COMMON CORE-ALIGNED TASK
WITH INSTRUCTIONAL SUPPORTS

Mathematics

GRADE 2 MATH: CAROL'S NUMBERS

UNIT OVERVIEW

The mathematics of the unit involves understanding the meaning of base ten and using that understanding to solve number and real life problems. The number line is used as a tool to help articulate understanding of base ten and to solve problems using addition and subtraction of numbers less than one hundred. The focus is on the big idea of going around groups of ten. Strategies will involve applying number properties including distributive, associative, and commutative.

TASK DETAILS

Task Name: Carol's Numbers

Grade: 2

Subject: Math

Task Description: The final performance assessment is entitled Carol's Numbers. The mathematics of the task involves understanding the meaning of base ten and using that understanding to compare the magnitude of numbers. The number line is used as a tool to help articulate understanding of base ten.

Standards Assessed:
2.NBT.1 Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:
2.NBT.1a 100 can be thought of as a bundle of ten tens – called a "hundred"
2.NBT.1b The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
2.NBT.3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.
2.NBT.4 Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.
2.NBT.6 Add up to four two-digit numbers using strategies based on place value and properties of operations.
2.MD.6 Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.

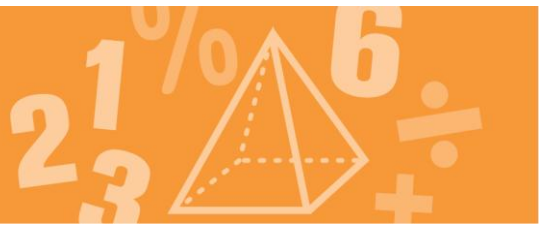
Standards for Mathematical Practice:
MP.1 Make sense of problems and persevere in solving them.
MP.3 Construct viable arguments and critique the reasoning of others.
MP.6 Attend to precision.

NYC Department of Education
Dennis M. Walcott, Chancellor

TABLE OF CONTENTS

The task and instructional supports in the following pages are designed to help educators understand and implement tasks that are embedded in Common Core-aligned curricula. While the focus for the 2011-2012 Instructional Expectations is on engaging students in Common Core-aligned culminating tasks, it is imperative that the tasks are embedded in units of study that are also aligned to the new standards. Rather than asking teachers to introduce a task into the semester without context, this work is intended to encourage analysis of student and teacher work to understand what alignment looks like. We have learned through the 2010-2011 Common Core pilots that beginning with rigorous assessments drives significant shifts in curriculum and pedagogy. Universal Design for Learning (UDL) support is included to ensure multiple entry points for all learners, including students with disabilities and English language learners.

Acknowledgements: This task and unit outline were developed by the Silicon Valley Mathematics Initiative and SCALE.



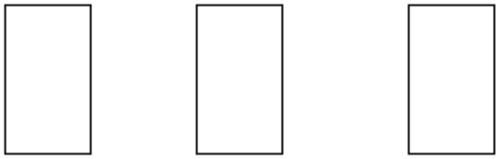
GRADE 2 MATH: CAROL'S NUMBERS PERFORMANCE TASK

NYC Department of Education
Dennis M. Walcott, Chancellor

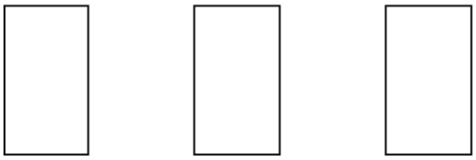
Carol's Numbers

Carol has three number cards.

1. What is the largest three-digit number Carol can make with her cards?



2. What is the smallest three-digit number Carol can make with her cards?



Explain to Carol how she can make the smallest possible number using her three cards.

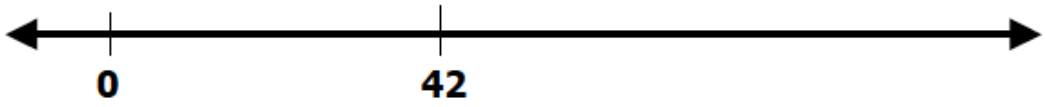
_____

_____

_____

_____

Carol's teacher drew a number line on the board.



3. About where would 85 be? Place 85 on the number line where it belongs.

4. About where would 21 be? Place 21 on the number line where it belongs.

5. About where would 31 be? Place 31 on the number line where it belongs.

Tell Carol how you knew where to place 31 and why.

_____

_____

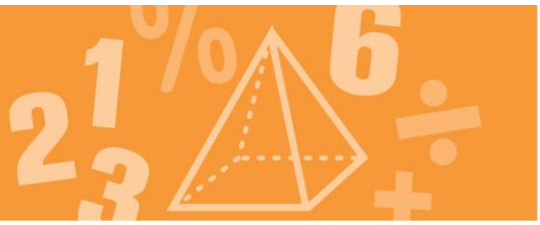
_____

_____

_____

COMMON CORE-ALIGNED TASK
WITH INSTRUCTIONAL SUPPORTS

Mathematics



GRADE 2 MATH: CAROL'S NUMBERS

UNIVERSAL DESIGN FOR LEARNING (UDL) PRINCIPLES

NYC Department of Education
Dennis M. Walcott, Chancellor

The goal of using Common Core Learning Standards (CCLS) is to provide the highest academic standards to all of our students. Universal Design for Learning (UDL) is a set of principles that provides teachers with a structure to develop their instruction to meet the needs of a diversity of learners. UDL is a research-based framework that suggests each student learns in a unique manner. A one-size-fits-all approach is not effective to meet the diverse range of learners in our schools. By creating options for how instruction is presented, how students express their ideas, and how teachers can engage students in their learning, instruction can be customized and adjusted to meet individual student needs. In this manner, we can support our students to succeed in the CCLS.

Below are some ideas of how this Common Core Task is aligned with the three principles of UDL; providing options in representation, action/expression, and engagement. As UDL calls for multiple options, the possible list is endless. Please use this as a starting point. Think about your own group of students and assess whether these are options you can use.

REPRESENTATION: *The "what" of learning.* How does the task present information and content in different ways? How students gather facts and categorize what they see, hear, and read. How are they identifying letters, words, or an author's style?

In this task, teachers can…

- ✓ **Offer alternatives for visual information** by allowing for a competent aide or "intervener" to read directions aloud.

ACTION/EXPRESSION: *The "how" of learning.* How does the task differentiate the ways that students can express what they know? How do they plan and perform tasks? How do students organize and express their ideas?

In this task, teachers can…

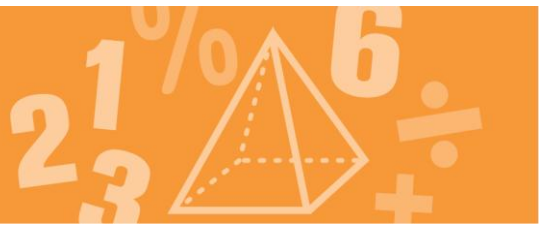
- ✓ **Vary the methods for response and navigation** by providing alternatives for physically responding to performance tasks in *Carol's Numbers* by using teacher-made number cards to demonstrate understanding of the tasks.

