $32. How many tubs did she sell?
Show how you figured it out.

__8__ tubs

4. Jermaine's mother loves oatmeal cookies. She has $20. to spend. What is the greatest number of tubs of Oatmeal Cookie Dough she can buy?

__18__ tubs

Explain how you figured it out.

I made a chart and
$3 \times 6 = 18$ and $3 \times 7 = 21$
it pasdof

Third Grade: Cookie Dough
Annotated Student Work

Cookie Dough

This problem gives you the chance to:
- choose and use number operations in context

Clear Creek School is fundraising. They are selling Cookie Dough in tubs.

Chocolate Chip Cookie Dough $5 a tub	Peanut Butter Cookie Dough $4 a tub	Oatmeal Cookie Dough $3 a tub

1. Jill sold 2 tubs of Oatmeal Cookie Dough. How much did she raise?

$ __6__

2. Joe sold 4 tubs of Peanut Butter Cookie Dough and 4 tubs of Chocolate Chip Cookie Dough. How much money did he raise in all? Show how you figured it out.

$ __34__

3. Jade sold only Peanut Butter Cookie Dough. She raised $32. How many tubs did she sell? Show how you figured it out.

__8__ tubs

4. Jermaine's mother loves oatmeal cookies. She has $20. to spend. What is the greatest number of tubs of Oatmeal Cookie Dough she can buy?

__6__ tubs

Explain how you figured it out.

I got the answer by counting by threes.

This student performed at Level 3 (Meeting Standards) with a rubric score of 5. The work shown is limited, but appears to show mathematical understanding.

The student was able to get the first part correct, and this indicates the student can reason quantitatively.

3.OA, MP2

The student provided incorrect answers for part 2 and showed no work, so it is not clear how the student arrived at $34.

In part 3, the student wrote a very faint number sentence that was correct 32÷4. Although the work does not show the actual calculation, the student did arrive at the correct answer. 3.OA.5&6

In part 4, the student found the correct answer by repeatedly counting groups of three. This involves reasoning quantitatively and understanding a problem situation. MP2, MP1

Level 3 Implications for Instruction

Students who met standard on the task can still improve their performance by being attentive to process and by making complete explanations and justifications. Students must learn to provide complete arguments when justifying why a statement is correct. It is equally important that students also explain why a value is not correct. When there is only one answer, the students should justify why. This requires the student not only to tell how an answer was arrived at, but also why another value cannot be a solution. In part four, students were asked to determine the greatest number of tubs of Oatmeal Dough that could be purchased with $20. A complete explanation isn't limited to the answer of number of tubs. It should also address why that answer is the maximum amount possible. Students need experiences in writing complete explanations and justification. They can benefit from reading other students' explanations and critique explanations to improve them. Students should follow the reasoning of sound justifications and write conclusions from deductive arguments. Routinely asking students to justify and explain their approaches, solutions, justifications and generalizations in class will foster the development of this skill. Students should be held accountable for complete explanations and must learn that mere answers are unsatisfactory. Explaining "why", "how you know" and "why an answer cannot be something else" should be part of every student's argumentation.

Third Grade: Cookie Dough
Annotated Student Work

Level 2: Performance below Standard (Score Range 2 - 3)

The student's response shows some of the elements of performance that the tasks demand and some signs of a coherent attack on the core of some of the problems. However, the shortcomings are substantial, and the evidence suggests that the student would not be able to produce high-quality solutions without significant further instruction. The student might ignore or fail to address some of the constraints. The student may occasionally make sense of quantities in relationships in the problem, but their use of quantity is limited or not fully developed. The student response may not state assumptions, definitions, and previously established results. While the student makes an attack on the problem it is incomplete. The student may recognize some patterns or structures, but has trouble generalizing or using them to solve their problem.

Cookie Dough

This problem gives you the chance to:
- choose and use number operations in context

Clear Creek School is fundraising. They are selling Cookie Dough in tubs.

Chocolate Chip Cookie Dough $5 a tub	Peanut Butter Cookie Dough $4 a tub	Oatmeal Cookie Dough $3 a tub

1. Jill sold 2 tubs of Oatmeal Cookie Dough. How much did she raise?

$ 6.00

2. Joe sold 4 tubs of Peanut Butter Cookie Dough and 4 tubs of Chocolate Chip Cookie Dough. How much money did he raise in all? Show how you figured it out.

$ 36.00

$$20$$
$$+16$$
$$\overline{\$36.00}$$

The student was able to get the correct answers to the first two parts of the task by showing a number sentence in the second part. 3.OA & MP4

3. Jade sold only Peanut Butter Cookie Dough. She raised $32. How many tubs did she sell? Show how you figured it out.

$$32$$
$$\times 3$$
$$\overline{96}$$

96 tubs

The student could not work backwards and did not use the inverse operation of multiplication (division). Instead the student incorrectly multiplied by 3. 3.OA.5 & 3.OA.6

4. Jermaine's mother loves oatmeal cookies. She has $20. to spend. What is the greatest number of tubs of Oatmeal Cookie Dough she can buy?

5 tubs

Explain how you figured it out.

Beucse 5x4=20

The student either misread or misinterpreted part 4. At times, when students are unsure, they look to friendly numbers to perform calculations. Here, the student showed no understanding for a sense of division, and remainders were involved. 3.OA.8&9

15

Level 2 Implications for Instruction

Students need help understanding the relationship between multiplicative operations (scaling up, repeated addition, skip counting up, multiplying and reasoning about the number of equal groups, etc.) and its inverse operation. The inverse process can be understood in several ways including, repeated subtraction, partitioning a set into equal groups, counting backwards, counting up knowing the termination number, and keeping track of groups, etc. Students come to understand that division is the inverse of multiplication, or from a student's perspective, division undoes what multiplication did.

Students at this level must understand the inverse relationship to be successful in parts 3 and 4. Students can create numeric tables, use number lines and skip count (one direction or the other), and/or use sets of counters to understand division. Students who were successful approached the problems using different strategies, yet regardless of the strategy, they all reasoned about the relationship of the numbers. Students who were unsuccessful merely chose operations to use. Therefore students need to learn the relationships of operations, what they mean, and how two quantities are related. Understanding fact families of multiplication and their related division relationship is an important tool students should know. Students should learn to reflect on their answers, check units of measure and their calculations to verify their solutions. This requires students to privately ask, "does this answer make sense in the context of the problem?"

Level 1: Demonstrates Minimal Success (Score Range 0 – 1)

The student's response shows few of the elements of performance the tasks demand. The work shows a minimal attempt on the problem, and a struggle to make a coherent attack on the problem. Communication is limited and work shows minimal reasoning. The student's response rarely uses definitions in their explanation. The student struggles to recognize patterns or the structure of the problem situation.

This student performed at Level 1, minimal success with a score of 2. The student answered two questions correctly, but the work showed misunderstandings and flaws in reasoning.