ENGAGEMENT: *The "why" of learning.* How does the task stimulate interest and motivation for learning? How do students get engaged? How are they challenged, excited, or interested?

In this task, teachers can…

- ✓ **Minimize threats and distractions** by varying the level of visual stimulation, such as limiting the number of items presented at a time.

Visit http://schools.nyc.gov/Academics/CommonCoreLibrary/default.htm to learn more information about UDL.



GRADE 2 MATH: TASK CAROL'S NUMBERS RUBRIC

The rubric section contains a scoring guide and performance level descriptions for the Carol's Numbers task.

Scoring Guide: The scoring guide is designed specifically to each small performance task. The points highlight each specific piece of student thinking and explanation required of the task and help teachers see common misconceptions (which errors or incorrect explanations) keep happening across several papers. The scoring guide can then be used to refer back to the performance level descriptions.

Performance Level Descriptions: Performance level descriptions help teachers think about the overall qualities of work for each task by providing information about the expected level of performance for students. Performance level descriptions provide score ranges for each level, which are assessed using the scoring guide.

Grade 2 Math: Carol's Numbers

Carol's Numbers Scoring Guide

Carol's Numbers
Mathematics Assessment Collaborative
Performance Assessment Rubric Grade 2

	Carol's Numbers: Grade 2	Points	Section Points
	The core elements of the performance required by this task are: • Understand the relative magnitude of whole numbers and the concepts of sequences, quantity, and the relative position of numbers • Use strategies to estimate and judge the reasonableness of results • Communicate reasoning using words, numbers or pictures Based on these credit for specific aspects of performance should be assigned as follow:		
1	Gives correct answer of : **742**	**1**	**1**
2	Gives correct answer of: **247** Gives correct explanation such as: Put the smallest number on the left, then the next smallest number and the largest number last	**1** **1**	**2**
3	Places 85 approximately twice the length of 42	**1**	**1**
4	Places 21 approximately one half the length of 42 (use a range from approximately 15 -25)	**2**	**2**
5	Places 31 approximately on half the length between 21 and 42 Dependent upon the correct placement- Gives correct explanation such as: 31 is almost in the middle of 21 and 42. Or Because 31 is 10 more than 21 or Because 31 is 11 less than 42	**1** **1**	**2**
Total			**8**

Grade 2 Math: Carol's Numbers

Rubric

Performance Level Descriptions and Cut Scores
Performance is reported at four levels: 1 through 4, with 4 as the highest.

Level 1: Demonstrates Minimal Success (0 – 1 point)
The student's response shows few of the elements of performance that the tasks demand as defined by the CCSS. The work shows a minimal attempt on the problem and struggles to make a coherent attack on the problem. Communication is limited and shows minimal reasoning. The student's response rarely uses definitions in their explanations. The students struggle to recognize patterns or the structure of the problem situation.

Level 2: Performance Below Standard (2 – 4 points)
The student's response shows some of the elements of performance that the tasks demand and some signs of a coherent attack on the core of some of the problems as defined by the CCSS. However, the shortcomings are substantial and the evidence suggests that the student would not be able to produce high-quality solutions without significant further instruction. The student might ignore or fail to address some of the constraints of the problem. The student may occasionally make sense of quantities in relationships in the problem, but their use of quantity is limited or not fully developed. The student response may not state assumptions, definitions, and previously established results. While the student makes an attack on the problem it is incomplete. The student may recognize some patterns or structures, but has trouble generalizing or using them to solve the problem.

Level 3: Performance at Standard (5 – 6 points)
For most of the task, the student's response shows the main elements of performance that the tasks demand as defined by the CCSS and is organized as a coherent attack on the core of the problem. There are errors or omissions, some of which may be important, but of a kind that the student could well fix, with more time for checking and revision and some limited help. The student explains the problem and identifies constraints. The student makes sense of quantities and their relationships in the problem situations. S/he often uses abstractions to represent a problem symbolically or with other mathematical representations. The student response may use assumptions, definitions, and previously established results in constructing arguments. S/he may make conjectures and build a logical progression of statements to explore the truth of their conjectures. The student might discern patterns or structures and make connections between representations.

Level 4: Achieves Standards at a High Level (7 - 8 points)
The student's response meets the demands of nearly all of the tasks as defined by the CCSS, with few errors. With some more time for checking and revision, excellent solutions would seem likely. The student response shows understanding and use of stated assumptions, definitions and previously established results in constructing arguments. The student is able to make conjectures and build a logical progression of statements to explore the truth of their conjecture. The student response routinely interprets their mathematical results in the context of the situation and reflects on whether the results make sense. The communication is precise, using definitions clearly. The student looks closely to discern a pattern or structure. The body of work looks at the overall situation of the problem and process, while attending to the details.

GRADE 2 MATH: CAROL'S NUMBERS
ANNOTATED STUDENT WORK

This section contains annotated student work at a range of score points and implications for instruction for each performance level (excluding the expert level). The student work and annotations are intended to support teachers, showing examples of student understandings and misundertandings of the task. The annotated student work and implications for instruction can be used to understand how to move students to the next performance level.

Level 4: Achieves Standards at a High Level (Score Range 7 - 9)

The student's response meets the demands of nearly the entire task, with few errors. With some more time for checking and revision, excellent solutions would seem likely. The student response shows understanding and use of stated assumptions, definitions and previously established results in constructing arguments. The student is able to make conjectures and build a logical progression of statements to explore the truth of their conjecture. The student response routinely interprets their mathematical results in the context of the situation and reflects on whether the results make sense. The communication is precise, using definitions clearly. The students look closely to discern a pattern or structure. The body of work looks at the overall situation of the problem and process, while attending to the details.

Student A – Level 4 (Score 8)

Carol's Numbers

Carol has three number cards.

4. What is the largest three-digit number Carol can make with her cards?

5. What is the smallest three-digit number Carol can make with her cards?

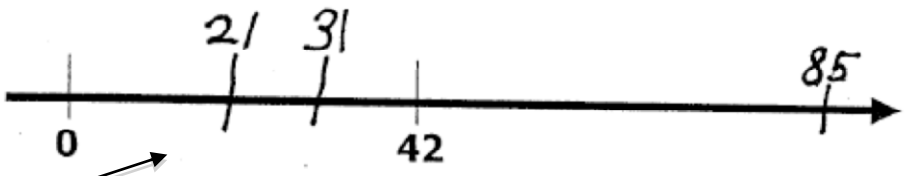
Explain to Carol how she can make the smallest possible number using her three cards.

You take the smallest number and place it in the front. Then you take the next smallest a place it behind the smallest, and so on.

Student A correctly creates the largest and smallest possible numbers out of the three digits. The student's explanation is clear and complete, indicating exactly how to find the smallest possible number. 2.NBT.1&4 MP3

Student A – Level 4 (Score 8) Page 2

Carol's teacher drew a number line on the board.



1. About where would 85 be? Place 85 on the number line where it belongs.

2. About where would 21 be? Place 21 on the number line where it belongs.

3. About where would 31 be? Place 31 on the number line where it belongs.

Tell Carol how you knew where to place 31 and why.

You place the 31 in the middle and a little smaller because 31 is ten more than 21 and is 11 less than 42.

The student correctly places the numbers on the number line and then explains the precise distances each number is from the other. 2NBT.4, 2MD.6 MP2, MP3.

Student B – Level 4 (Score 8)

Carol's Numbers

Carol has three number cards.

4 7 2

4. What is the largest three-digit number Carol can make with her cards?

7 4 2

5. What is the smallest three-digit number Carol can make with her cards?

2 4 7

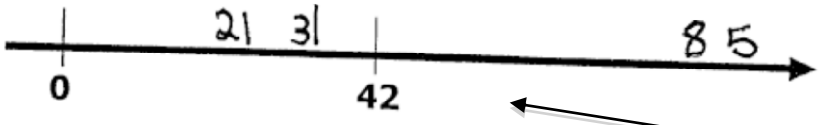
Explain to Carol how she can make the smallest possible number using her three cards.

She has the numbers 472, so her smallest number is 2 so she can put it in the hundreds. She has now 74 so now her smallest number is 4 so she puts it in the tens. Now her only number is 7 so she puts it in the ones.

The student's explanation of how to construct the smallest number indicates strong knowledge of place value. Articulating the process of determining the smallest digit available and then putting in the hundred's place is sophisticated thinking at this grade. 2.NBT.1&4 MP3

Student B – Level 4 (Score 8) Page 2

Carol's teacher drew a number line on the board.



1. About where would 85 be? Place 85 on the number line where it belongs.

2. About where would 21 be? Place 21 on the number line where it belongs.

3. About where would 31 be? Place 31 on the number line where it belongs.

Tell Carol how you knew where to place 31 and why.

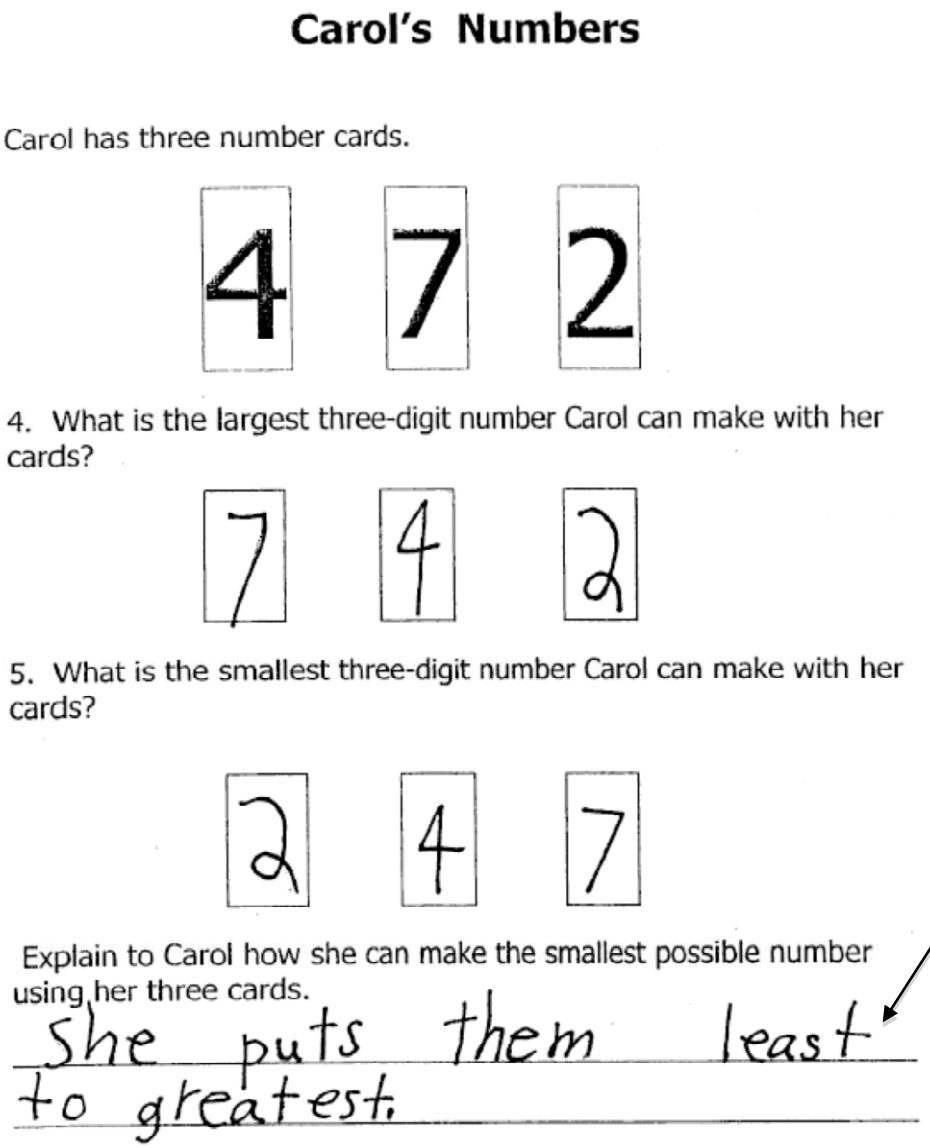
31 is almost between 21 and 42.

The student accurately places the numbers on the number line and then explains that 31 is almost (in the middle) between 21 and 42, indicating knowledge of quantities. 2NBT.4, 2MD.6, MP2

Level 3: Performance at Standard (Score Range 5 - 6)

For most of the task, the student's response shows the main elements of performance that the tasks demand and is organized as a coherent attack on the core of the problem. There are errors or omissions, some of which may be important, but of a kind that the student could well fix, with more time for checking and revision and some limited help. The student explains the problem and identifies constraints. The student makes sense of quantities and their relationships in the problem situations. S/he often uses abstractions to represent a problem symbolically or with other mathematical representations. The student response may use assumptions, definitions, and previously established results in constructing arguments. They may make conjectures and build a logical progression of statements to explore the truth of their conjectures. The student might discern patterns or structures and make connections between representations.

Student C – Level 3 (Score 6)



Carol's Numbers

Carol has three number cards.