Clear Creek School is fundraising. They are selling Cookie Dough in tubs.

Chocolate Chip Cookie Dough $5 a tub	Peanut Butter Cookie Dough $4 a tub	Oatmeal Cookie Dough $3 a tub

1. Jill sold 2 tubs of Oatmeal Cookie Dough. How much did she raise?

$$3 + 3.00 = \$6.00$$

$ \$6.00

2. Joe sold 4 tubs of Peanut Butter Cookie Dough and 4 tubs of Chocolate Chip Cookie Dough. How much money did he raise in all? Show how you figured it out.

$\$41.00

$$4+4+4+4 = 16$$
$$5+5+5+5+5 = 25 \quad +16 \, 25 = \$41.00$$

In part two, the student does use repeated addition to find 4x4 but then models an incorrect number sentence for the Chocolate Chip Cookie Dough. The student does seem to know the two amounts that need to be added. MP2.

3. Jade sold only Peanut Butter Cookie Dough. She raised $32. How many tubs did she sell? Show how you figured it out.

___8___ tubs

4. Jermaine's mother loves oatmeal cookies. She has $20. to spend. What is the greatest number of tubs of Oatmeal Cookie Dough she can buy?

___7___ tubs

In part 4, the student appears to misinterpret the problem. Instead of finding how many Oatmeal Cookie tubs you can buy for $20, the student tries to find numbers of 4 in 20. The student's recording is not clear for even the student, and he/she arrives at 7 in 20. The student has trouble reasoning quantitatively.

Explain how you figured it out.

$$4+4=8+8=16+4=20$$

Third Grade: Cookie Dough
Annotated Student Work

Level 1 Implications for Instruction

Students need support with reasoning quantitatively. They may need to start with repeated addition to understand what the total of several equal groups mean. Having student use counters to understand skip counting may be a foundational experience for the students.

Part two requires a more complex chain of reasoning. Students need to sort and reason about two different types of cookie dough, creating number sentences to find 4 x $4 and 4 x $5. After arriving at those products (or sums using repeated addition), the student must perform a third step adding those two amounts to get a total. Students learn to solve multi-step problems only by engaging in multi-step problems. All students need to regularly experience solving problems with more than one or two steps and need to experience problem solving that requires longer chains of reasoning.

Students need experience in creating mathematical models for contextual situations. The modeling experience in third grade may involve writing number sentences, making numerical tables, creating charts or diagrams and/or drawing pictures to characterize a situation. Students will benefit from connecting and linking the representations to make sense of how the representations model the situation. Students learn to model situations by making sense of the problem and then tinkering with representations through trial and error.

Students need to talk through why a mathematical model makes sense in representing a problem. This can be done in pairs, small groups or by the whole class. More instructional emphasis should be placed on finding and understanding why a representation makes sense, than merely finding the answer to problems in context.

GRADE 3 MATH: COOKIE DOUGH INSTRUCTIONAL SUPPORTS

The instructional supports on the following pages include a unit outline with formative assessments and suggested learning activities. Teachers may use this unit outline as it is described, integrate parts of it into a currently existing curriculum unit, or use it as a model or checklist for a currently existing unit on a different topic.

NYC Department of Education
Dennis M. Walcott, Chancellor

Unit Outline Template

INTRODUCTION: This unit outline provides an example of how teachers may integrate performance tasks into a unit. *Teachers may (a) use this unit outline as it is described below; (b) integrate parts of it into a currently existing curriculum unit; or (c) use it as a model or checklist for a currently existing unit on a different topic.*

Grade 3: Interpreting Multiplication and Division

UNIT TOPIC AND LENGTH:

- This unit should run between 20 and 25 standard periods of instruction. One of the periods will involve the pre-assessment (0.5 period), introducing and supporting problem solving on the long lesson (2 periods), teaching the formative assessment lesson (2.5 periods) and the final assessment (0.5 period). This unit should be taught after or during the time students have learned about multiplication and division.

COMMON CORE LEARNING STANDARDS:

- **3.OA.1** Interpret products of whole numbers, e.g., interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each.
- **3.OA.2** Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects and partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each.
- **3.OA.3** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
- **3.OA.4** Determine the unknown whole number in a multiplication or division equation relating three whole numbers.
- **3.OA.5** Apply properties or operations as strategies to multiply and divide.
- **3.OA.6** Understand division as an unknown-factor problem.
- **3.OA.7** Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 x 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.
- **3.OA.8** Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
- **3.OA.9** Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.
- **MP.1** Make sense of problems and persevere in solving them.
- **MP.2** Reason abstractly and quantitatively.
- **MP.3** Construct viable arguments and critique the reasoning of others.
- **MP.4** Model using mathematics.

BIGIDEAS/ENDURING UNDERSTANDINGS: Student will understand: ➢ How to match equations to language descriptions. ➢ How to describe multiplicative and division relationships with area models and discrete models. ➢ How to connect naked number equations to contextual word problems. ➢ How to convince others that different multiplication and division representations can be equivalent.	**ESSENTIAL QUESTIONS:** ➢ What are the various representations of multiplication and division (*finding an unknown product, and finding an unknown factor*)?
CONTENT: ➢ The big idea of the unit is to understand the meaning of multiplication and its inverse relationship to division. The understanding should include the ability to translate between different representations of multiplication and division including the language of mathematics: understanding equations written in terms of equal-sized groups, area models, discrete models, and using contextual word situations to model multiplication and division. ➢ Students use understanding of equivalency to match multiplication and division equations to various visual representations and verbal descriptions. ➢ Students demonstrate knowledge through the expert investigation, the performance assessment task in the formative assessment lesson and the final assessment.	**SKILLS:** ➢ The targeted proficiencies; technical actions and strategies. Starting with an action verb. ➢ Students match multiplication and division equations with the language of mathematics, area models, discrete models, and contextual word problems. ➢ Students apply what they know about the meaning of multiplication and division to generate visual, verbal and numerical representations. ➢ Students look at problem situations and translate them into numerical and visual representations. ➢ Students apply the new knowledge to constructing arguments, making connections between representations and carefully evaluate the arguments and justifications of others.

Assessment evidence and activities:

Initial Assessment :
The unit begins with the performance task *Sponsored Walk*. The task is designed to measure what students bring to the unit with regard to their knowledge and skill at working with multiplication and division. Please reference *Sponsored Walk* for full details.

Formative Assessment:
About 3/4 of the way through the unit, teachers would use the *formative assessment lesson(FAL)*. The FAL is entitled *Interpreting Multiplication and Division*. A different pre-assessment task should be administered in class at least two days prior to the two-day lesson. Students should spend no more than 20 minutes on the task. Teachers should review the student work prior to teaching the lesson. The FAL comes with complete teacher notes and student pages. Please reference *Interpreting Multiplication and Division* for full details.