4 7 2

4. What is the largest three-digit number Carol can make with her cards?

7 4 2

5. What is the smallest three-digit number Carol can make with her cards?

2 4 7

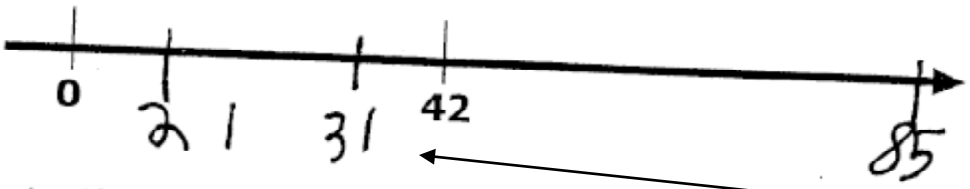
Explain to Carol how she can make the smallest possible number using her three cards.

She puts them least to greatest.

The student correctly creates the largest and smallest numbers possible out of the three cards. The explanation is short but does communicate least to greatest, indicating understanding and good use of vocabulary. 2NBT.4

Student C – Level 3 (Score 6) Page 2

Carol's teacher drew a number line on the board.



1. About where would 85 be? Place 85 on the number line wher it belongs.

2. About where would 21 be? Place 21 on the number line wher it belongs.

3. About where would 31 be? Place 31 on the number line where it belongs.

Tell Carol how you knew where to place 31 and why.

I put is ih the middle of 21 and 42 because it is ih beetween the to numbers.

The student's location of 21 indcates fragile understanding of numbers on the number line. It should be located half way between 0 and 42. The student does show some understanding by correctly locating 31. The student explains that 31 is between 21 and 42. The student reasons quantitatively and provides a clear explanation of how they thought. MP2 and MP3

Student D – Level 3 (Score 5)

Carol's Numbers

Carol has three number cards.

4. What is the largest three-digit number Carol can make with her cards?

5. What is the smallest three-digit number Carol can make with her cards?

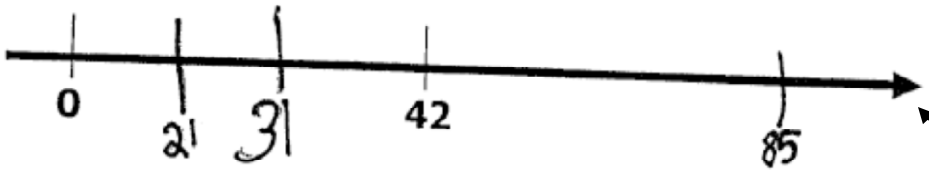
Explain to Carol how she can make the smallest possible number using her three cards.

She can put the smallest number first and the biggest number last.

The digits are correctly placed to make the largest and smallest possible numbers. The explanation addresses the smallest and largest digits, leaving the third digit to be placed by default. The student implies place value and comparison. 2NBT.1 & 4

Student D – Level 3 (Score 5) Page 2

Carol's teacher drew a number line on the board.



1. About where would 85 be? Place 85 on the number line where it belongs.

2. About where would 21 be? Place 21 on the number line where it belongs.

3. About where would 31 be? Place 31 on the number line where it belongs.

Tell Carol how you knew where to place 31 and why.

I knew I had to place 31 before and close to 42 because it less than 42 and close.

The student misplaced 21, but did place 31 about halfway between 21 and 42. The student's explanation indicated all the reasoning was concentrated on placing 31 correctly. 2MD.6 MP2

Student E – Level 3 (Score 5)

Carol's Numbers

Carol has three number cards.

4. What is the largest three-digit number Carol can make with her cards?

5. What is the smallest three-digit number Carol can make with her cards?

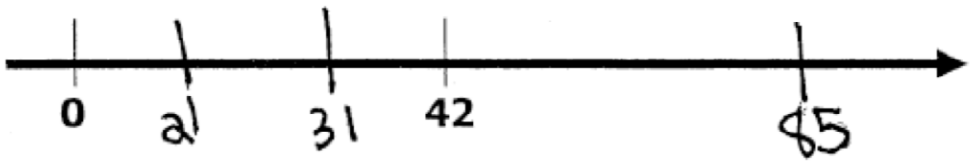
Explain to Carol how she can make the smallest possible number using her three cards.

She could put the card with the number 2 to the left, 4 to the middle, and 7 on the right.

The student found the largest and smallest 3 digit numbers. The explanation merely recalls the process the student used to make the smallest number, but does not explain why. The student needs to by indicate ideas such as least or most. The students needs to show more quantitative reasoning. MP2

Student E – Level 3 (Score 5) Page 2

Carol's teacher drew a number line on the board.



1. About where would 85 be? Place 85 on the number line where it belongs.

2. About where would 21 be? Place 21 on the number line where it belongs.

3. About where would 31 be? Place 31 on the number line where it belongs.

Tell Carol how you knew where to place 31 and why.

I knew because 31 is before 42 and after 21.

21 is misplaced. The student does explain why s/he placed 31 correctly on the number line. The student needs additional experiences in finding the mid-point between a larger number and zero. 2MD.6

Level 3 Implications for Instruction

Students who met standard on the task can still improve their performance by being attentive to precession and by making complete explanations. Students must learn to provide complete explanations of their process and why it makes sense. Many of the students who met level 3 failed to place 21 correctly on the number line. They seemed to fail to understand that 21 should be half way between 0 and 42. On the other hand, most students could accurately use their placement of 21 to find a mid-way mark between its placement and 42 to locate 31. This may indicate that students have quantative reasoning about length when comparing smaller lengths (20) apart. These same students had trouble with longer distances (42) and perhaps dealing with 0 and an ending length.

Therefore, students need more practice with finding lengths on the number line. Student should locate numbers on an open number line, learning to make relative judgements base on benchmark amounts. Students will benefit from engaging in number line math talks and measuring along a number line when working with numbers.

Level 2: Performance below Standard (Score Range 2 - 4)

The student's response shows some of the elements of performance that the tasks demand and some signs of a coherent attack on the core of some of the problems. However, the shortcomings are substantial, and the evidence suggests that the student would not be able to produce high-quality solutions without significant further instruction. The student might ignore or fail to address some of the constraints. The student may occasionally make sense of quantities in relationships in the problem, but their use of quantity is limited or not fully developed. The student response may not state assumptions, definitions, and previously established results. While the student makes an attack on the problem it is incomplete. The student may recognize some patterns or structures, but has trouble generalizing or using them to solve the problem.

Student F – Level 2 (Score 4)

Carol's Numbers

Carol has three number cards.

4. What is the largest three-digit number Carol can make with her cards?

5. What is the smallest three-digit number Carol can make with her cards?

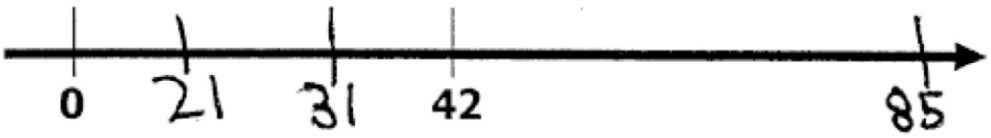
Explain to Carol how she can make the smallest possible number using her three cards.

247 is less then 427 and 742.
That why 247 is less.

The student's explanation restates what was shown in part 4 and 5. It does not explain how to create the smallest number or how the student knows it is the smallest number. The student needs more instruction around creating clear and complete mathematical explanations. MP3

Student F – Level 2 (Score 4) Page 2

Carol's teacher drew a number line on the board.



1. About where would 85 be? Place 85 on the number line where it belongs.

2. About where would 21 be? Place 21 on the number line where it belongs.

3. About where would 31 be? Place 31 on the number line where it belongs.

Tell Carol how you knew where to place 31 and why.

31 Was less then 42. SO
31 is betwene 21 42.

This work continues to show the trend of not being able to accurately place 21 half way between 0 and 42, but still showing that 31 is about mid-way between 21 and 42. More quantitative reasoning is needed. 2MD.6, MP2

Level 2 Implications for Instruction

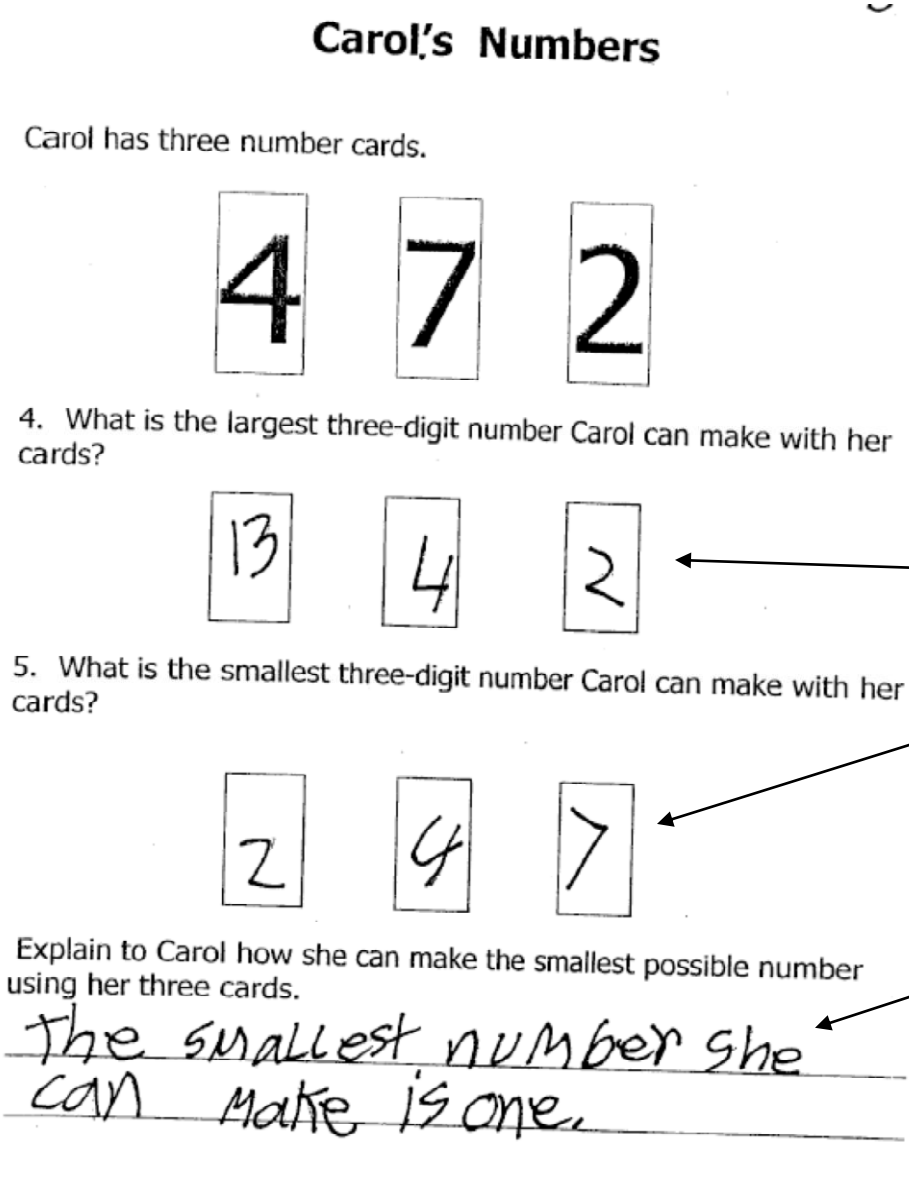
Students need help in comparing the placement of numbers using 0 as a benchmark. The explanations at this level are either incomplete or not focused on mathematical reasoning the make sense for the situation. The students need learning experiences with number lines that start with zero.

Students can experiment with these ideas to develop a deeper conception of numbers, so that they may more flexibly reason quantitatively. Instruction should involve more work on defining the relationships between length and the size of numbers. Students need experiences to construct learning for themselves. Students should be asked to explain and justify their answers regularly in class to develop mathematical argumentation. Examining and analyzing other students' explanations is an important experience for students. It provides models and helps students discern important elements of convincing arguments.

Level 1: Demonstrates Minimal Success (Score Range 0 – 1)

The student's response shows few of the elements of performance that the tasks demand. The work shows a minimal attempt on the problem and struggles to make a coherent attack on the problem. Communication is limited and shows minimal reasoning. The student's response rarely uses definitions in their explanations. The student struggles to recognize patterns or the structure of the problem situation.

Student G – Level 1 (Score 1)



Carol's Numbers

Carol has three number cards.

4 7 2

4. What is the largest three-digit number Carol can make with her cards?

13 4 2

5. What is the smallest three-digit number Carol can make with her cards?

2 4 7

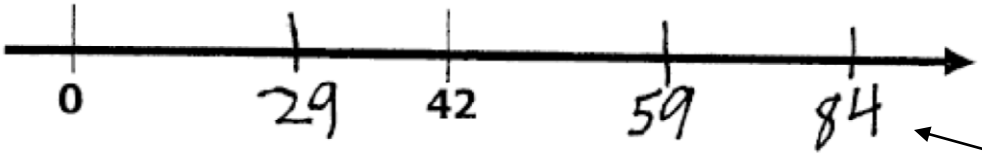
Explain to Carol how she can make the smallest possible number using her three cards.

The smallest number she can make is one.