Final Performance Task:

The final performance assessment is entitled *Cookie Dough*. It should be administered during a class period. Most students will complete the task in about 10 – 20 minutes, although time should not be a factor. The teacher should provide a reasonable amount of time for all students to finish. The students should be allowed to use any tools or materials they normally use in their classroom. The task can be read to the students and all accommodations delineated in an IEP should be followed. Please reference *Cookie Dough* for full details.

Learning Plan & Activities:

➢ The unit is designed with a pre-assessment task, an expert task/investigation, a formative assessment lesson and a final assessment. The mathematics of the unit involves understanding the meaning of multiplication and division, as well as interpreting and translating between representations. Students will work with multiplication and division using equations, mathematical language, area models, discrete representations, number lines, and contextualized word problems to think about the meaning of multiplication and its inverse, division. Also included are teacher notes, rubrics, and sample student work with commentary. This unit is designed to accompany the curriculum a teacher currently uses to teach the topics listed. The elements in the unit will provide activities to foster formative assessment practices, conceptual understanding and non-routine problem solving.

➢ The expert investigation is entitled *Squirreling it Away*. It contains three separate but mathematically related problems labeled Part A, Part B, and Part C. All students should start with the Part A task and then proceed at their own speed to Part B and Part C. It is more important for the student to work deeply on a part and complete a write up, than to merely work through and find answers. It is the student's responsibility to be reflective and thorough in their explanations, findings and justifications. The investigation comes with

administration, teacher notes, the expert investigation, report guidelines and rubric.

➢ **Re-engagement:** The unit begins with a pre-assessment called sponsored walk. After the teacher analyzes the student performances, a decision of how to proceed through the lesson is important. Instead of going back to re-teach skill and concepts lacking, it is much more powerful for the teacher to use the work students have already done on a contextual problem, to help them build upon their understanding from previous thinking. That process is called "re-engagement". It is powerful to use student work because students become very engaged in the process of figuring out what someone else is thinking. This process of analyzing and contrasting student thinking raises the cognitive demand for students and supports them to be more reflective about their own thinking. The re-engagement lesson will depend upon the results from the students in each individual class. Thus, each lesson will look very different from class to class. Students have already done the task on their own and now the important ideas need to be brought out and examined. In the process, students must have the opportunity to confront and understand the error in the logic of their misconceptions. Often, as teachers, we try to prevent errors by giving frequent reminders, such as "line up the decimal point," but actually errors provide great learning opportunities for all students. Students don't let go of misconceptions until they understand why they don't make sense. For the student, there is underlying logic to their misconceptions.

Re-engagement – Confronting misconceptions, providing feedback on thinking, going deeper into the mathematics.
1. Start with a foundational problem to bring all the students along; this allows students to clarify and articulate important mathematics in order to better understand the entirety of the task.
2. Share different student approaches and ask all students to make sense of each strategy. Have all students compare the strategies to look for the mathematical connections and relationships.
3. Have students analyze misconceptions and discuss why they don't make sense. In the process students can let go of misconceptions and clarify their thinking about big mathematical ideas.
4. Have students determine how a strategy could be modified to get the correct solution. Have students look for the seeds of mathematical thinking in the selected student work.

➢ **Number Talks** A daily ritual with the entire class for the purposes of developing conceptual understanding of numbers, operations and mathematics. Number talks are used to:
   •Review and practice operations, procedures and concepts of numbers.
   •Introduce concepts and properties about numbers.
   •Reinforce procedures and number concepts.
   •Explore connections about numbers.
Do a number talk every day but for only 10 minutes. A few minutes more often is better than a lot of minutes infrequently.
1. Ask questions such as
   • How did you think about that?
   • How did you figure it out?
   • What did you do next?
   • Why did you do that? Tell me more.

- Who would like to share their thinking?
- Did someone solve it a different way?
- Who else used this strategy to solve the problem?
- What strategies do you see being used?
- Which strategies seem to be efficient, quick, and simple?

2. Give yourself time to learn how to
   - Record student solutions
   - Listen to and observe students
   - Collect notes about student strategies and understanding

3. Name/label the strategies that emerge from your students:
   - Use doubles
   - Break apart numbers
   - Make it simpler
   - Use landmark numbers (25, 50, 75, 200, etc)
   - Use a model to help
   - Use what you already know
   - Make a "10"
   - Start with the 10's
   - Think about multiples
   - Think about money
   - Traditional algorithm
   - Counting on

4. Create a safe environment. When students feel safe, they are comfortable sharing answers even when it's different from everyone else's.

5. Give opportunities for students to "think first."

6. Encourage self-correction; it's okay to change your mind, analyze your mistake, and try again.

7. Give number talks time to become part of your classroom culture. Expect them to follow the usual learning curve stages. "Keep on keeping on" and you will get positive results.

➢ **Think/Write/Pair/Share** is a high leverage strategy that respects individual time to process and organize ideas before engaging in peer-to-peer discussions. This process can be used throughout the unit as a vehicle for students to self reflect, construct new meaning by building on the ideas of others, and strengthen their arguments.

➢ **Journal Entries for Reflection**: Using a prompt such as, *"How has my thinking changed as a result of what I have discussed with my peers?" or "How can I improve my argument or explanation using evidence and content vocabulary?"* can provide valuable opportunities for students to tweak their own solutions, during class or for homework, and subsequently, deepen their understanding of content.

➢ **Purposeful Questioning and Feedback** are instructional supports that can help refocus students' attention to specific aspects of their work. There are some suggestions based on some common difficulties. Although these error patterns/questions relate to the pre-task *Sponsored Walk*, they can be easily modified to address similar misconceptions that are revealed from any other problems or tasks used:

Poin ts	Understandings	Misunderstandings	Suggested Prompts and Questions
0	95% of the students with this score attempted the task.	Students had difficulty choosing the correct operation needed to solve the problem. Many students chose addition for all problems or multiplication for all problems. *Example: 6 laps x $30 total = $180 per lap*	• Why did you pick that operation for the problem? • How did you know when to add and when to multiply?
2	Students with this score generally knew to add the amount of money per lap for Jack and Bill, and could total that to $10.	The students did not know to take this rate and multiply it by the number of laps to find the amount of money earned.	• How do you find how much money was earned? • How much would be earned if they did just two laps?
4	Students could use all the facts in part one to find the total amount of money earned by Jack and Bill and show all their steps.	More than 15% of the students had a final answer of $10 for part 1. 6% of the students added all the numbers in part 1 to get an answer of 15. Adding dollars and laps together didn't bother students.	• Does it make sense to add dollars and laps together? • What would you label your answer if you did?
6	Students could find the total money earned by Jack and Bill. They could also work backwards from the total amount of money to find the number of laps Maria ran.	8% of the students multiplied instead of divided in part 2. 13% of the students could write a correct number sentence for part 2, but could not decipher which part of the number sentence represented the answer (picking either the 30 or 6 instead of the 5). 7% of the students just added the numbers in the problem, not recognizing the operation appropriate for solving the question.	• Why did you multiply those values, what did you think you would find out? • What might a number sentence look like for the total money being earned by Jack and Bill? • How might you check your answer to see if it is right?
7	Students could understand the context of part 3, but weren't successful with interpreting the constraint "at least" or understanding how to interpret a remainder. Many students used multiplication to think about the problem and arrive at a correct or nearly correct solution.	Students were thinking about 6 x 3 = 18 and 7 x 3= 21. They picked 6 laps because they didn't want to go over $20. They did not understand how the remainder in 20 divided by 3 applied to the context of earning a given amount of money.	• What does the remainder 2 mean in this problem? • What does "at least" mean for this problem? • If Sarah does 6 laps how much money will she raise? Is that enough? • Is this easier to think about using multiplication or division? Why?