The student must have added the original digits to find 13 and then used it to indicate the largest digit. The student was able to determine the smallest (three digit) number, but goes on to explain that 1 is the smallest possible number. The student's explanation showed a lack of understanding what the problem entailed. The students needs to focus on what the prompts are asking.

Student G – Level 1 (Score 1) Page 2

Carol's teacher drew a number line on the board.



1. About where would 85 be? Place 85 on the number line where it belongs.

2. About where would 21 be? Place 21 on the number line where it belongs.

3. About where would 31 be? Place 31 on the number line where it belongs.

Tell Carol how you knew where to place 31 and why.

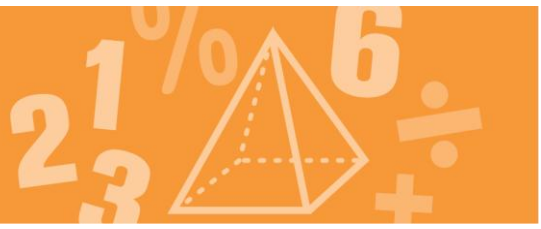
I cawnted It wood be eser,

The student showed little understanding of the prompt and seem to ignore the values 21, 31 and 85. The student placed random numbers on the number line. They did seem to do so in an accurate manner though. The explanation was off the mark. This student needs additional instruction in understanding the task prompts and addressing them.

Level 1 Implications for Instruction

Students need support in reasoning quantitatively. They may need to start with number lines and the idea of length equaling the size of a number. Students need experiences connecting numbers to their location on the number line. Having students divide the distance between zero and the number to find the location is important. The location of the number on the line is the equal to the number of partitioned segments of length 1. Students need experiences in reasoning quantitatively about benchmark numbers and their relate size to other numbers. One method successful students use is to determine midway points as benchmark. They estimate about half to compare relative magnitude of different lengths. Students need learning experiences with number lines, including both closed and open number lines.

Students need additional instruction and experiences in writing explanations that fully articulate their understanding and justify their findings. Sharing models of good explanations are helpful. Having students rewrite and revise explanations is essential. Having students read others' explanations and critiquing their reasoning raises the cognitive demand and helps students create sounder arguments.



GRADE 2 MATH: CAROL'S NUMBERS INSTRUCTIONAL SUPPORTS

The instructional supports on the following pages include a unit outline with formative assessments and suggested learning activities. Teachers may use this unit outline as it is described, integrate parts of it into a currently existing curriculum unit, or use it as a model or checklist for a currently existing unit on a different topic.

Grade 2 Math Unit Outline

INTRODUCTION: This unit outline provides an example of how teachers may integrate performance tasks into a unit. *Teachers may (a) use this unit outline as it is described below; (b) integrate parts of it into a currently existing curriculum unit; or (c) use it as a model or checklist for a currently existing unit on a different topic.*

Second Grade Math: Strategies Using Base Ten

UNIT TOPIC AND LENGTH:

- The mathematics of the unit involves understanding the meaning of base ten and using that understanding to solve number and real life problems. The number line is used as a tool to help articulate understanding of base ten and to solve problems using addition and subtraction of numbers less than one hundred. The focus is on the big idea around groups of ten. Strategies will involve applying number properties including distributive, associative, and commutative. The units should run between 20 and 25 standard periods of instruction. Five of the periods will involve the pre-assessment (0.5 periods), introducing and supporting problem solving on the investigation (2 periods), teaching the formative assessment lesson (2 periods, broken into smaller segments at the beginning of several lessons to adjust for grade appropriate attention span and learning styles) and the final assessment (0.5 periods). This unit should be taught after or during the time students have learned about base ten concepts, place value and operations on numbers.

COMMON CORE LEARNING STANDARDS:

- **2.NBT.1** Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:
  - **1a** 100 can be thought of as a bundle of ten tens – called a "hundred"
  - **1b** The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
- **2.NBT.3** Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.
- **2.NBT.4** Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.
- **2.NBT.6** Add up to four two-digit numbers using strategies based on place value and properties of operations.
- **2.MD.6** Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.
- **MP.1** Make sense of problems and persevere in solving them.
- **MP.3** Construct viable arguments and critique the reasoning of others.
- **MP.6** Attend to precision

BIG IDEAS/ENDURING UNDERSTANDINGS:

- The big idea of the unit is to understand the meaning of place value and properties of operations to develop strategies for adding and subtracting.
- The number line will serve as one model for helping to ground student thinking about place value and landmark values of groups of tens.
- Students will be able to justify strategies using properties and models.
- Students will represent strategies using number sentences, models, and drawings.
- Students will apply strategies to solve one and two- step problems and express their ideas using number sentences.

ESSENTIAL QUESTIONS:

- What are the ways to compose and decompose numbers, using place value, to aid in adding and subtracting numbers with ease? What situation types represent addition and subtraction?

CONTENT:

The students will use knowledge of mathematics to:

- Think of numbers as sets of ten and extras
- Fluently add and subtract within 20 using mental strategies
- Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction
- Translate between models.
- Add and subtract using concrete models or drawings including the number line.
- Explain why addition and subtraction strategies work, using place value and properties of operations.
- Construct valid arguments.
- Critique the validity of arguments.

SKILLS:

Students will understand:

- How to represent and solve problems involving addition and subtraction and understand the relationship between the two operations.
- How to add and subtract within 20
- How to use place value understanding and properties of operations to add and subtract within 100
- How to interpret the action of a situation to choose an appropriate operation for solving a problem.
- Make convincing arguments for why strategies work using knowledge of place value and operations properties.

ASSESSMENT EVIDENCE AND ACTIVITIES:

INITIAL ASSESSMENT :

The unit begins with the performance task ***Student Store***. The task is designed to measure what students bring to the unit in regards to their knowledge and skill at adding and subtracting two-digit numbers and the relationship between operations. The task allows the teacher to see what strategies the students know and to think about how they might be developed. The task also allows misconceptions about place value to surface. Please reference ***Student Store*** for full details.

FORMATIVE ASSESSMENT:

The Formative Assessment Lesson is entitled ***Strategies in Base Ten: Base Ten Number Talks***. To be grade appropriate, the FAL is a series of seven number talks to be done over the course of the instruction unit as warm ups before the regular instruction for the day. This allows students to digest the material over time. The FAL comes with complete teacher notes and the student pages. Please reference ***Base Ten Number Talks*** for full details.

FINAL PERFORMANCE TASK:

The final performance assessment is entitled ***Carol's Numbers***. It should be administered during a class period. Most students will complete the task in about 20 – 25 minutes, although time should not be a factor. The teacher should provide a reasonable amount of time for all students to finish. The students should be allowed to use any tools or materials they normally use in their classroom. The task can be read to the students and all accommodations delineated in an IEP should be followed. Please reference Carol's Numbers for full details.