RESOURCES:

- ➢ Normal materials used in math class include manipulatives such as cards for matching activities, square tiles, counters, and cm graph paper.
- ➢ All the materials referenced in the assessments, formative assessment lesson, and expert investigation are included. Most supplementary materials are located in the appendix, including the established scored benchmark papers and some student work examples.
- ➢ What isn't included in print materials can be found on a TBA websites.

GRADE 3 MATH: COOKIE DOUGH

INITIAL TASK: SPONSORED WALK

Sponsored Walk

This problem gives you the chance to:
• choose and use number operations in a real context

Students at the Mountain View Elementary School do a sponsored walk.

1. Jack is sponsored for $6 for each lap.

   Bill is sponsored for $4 for each lap.

   Jack and Bill each do 5 laps.

   How much money do Jack and Bill raise in all? $ _____

   Show your work.

2. Maria does 6 laps.

   She raises $30.

   How much for each lap was she sponsored? $ _____

   Show how you figured it out.

Sponsored Walk   Test 3

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3. Sarah wants to raise at least $20.

   She is sponsored for $3 for each lap.

   What is the least number of whole laps she must walk? _____

   Explain how you figured it out.

   _____

   _____

   _____

Sponsored Walk   Test 3

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GRADE 3 MATH: COOKIE DOUGH

FORMATIVE ASSESSMENT LESSON:
INTERPRETING MULTIPLICATION AND DIVISION

Interpreting Multiplication and Division

Silicon Valley Mathematics Initiative- 2011

Formative Assessment Lesson

Interpreting Multiplication and Division

Mathematical Goals

This lesson unit is designed to assess students' understanding of multiplication and division and the relationship between these operations, as well as students' ability to translate between different representations or models of these operations. The representations include symbolic, concrete, bar and area, number line, and contextual models of these number operations. The lesson design facilitates identification of students who have difficulty:

- recognizing multiplicative relationships and transitioning from additive to multiplicative solution strategies
- understanding and using the language of "equal groups of" and "equal parts of" to make sense of multiplication and division
- translating between models
- understanding the meanings of the words factor, multiple, product, and quotient

Standards Addressed:

- Third Grade Operations and Algebraic Thinking- Represent and solve problems involving multiplication and division
- Third Grade Operations and Algebraic Thinking– Understand properties of multiplication and the relationship between multiplication and division
- Third Grade Operations and Algebraic Thinking – Multiply and divide within 100
- Third Grade Operations and Algebraic Thinking – Solve problems involving the four operations and identify and explain patterns in arithmetic

This lesson also relates to the following Mathematical Practices in the Common Core State Standards for Mathematics:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.

> ***Students will be asked to do a written reflection on how they were demonstrating these Practice Standards throughout their work in this unit.***

Essential Understandings:

The big idea of the unit is to understand the meaning of multiplication and its inverse relationship to division. The understanding should include the ability to translate between different representations of multiplication and division. This lesson is designed to facilitate the following understandings:

- Student will understand:
  - How to match equations to language descriptions
  - How to describe multiplicative and division relationships with area models and discrete models
  - How to connect naked number equations to contextual word problems

o   How to convince others that different multiplication and division representations can be equivalent

Introduction:

This lesson unit is structured in the following way:

- Students work on their own, completing an assessment task that is designed to reveal their current understandings and misconceptions.
- Students work in pairs or threes on collaborative discussion tasks. As they work, they translate between the symbols and models of multiplication and division and build understanding of the relationships between these operations through discussion as they make sense of the models.
- Students return to their original work and try to improve their own responses.

Materials required:

Each student will need one copy of the assessment task:

Each pair will need the following: (the cards should be cut up and packaged beforehand).

- Card Set A Equations; Card Set B Verbal (Explanations in Words); Card Set C Area Models; Card Set D Concrete Models; Card Set E Contextual Models; Card Set F Number Line Models
- Pattern blocks, Cuisenaire rods, counters, grid paper, and blank paper should be made accessible to students
- A glue stick, a felt tipped pen, a large sheet of paper or card for making a poster- one per pair
- Envelopes and paper clips for storing matched cards in between lessons if time is short
- Additional supplies dependent on procedures used for sharing and debriefing student work, such as overhead transparencies, document reader, additional chart paper

Time needed:

The lesson will need <u>at least</u> two one-hour sessions. Timings given are only approximate. Exact timings will depend on the needs of the class.

Before the lesson:

Individual Assessment Task: *"Baking Cookies"*

The assessment task, *"Baking Cookies"* should be completed before the lesson. Ask students to attempt the task on their own. Explain that they should not worry too much if they cannot understand or do everything, because you plan to teach a lesson using a similar task which should help them.

> It is important that students are allowed to answer the questions without assistance, as far as possible. If students are struggling to get started, ask questions that help them understand what is required, but don't do it for them!

Department of Education
Dennis M. Walcott, Chancellor

Assessing student responses:

Collect a sample of students' responses to the task and make some notes on what their work reveals about their current levels of understanding. The purpose of doing this is to forewarn you of the difficulties students may experience during the lesson itself, and so that you may prepare carefully. Do not grade students' work at this stage. Research shows this could be counterproductive as it will encourage students to compare their grades, and will distract their attention from the mathematics. Instead, try to understand their reasoning and think of ways in which you can help them.

Suggested Lesson Outline

Class introduction- Math Talk (15 minutes)

[Please see Math Talk Format below for support]

MATH TALK FORMAT: IMPLEMENTING NUMBER TALKS

Helpful Hints

- Do number talks every day, but for only 10 minutes. A few minutes more often is better than a lot of minutes infrequently.
- Ask questions such as….
    - How did you think about that?
    - How did you figure it out?
    - What did you do next?
    - Why did you do that? Tell me more.
    - Who would like to share their thinking?
    - Did someone solve it a different way?
    - Who else started the problem this way?
    - Who else used this strategy to solve the problem?
    - What strategies do you see being used?
    - Which strategies seem to be efficient, quick, simple?
- Experiment with using the overhead, the whiteboard, chart paper, etc.
- Consider having students "circle up" in chairs or on the floor.
- Give yourself time to learn how to….
    - record student solutions
    - listen to and observe students
    - collect notes about student strategies and understanding
- To help determine what numbers or problems you select use what you learn from previous number talks as well as the focus of your daily classroom instruction.
- Do number talks with yourself and others to try new strategies and increase your own confidence.
- Name/label the strategies that emerge from your students:

• Use doubles	• Break apart numbers	• Make it simpler
• Use landmark numbers (25, 50, 75, 200 etc.)	• Use a model to help	• Use what you already know

• Make a "10"	• Start with the 10's	• Think about multiples
• Think about money	• Traditional algorithm	• Counting on

- Use related problems: 3 x 14, 3 x 114 , 3 x 1014 <u>or</u> 7 + 8, 27 + 8, 107 + 8, <u>or</u> 3 x 7 , 6 x 7
- Do number talks in small groups
- Ask students to: "Do as much of the problem as you can."
- Give students lots of practice with the same kinds of problems.
- Use numbers for subtraction and addition that require students to work past a ten or hundred.
  Example: 56 + 7 = 87 – 9 = 25 + 6 = 94 + 8 = 106 – 8 =
- Give students opportunities to add and subtract 9, then 8 etc. using 10 as a friendly number to work with.
  Example: 68 + 10 = 78 so 68 + 9 = 77
- Expect students to break apart numbers, not count on their fingers. Show them how.
  6 + 8 (think of 6 as 4 + 2; add the 2 to 8 to get 10 and just add the remaining 4 to get 14)
- Show the strategy you used. Make sure they know it's not "the" way, just another strategy.
- Give students larger numbers so they can give "estimates."
- If you use chart paper, write down the student's name next to their solution. Keep track of who is participating and their strategies. Use the following as a "sorting" or assessment guide:
  - Can they figure it out (by counting on, using an involved strategy, etc.)
  - Begin to use efficient strategies (can they complete some of the problem efficiently)
  - Do they just know or are they using efficient strategies
- Create a safe environment. When children feel safe, they are comfortable sharing an answer even when it's different from everyone else's.
- Provide concrete models (snap cube "trains", base 10 blocks, money etc.)
- Give opportunities for children to "think first" and then check with the models.
- Have students occasionally record their thinking and the steps they use to solve a problem.
- Encourage self-correction; it's okay to change your mind, analyze your mistake, and try again.
- Provide number stories.
- Be curious; avoid making assumptions.
- Give number talks time to become part of your classroom culture. Expect them to follow the usual learning curve stages. "Keep on keeping on" and you will get positive results!

KIDS LIKE NUMBER TALKS!

Present the following multiplication problem to students using a Math Talk Format. Tell them that you are not interested in the answer as much as how they figure it out.

> *Mrs. Clark's class has 36 students. She wants to put their desks into 6 rows. How many student desks will she need to put in each row?*

First, ask students to use a manipulative to demonstrate their thinking at their desks. These could be color tiles or cm cubes. Have students share with their shoulder partner and have a conversation about the similarities or differences between the two representations.

Ask for student volunteers to share one of their solution strategies. Have each volunteer, use his/her white board to explain one strategy/model. Allow students to ask questions of the student presenting to ensure clarification of the solution strategies. If students show any incorrect answers, write the correct answer on the board and discuss the reasoning as a group.

Second, ask students to write an equation that could be used to solve the answer on a white board or facsimile thereof, [teachers have put tag board inside of a plastic sleeve and given students overhead transparency pens or washable markers to use to write on these.]

>   *On your mini-white boards, show me………*

Put their ideas on the classroom whiteboard/document reader/smart board and hold a discussion about which one[s] represent the mathematics of the problem and could be used to find the answer.

Show students an area model representation of this problem and ask students if this model represents the given multiplication word problem. Give them a few minutes of think time, and then ask them to discuss with their elbow partner.   Have a whole group discussion to surface understandings and misconceptions.

If needed, show another area model and ask students to write an equation to represent this new area model. Likewise, having students write a contextual problem with a partner would connect these representations together.

Lesson: To deepen student understanding of multiplication and division, use the terms *factor, multiple, product and quotient* during discussions of the different representations i.e., symbols, words, and concrete/bar/area/number line models*.*

<u>Collaborative Task # 1</u>: Matching symbolic notation of equations (Card Set A) with verbal explanations (Card Set B) (15-20 minutes)

The first task is designed to help students identify and interpret equations involving multiplication and division and to match the equations with verbal descriptions [explanations in words], using such language as "groups of" and "as big as." Organize students into pairs and give out Card Set A – *Equations*, and Card Set B – *Verbal Explanations*.

>   *Your task is to match the equations (symbolic notations)* **Card Set A** *with the explanations in words* **Card Set B***.*

>   *Take turns with your partner choosing an equation, and the verbal description to match it.*

>   *Place these side by side on the table and explain to your partner[s] how they match.*

>   *If you cannot find a matching card, you have to write your own using one of the blank cards.*

>   *Take turns choosing <u>an equation</u> and <u>the verbal description</u> to match it with your partner. Place these side by side on the table and explain to your partner[s] how they match.*

It is extremely important to model this behavior with students in the class. This may be a student's first experience having a structured mathematical dialogue with another student.

There is a one to one correspondence between sets. The blank cards are an integral part of the sets because some students may not see the corresponding verbal description and will choose to use the blank cards to complete their match. This provides invaluable information to you, the teacher, about student understanding and misconceptions.

Furthermore, to increase the cognitive demand of these collaborative tasks, selecting a few cards of your choice to be removed from one or more sets of representations, will force students to fill in the missing representation[s] on their own, giving deeper insight into student thinking and understanding.

Whether you use the complete set of cards or you remove some, letting students know that there are blank cards available is important because it will reveal additional information of students' understandings and misconceptions if they use them.

Students may notice that some of the equations are related. You do not need to comment on this now, but this is an important connection that warrants discussion as the students progress through the matches. These relationships will be emphasized during the summary processing discussion.

Collaborative Task # 2: **Matching symbolic notations (**Card Set A), **explanations in words** (Card Set B), **and area models** (Card Set C) **for multiplication/division** (15-20 minutes)

Give each pair of students a copy of ***Card Set C,*** a glue stick.

> *Your task is to match the equations in* **Card Set A** *with the verbal explanations in* **Card Set B** *and select the proper area model in* **Card Set C**.

> *Take turns with your partner choosing an area model Card C, to match one set of Cards A and B.*

> *Place the chosen C card along side the A and B card set on the table and explain to your partner how they match.*

> *If you cannot find a matching card, write your own using one of the blank cards.*

Collaborative Task # 3: **Matching symbolic notations** (Card Set A), **verbal explanations** (Card Set B), **area models (**Card Set C), **and concrete models** (Card Set D) **for multiplication/division** (15 to 20 minutes)

> *Your task is to match the equations in* **Card Set A**, *the verbal explanations in* **Card Set B**, *the area models in* **Card Set C** *with the concrete models in* **Card Set D**.