LEARNING PLAN & ACTIVITIES:

The investigation is entitled *Got Your Number*. It contains three separate but mathematically related problems labeled Level Primary A, Level A, and Level B. All students should start with the Level Primary-A task and then proceed at their own speed to Level A and Level B. It is more important for the student to work deeply on a level and complete a write up than to merely work through and find answers. It is the student's responsibility to be reflective and thorough in their explanations, findings and justifications. The investigation comes with administration and teacher notes, the expert investigation, report guidelines and rubric.

Number Talks A daily ritual with the entire class for the purposes of developing conceptual understanding of numbers, operations and mathematics. Number talks are used to:

- Review and practice operations, procedures and concepts of numbers.
- Introduce concepts and properties about numbers.
- Reinforce procedures and number concepts.
- Explore connections about numbers.

Do a number talk every day but for only 10 minutes. A few minutes more often is better than a lot of minutes infrequently.

1. Ask questions such as:
   - How did you think about that?
   - How did you figure it out?

- What did you do next?
- Why did you do that? Tell me more.
- Who would like to share their thinking?
- Did someone solve it a different way?
- Who else used this strategy to solve the problem?
- What strategies do you see being used?
- Which strategies seem to be efficient, quick, and simple?

2. Give yourself time to learn how to:
   - Record student solutions
   - Listen to and observe students
   - Collect notes about student strategies and understanding
3. Name/label the strategies that emerge from your students:
   - Use doubles
   - Break apart numbers
   - Make it simpler
   - Use landmark numbers (25, 50, 75, 200, etc)
   - Use a model to help
   - Use what you already know
   - Make a "10"
   - Start with the 10's
   - Think about multiples
   - Think about money
   - Traditional algorithm
   - Counting on
4. Create a safe environment. When students feel safe, they are comfortable sharing answers even when it's different from everyone else's.
5. Give opportunities for students to "think first."
6. Encourage self-correction; it's okay to change your mind, analyze your mistake, and try again.
7. Give number talks time to become part of your classroom culture. Expect them to follow the usual learning curve stages. "Keep on keeping on" and you will get positive results.

Think/Write/Pair/Share is a high leverage strategy that respects individual time to process and organize ideas before engaging in peer-to-peer discussion. This process can be used throughout the unit as a vehicle for students to self reflect, construct new meaning by building on the ideas of others, and strengthen their arguments.

Journal Entries for Reflection Using a prompt such as, *"How has my thinking changed as a result of what I have discussed with my peers?"* or *"How can I improve my argument or explanation using evidence and content vocabulary?"* can provide valuable opportunities for students to tweak their own solutions during class, or for homework, and subsequently, to deepen their understanding of content.

Purposeful Questioning and Feedback are instructional supports that can help refocus students' attention to specific aspects of their work. Suggestions based on common difficulties students faced in their pre-assessment task, ***School Store*** for example can be easily modified to address similar misconceptions revealed from other problems or tasks used. Please see the chart containing suggested questions and prompts based on understandings/misunderstandings for reference.

Points	Understandings	Misunderstanding	Suggested Prompts and Questions.
0	Less than one-half of a percent of the students did not attempt this problem.	Calculating the correct total when adding and subtracting was a struggle for these students. Many did not give a total for the two items added together nor did they explain how they got their answers for problem 1 or 2.	• How can you find how much it costs all together? • Why did you pick the items you did? • How much total money does Alisha have? • Tell me how much each item costs?
1-2	Most of these students listed the names of two items that cost less than 70 cents when added together.	Many of these papers reflected errors in calculations of addition and subtraction. Either no attempt was made to explain the answer to either the question or to prove how they knew that their answer was correct.	• Show me how you added those two amounts? • How did you find this answer? • Tell me how you figured it out? • Can you check your answer to tell whether you added correctly?
3-4	Students scoring in this range could find two items and a correct total that was less than 70 cents.	Finding correct change was a struggle for these students. They subtracted one item from another and erred when it was necessary to count up to 70 cents, or subtracted incorrectly when they needed to rename 70 cents to subtract.	• Show me with money how you too much was left over after the items were paid for? • Is there a way to figure that out using operations? • Can you check your answer to tell whether you subtracted correctly?
5-6	In most cases, these students were able to find the correct change from 70 cents. Occasionally, their calculations where incorrect, but they correctly showed a method to arrive at the calculation.	Most of these students were unable to prove how they arrived at the change from 70 cents, (i.e. an answer was given, but no work was shown).	• How did you figure this out, what did you do? • Write down what you were thinking about when you found your answer? • Can you show me a number sentence that you used to find your answer?

Re-engagement: The unit begins with a pre-assessment called *School Store*. After the teacher analyzes the student performances, deciding how to proceed through the lesson is important based

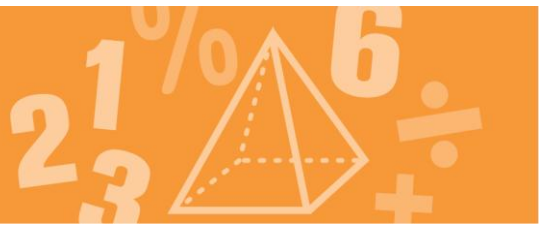
on their performance. Instead of going back to re-teach skill or concepts lacking, it is much more powerful for the teacher to use the work students have already done on a contextual problem to help them build upon their understanding from previous thinking. This process is called "re-engagement". It is especially powerful to use student work because students become very engaged in the process of figuring out what someone else is thinking. The process of analyzing and contrasting student thinking encourages cognitive demand and supports students to be more reflective about their own thinking. The re-engagement lesson will depend upon the results from students in each individual class. Thus, each lesson will look very different from class to class. Students have already completed the task on their own and now the important ideas need to be brought out and examined. In the process, students must have the opportunity to confront and understand the error in the logic of their misconceptions. Often, as teachers, we try to prevent errors by giving frequent reminders, such as "line up the decimal point." However, errors actually provide great opportunity for learning on the part of all students. Especially because students don't let go of misconceptions until they understand why they don't make sense. For the student, there is underlying logic to their misconceptions.

Re-engagement – Confronting misconceptions, providing feedback on thinking, going deeper into the mathematics.

1. Start with a foundational problem to bring all the students along; this allows students to clarify and articulate important mathematics in order to better understand the entirety of the task.
2. Share different student approaches and ask all students to make sense of each strategy. Have all students compare the strategies to look for the mathematical connections and relationships.
3. Have students analyze misconceptions and discuss why they don't make sense. In the process students can let go of the misconceptions and clarify their thinking about big mathematical ideas.
4. Have students determine how a strategy could be modified to get the correct solution. Have them look for the seeds of mathematical thinking within selected student work.

RESOURCES:

- Normal materials used in math class include manipulatives such as cards for matching activity, square tiles, counters, and cm graph paper.
- All the materials referenced in the assessments, formative assessment lesson and expert investigation are included. Most supplementary materials are located in the appendix, including the established scored benchmark papers and some student work examples.