Take turns with your partner choosing a concrete model Card D to match one set of Cards A, B, and C.

Place the chosen D card along side the A, B, and C card set on the table and explain to your partner how they match.

If you cannot find a matching card, write your own using one of the blank cards.

Collaborative Task # 4: Matching symbolic notations (Card Set A), verbal explanations (Card Set B), area models (Card Set C), concrete models (Card Set D), and contextual (word problems) (Card Set E) for multiplication/division (15 to 20 minutes)

*Your task is to match the equations in **Card Set A***, *the verbal explanations in **Card Set B**, the area models in **Card Set C**, the concrete models in **Card Set D** with the contextual models in **Card Set E.***

Each of the Set E cards is a representation of a multiplication or division or story. You are to match these situation cards to those already on the table. Remember, if you cannot find a matching card, write your own using one of the blank cards.

Take turns with your partner choosing a number line Card E to match one set of Cards A, B, C, and D.

Place the chosen E card along side the A, B, C, and D card set on the table and explain to your partner[s] how they match.

If you cannot find a matching card, write your own using one of the blank cards.

Collaborative Task #5: Matching symbolic notations (Card Set A), verbal explanations (Card Set B), area models (Card Set C), concrete models (Card Set D), and contextual models (Card Set E) for multiplication/division number line models (Card Set F), (15 to 20 minutes)

*Your task is to match the equations in **Card Set A***, *the verbal explanations in **Card Set B**, the area models in **Card Set C**, the concrete models in **Card Set D**, the contextual models in **Card Set E**, with the number line models in **Card Set F**.*

Take turns with your partner choosing a contextual model Card F to match one set of Cards A, B, C, D, and E.

Place the chosen F card along side the A, B, C, D, and E card set on the table and explain to your partner[s] how they match.

When your group reaches agreement, paste down your final arrangement onto the large sheet, creating a poster.

These posters will be used for the summary class discussion. As students do the matching and pasting or gluing, go around and encourage students to explain why particular cards go together.

Summary Class Work

First, give the following prompt questions to the class. Each pair of students will prepare their responses before presenting their poster and responses to the whole class. **Collect these for your information.**

> *Individually, think and write about these questions I am going to give you. Share your thoughts with your partner and agree on one response to present to the class. Remember to use mathematical language in your presentation: factor, multiple, product, quotient, "equal groups of", "as big as", etc.*
>
> *What determined how you matched the cards?*
>
> *Which representation[s] made the most sense to you? Why?*
>
> *Which representation[s] was/were the most difficult for you to understand? Why?*

Have each pair of students present their poster and responses to the whole class. If possible, limit presentations to about 2 minutes each. Whether or not clarifying questions from the audience are allowed, is dependent upon the current classroom norms, culture and expectations in individual classrooms.

Second, after the completion of presentations and collection of individual responses, re-visit the Standards for Mathematical Practices listed in the beginning of this unit with your students. Ask them to reflect upon these practices and write about how this unit helped them practice one or more of these standards.

Individual work (15-20 minutes)
Lastly, students need to revisit the original task, ***Baking Cookies***. Giving students a new, clean copy of this task allows students the opportunity to share their new learning. A potential follow up activity is to have students look at their first work on the task, and compare and contrast it to their second attempt. This affords them the opportunity to articulate and summarize their mathematical understandings by re-engaging in the mathematics of the task. Another option is to have students edit their original paper with a pen. Editing is an invaluable skill used quite effectively in language arts and very little, if at all, in mathematics. Many students struggle with editing their original task work, yet both teachers and students alike find more evidence of learning when students have the opportunity to re-work a clean copy of the original task. This is not to say that teachers and students should not learn to persevere when it comes to editing mathematics. In fact, teachers should attempt to create more opportunity for students to edit and re-edit their mathematics within their classwork.

Solutions:

This table is for your convenience only. Rather than referring to the cards by letter and number, the connections should be made by the content of the cards as there are different possible groupings of six representations.

Equation	Verbal	Area	Concrete	Number Line	Contextual
A1	W7, W8	AM8	CM4, 6, 11	W1	NL7
A2	W 9	AM9	CM8	W2	NL9
A3	W 6	AM6	CM3,CM10	W3	NL6
A4	W1, W4, W12	AM10	CM1	W4	NL11, NL12
A5	W1	AM1	CM12	W5	NL1
A6	W10	AM11	CM9	W6	NL10
A7	W11	AM5	CM2	W7	NL11
A8	W8	AM12	CM11	W8	NL7, NL8
A9	W5, W6	AM6	CM3, CM10	W3	NL5, NL6
A10	W13	AM3, AM7	CM5, CM7	W9	NL4
A11	W1, W2	AM1, AM4	CM11	W5, W10	NL2
A12	W3, W4	AM3	CM7	W9	NL3 with discussion
A13	W4, W13	AM2	CM7	W12	NL3

Answer Key for #s 1 & 2
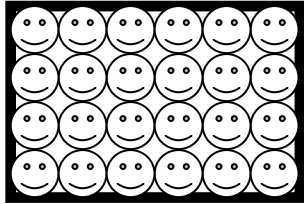
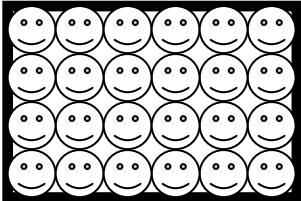### *See Rubric for "Baking Cookies*

Rubric

Baking Cookies

The core elements of performance required by this task are:
    • work with quantities in a contextual situation
    • use different representations for multiplication
 	
Based on these, credit for specific aspects of performance should be assigned as follows	

1. Gives correct answer: $ **48.00** *Shows work such as: (award one point for each correct solution strategy)* 4 x $12.00 = $48.00 $12.00 + $12.00 + $12.00 + $12.00 = $48.00 *(Accept any correct solution strategy including concrete, number line, and/or bar/area models.)*	1 2x1
2. Gives correct answer: **6 rows** *Shows work such as:* $30 \div 5 = 6$ and/or *(Accept any correct solution strategy including concrete, number line, and/or bar/area models.)*	1 2x1

3.   See ***Answer Key*** on next page showing multiple representations/models for this problem.
 	
   7 or 8 correct answers
   5 to 6 correct answers
   4 to 5 correct answers
   3 correct answers
	
	4 pts
	3 pts
	2 pt
	1 pt

Total Points	**10**

Answer Key for #3

Baking Cookies

Answers may vary……

$\boxed{}$ X 6 = 24	4 X $\boxed{}$ = 24	4 X 6 = $\boxed{}$
Jill baked 24 cookies for her friends. She had 6 cookies in each row in her pan, How many rows of cookies were in the pan?	**Jill baked 24 cookies for her friends. She had 4 rows of cookies in each pan. How many cookies were in each row?**	**Jill baked cookies for her friends. She had 4 rows of 6 cookies in the pan How many cookies did she bake?**
24 cookies	**24 cookies**	
6 **?** **24**	**?** **4** **24**	**6** **4** **?**

Student Materials

- **Pre/Post Assessment**

Performance Task
Baking Cookies
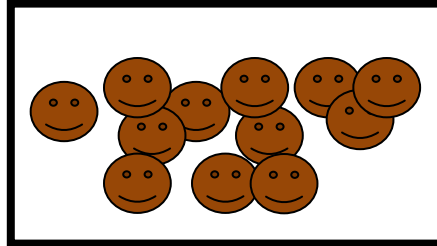
1. Jill is baking "Happy Face" cookies for 4 of her special friends. If she wants each friend to have 12 cookies, how many cookies will she need to bake?

_____cookies

Show how you got your answer.

Show a different way to get your answer.
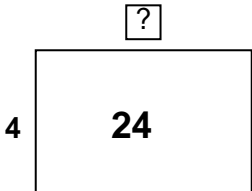
2. Jill decides to give each of her 30 classmates a cookie. If she can fit 5 cookies in a row on her baking pan, how many rows of cookies will she need to make?

_____rows

Show how you got your answer in two different ways.

3. Jill decides to only bake cookies for her **4** special friends. She is using a pan that holds **4 rows of 6 cookies** or **24 cookies total**. Let's pretend that **one piece** of this information is missing, and you have to solve the problem for that missing number.

In each **column** of the chart below, a different piece of this information is missing, and this is the number you have to find. Each **row** of the chart is a different representation. If the word problem is missing, make up a problem to match the clue given. If a different representation is missing, use the clues in the chart below to fill in the other representations.

Equation			4 X 6 = ☐
Word Problem		Jill baked 24 cookies for her friends. She had 4 rows of cookies in each pan. How many cookies were in each row?	
Concrete Model	24 cookies (☺☺☺☺☺☺)		
Area Model		? / 4 | 24	

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Rubric

Baking Cookies

The core elements of performance required by this task are:
    • work with quantities in a contextual situation
    • use of different representations for multiplication
 	
Based on these, credit for specific aspects of performance should be assigned as follows	

1. Gives correct answer: $ **48.00** *Shows work such as: (award one point for each correct solution strategy)* 4 x $12.00 = $48.00 $12.00 + $12.00 + $12.00 + $12.00 = $48.00 *(Accept any correct solution strategy including concrete, number line, and/or bar/area models.)*	1 2x1
2. Gives correct answer: **6 rows** *Shows work such as:* 30 ÷ 5 = 6 and/or *(Accept any correct solution strategy including concrete, number line, and/or bar/area models.)*	1 2x1

3.   See **_Answer Key_** on next page showing multiple representations/models for this problem.
 	
   7 or 8 correct answers
   5 to 6 correct answers
   4 to 5 correct answers
   3 correct answers
	4 pts
	3 pts
	2 pt
	1 pt

Total Points	**10**

Answer Key

Baking Cookies

Answers may vary……

☐ X 6 = 24	4 X ☐ = 24	4 X 6 = ☐
Jill baked 24 cookies for her friends. She had 6 cookies in each row in her pan, How many rows of cookies were in the pan?	**Jill baked 24 cookies for her friends. She had 4 rows of cookies in each pan. How many cookies were in each row?**	**Jill baked cookies for her friends. She had 4 rows of 6 cookies in the pan How many cookies did she bake?**
24 cookies	**24 cookies**	
6 ? **24**	? 4 **24**	6 4 **?**

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Card Set A-Equations

E1 20 ÷ ☐ = 4	**E2** 7 x 8 = ☐
E3 5 x ☐ = 15	**E4** 24 ÷ 4 = ☐
E5 4 x 5 = ☐	**E6** 8 x 9 = ☐
E7 6 x ☐ = 24	**E8** 20 ÷ 5 = ☐
E9 15 ÷ 5 = ☐	**E10** 3 x 12 = ☐
E11 5 x 4 = ☐	**E12** ☐ + ☐ = 36
E13 36 ÷ ☐ = 3	**E14**

Card Set B – Explanations in Words
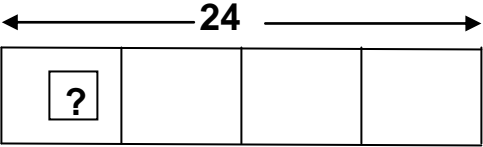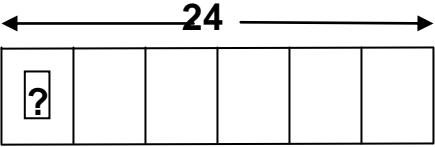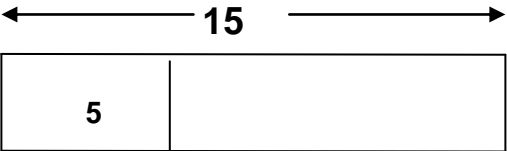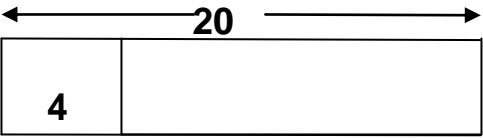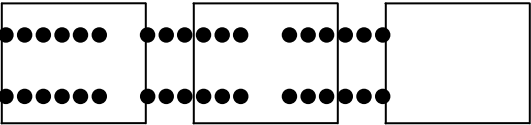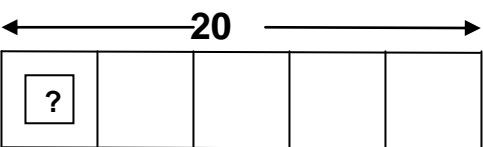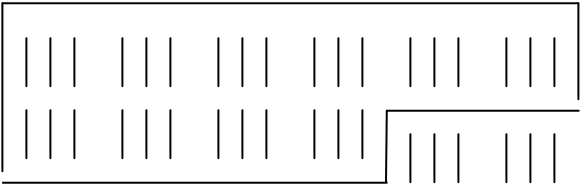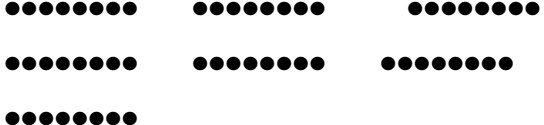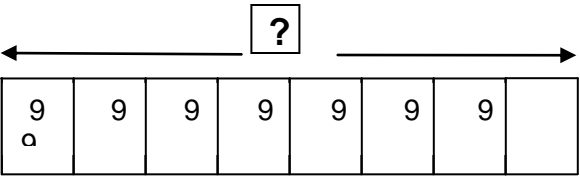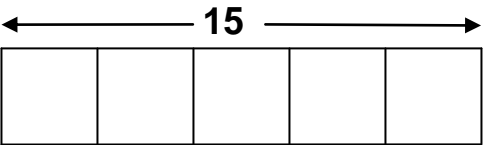
W1 **4 times as big as 5**	**W2** **5 groups of 4**
W3 **10 groups of 3 and 2 groups of 3**	**W4** **3 groups of 12**
W5 **?** **number of equal groups of 3 in 15**	**W6** **5 equal groups of** **?** **in 15**
W7 **?** **number of equal groups of 4 in 20**	**W8** **20 divided into 5 equal groups**
W9 **7 groups of 8**	**W10** **8 times as big as 9**
W11 **6 groups of** **?** **is 24**	**W12** **24 is divided into 4 equal groups of** **?**
W13 **36 is divided into 3 equal groups of** **?**	**W14**

A1 Card Set C - Area Model

AM1 4 [5×4 grid] 5	**AM2** 3 ? [36]
AM3 10 2 3 [ ]	**AM4** 4 5 [?]
AM5 ? 6 [24]	**AM6** ? 5 [15]
AM7 12 3 [?]	**AM8** 4 ? [20]
AM9 8 7 [?]	**AM10** ? 4 [24]

AM 11   9

8

AM12

?

5   20

AM13

AM14

Card Set D –Concrete Models

CM1 ← 24 → [?][ ][ ][ ]	**CM2** ← 24 → [?][ ][ ][ ][ ][ ]
CM3 ← 15 → [5][ ] [?] groups of 5	**CM4** ← 20 → [4][ ] [?] groups of
CM5	**CM6** ← 20 → [?][ ][ ][ ]
CM7	**CM8**
CM9 [?] ← → [9][9][9][9][9][9][9][ ]	**CM10** ← 15 → [ ][ ][ ][ ][ ]
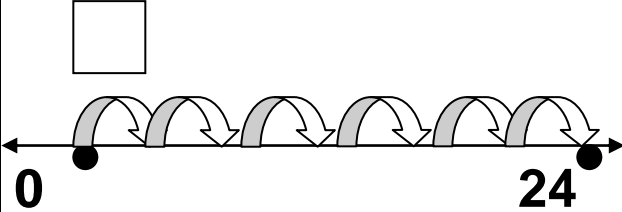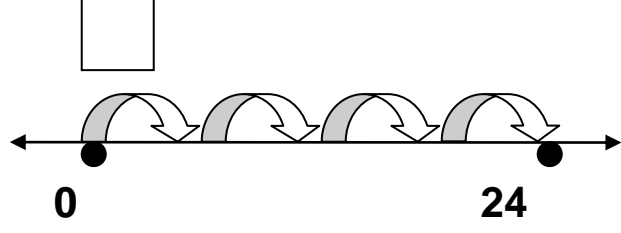
CM11

.

●●●●   ●●●●   ●●●●   ●●●●   ●●●●

CM12

●●●●●

●●●●●

●●●●●

●●●●●

CM13

CM14

Card Set E – Contextual Problems

W1 Joe has 20 crayons. How many crayons does he put in each box if he gets 4 boxes for 20 crayons?	**W2** Susie wants to give her 7 friends 8 pieces of candy each. How many pieces of candy will she need to buy?
W3 Polly's mom has planted 15 plants in 5 rows. How many plants are there in each row?	**W4** Sam's dad bought 24 hotdogs for Sam and his 3 friends. How many hot dogs can they each have?
W5 Debbie wants to give her 5 friends 4 balloons each. How many balloons must she buy?	**W6** Manson wants to give his 8 friends 9 baseball cards each. How many baseball cards must he buy?
W7 Sarah buys 6 pieces of bubble gum for 24 cents. How much does each piece cost?	**W8** Caitlin buys 5 pies for $20 each. How much does one pie cost?
W9 Daniel buys 3 dozen donuts. How many donuts does he buy?	**W10** David wants to give his 4 friends 5 books each. How many books will he need?
W11	**W12** Mardi plants 36 plants in 3 rows. How many plants in each row?

Card Set F - Number Line Model

NL 1

5   5   5   5

0   ☐

NL 2

4   4   4   4   4

0   ☐

NL 3

☐

0   ☐   36

NL 4

12

0   ☐

NL 5

5   ? groups of 5

0   15

NL 6

☐

0   15

NL 7

5   ? groups of 5

0   20

NL 8

☐

0   20

NL 9

8   8   8   8   8   8   8

☐

NL 10

9   9   9   9   9   9   9   9

☐

W11

0   24

W12

0   24

W13

W14

GRADE 3 MATH: COOKIE DOUGH

EXPERT INVESTIGATION: SQUIRRELING IT AWAY

Department of Education
Dennis M. Walcott, Chancellor

Expert Task
Squirreling It Away

Level A:

Austin has a bag of 17 acorns. Eight squirrels came up to him. He gave each squirrel an acorn. Then five more squirrels came up to him and he gave away one acorn to each of them. How many more squirrels can he still feed?

Show how you figured it out.

How do you know you have the right answer?

Expert Task
Squirreling It Away

Level B:

Austin likes to watch squirrels find and store acorns for the winter. Brown Squirrels can carry two acorns at a time. Gray Squirrels can carry three acorns at a time and Black Squirrels can carry five acorns at a time. There is a pile of 24 acorns.

How many trips would a Brown Squirrel need to make to store all of the acorns in the pile?

How many trips would a Gray Squirrel need to make to store all of the acorns in the pile?

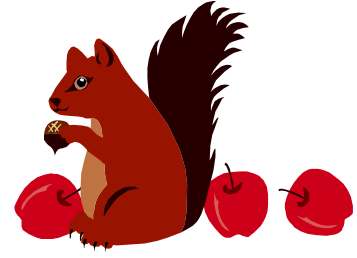How many trips would a Black Squirrel need to make to store all of the acorns in the pile?

If all three squirrels worked together to store the acorns how many trips would the squirrels need to make to store all of the acorns?

Explain your solution.

Expert Task
Squirreling It Away

Level C:

> **Brown Squirrels can carry 2 acorns at a time.**
> **Gray Squirrels can carry 3 acorns at a time.**
> **Black Squirrels can carry 5 acorns at a time.**

Suppose all three squirrels want to store acorns for the winter. Depending on how motivated each squirrel is they would end up with different amounts. For instance, suppose the Brown Squirrel took 4 trips, the Gray Squirrel took 2 trips, and the Black Squirrel took 2 trips. The Brown Squirrel would end up with 8 acorns, the Gray Squirrel would have 6 acorns and the Black Squirrel would have 10. If between them all they took every one of the 24 acorns:

How many different ways could the three squirrels divide up the 24 acorns and not leave any left over? Each squirrel must carry their maximum load on each trip.

How do you know that you have found all of the ways?

Expert Task
Squirreling It Away

Level D:

The squirrels are rather smart. They realize that they can carry less than their maximum loads. How many different ways could the squirrels divide up the 24 acorns?

Explain your solution.