GRADE 2 MATH: CAROL'S NUMBERS

INITIAL TASK: STUDENT STORE

Student Store

Alisha went to the student store at lunch.

Student Store	
Pen	39¢
Stickers	22¢
Eraser	42¢
Pencil	29¢
Paper	32¢

She had 70¢. She bought 2 items. When she left the store, she had some change left.

1. Which two items could she have bought? _____ and _____.

Show how much they cost all together.

2. How much change did she have left? _____

Show how you figured it out.

GRADE 2 MATH: CAROL'S NUMBER

FORMATIVE ASSESSMENT LESSON: BASE TEN NUMBER TALKS

Base Ten Number Talks

Silicon Valley Mathematics Initiative- 2011

Formative Assessment Lesson

Grade 2

Mathematical goals

This lesson unit is intended to help you assess how well students are able to use addition and subtraction strategies that relate to place value and to the properties of operations. Students should develop a range of strategies and be able to explain why the strategies work. In particular this unit aims to identify and help students who have difficulties with:

- Choosing an appropriate operation for a problem situation
- Understanding regrouping with addition and subtraction
- Estimating the reasonableness of answers
- Generalizing using numerical and visual tools
- Describing and explaining solutions clearly and effectively

Standards addressed

This lesson relates to the following Common Core State Standards:

Second Grade Number and Operations in Base Ten: Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

Second Grade Number and Operations in Base Ten: Explain why addition and subtraction strategies work, using place value and the properties of operations.

This lesson relates to the Mathematical Practices:

- Modeling: Mathematically proficient students can apply the mathematics they know to solve problems. This might mean at early grades being able to write an equation or problem to demonstrate a situation.
- Constructing Viable Arguments and Critiquing the Reasoning of Others: At the early grades, this might mean that students can compare their results with others. They might find ways to order and/or generalize about solutions. Others may find more than one solution path to a given problem.

Introduction

This lesson unit is structured in the following way:

- Students work on their own, completing an assessment task that is designed to reveal their current understanding and difficulties.
- Students work in seven number talk segments or warm-up sessions to be used over the course of the unit of study in number operations in base ten.
    1. During the number talks, students have individual think time to develop strategies on their own, actively listen to the strategies of others and ask clarifying questions, learn to explain why their strategies work or suggest logical reasons why a strategy won't work for all cases.
    2. Students are asked to “try on” key strategies to check for understanding.
    3. Students are confronted with examples of student work and asked to make sense of a strategy and justify why it does or does not work.
- Students return to their original task and try to improve their own responses.

Materials required

Each student will need one copy of the assessment task: Grandmother’s Birthday

Each pair of learners with need the following:

- Manipulatives used regularly in math class to facilitate number, operations and place values

such as: paper and pencil, mini-white boards, base ten models, 100s and/or 0-99 charts, ten frames, counters, cubes.
- Index cards or half sheets of paper to collect at least one formative assessment of number talk ideas

Resources

Books

Carpenter, Thomas P., Franke, Megan Loef, and Levi, Linda 2003 **Thinking Mathematically: Integrating Arithmetic & Algebra in Elementary School** Portsmouth, New Hampshire: Heinemann

Corwin, Rebecca B. 1996 **Talking Mathematics: Supporting Children's Voices** Portsmouth,New Hampshire: Heinemann

Fosnot, Catherine Twomey and Dolk, Maarten 2001 **Young Mathematicians at Work: Constructing Number Sense, Addition, and Subtraction** Portsmouth, New Hampshire: Heinemann

Heibert, James 1997 **Making Sense** Portsmouth, New Hampshire: Heinemann

Van de Walle, John A. 2004 **Elementary and Middle School Mathematics: Teaching Developmentally** Boston, MA Pearson Education, Inc.

Van de Walle, John A. 2006 **Teaching Student Centered Mathematics: Grades K-3** Boston, MA Pearson Education, Inc

Videos

"How to Teach Math as a Social Activity" Edutopia.org video on building community norms around math discussions.

Video episodes through this resource show students formulating conjectures around properties of numbers and operations: Carpenter, Thomas P., Franke, Megan Loef, and Levi, Linda 2003 **Thinking Mathematically: Integrating Arithmetic & Algebra in Elementary School** Portsmouth, New Hampshire: Heinemann

Silicon Valley Mathematics Initiative Number Talk Videos, Grade 2 (needs to be added)

Time needed

The lesson will need one 20-25 minute pre-assessment session, at least seven 15-20 minute warm-up Number Talk sessions and a 15-20 minute student editing session to revise initial pre-assessment. Timings given are only approximate. Exact timings will depend on the needs of the class.

Before the lesson

Individual Assessment Task: *Grandmother's Birthday*

The assessment task, Grandmother's Birthday should be completed before the lesson. If needed, the task may be read to students. Ask students to attempt the task on their own. Explain that they should not worry too much if they cannot understand or do everything, because you plan to teach several number talk lessons which should help them.

It is important that students are allowed to answer the questions without assistance, as far as possible. If students are struggling to get started then ask questions that help them understand what is required, but don't do it for them!

Assessing students responses

Collect a sample of students' responses to the task and make some notes on what their work reveals about their current levels of understanding. The purpose of doing this is to forewarn you of the difficulties students may experience during the lesson itself and so that you may prepare carefully. Do not grade students' work at this stage. Research shows that this will be counterproductive, as it will encourage students to compare their grades and distract their attention from the mathematics. Instead, try to understand their reasoning and think of ways in which you can help them. Wait to grade this task until it is revised by students at the end of this lesson.

Three addendums to this unit are attached: **Sample Number Talk – Using Properties of Operations,** Addendum 1, provides an introduction to the processes and procedures of Number Talks; **Strategies and Solution Paths around Number, Operations and Place Value**, Addendum 2, provides a Common Issues of Number Talks chart with suggested prompts and questions and with references to the Common Core Mathematics Practices. **Student Responses to Discuss**, Addendum 3, includes the student work to discuss on Days 3 and 7 as well as questions and prompts for those days and recording strategies that can be adapted to the problems on any Number Talk Day.

Suggested lesson outline

Number talks begin with the teacher presenting a problem to the whole class. It is helpful to record the problems horizontally to encourage solution strategies other than the standard algorithm. Give students individual "think time" and ask them to signal when they have an answer (e.g. thumbs up). Students may (should) be asked to "pair share" their thinking before sharing with the large group. The teacher records the students' ideas on chart paper so that strategies can be referenced later. A good "general question" to ask students is, "Does my picture show what you were thinking?" Please note that the sample work below is by no means complete, students will always come up with strategies that are new to us!

Number Talk 1: Class introduction (15 minutes)

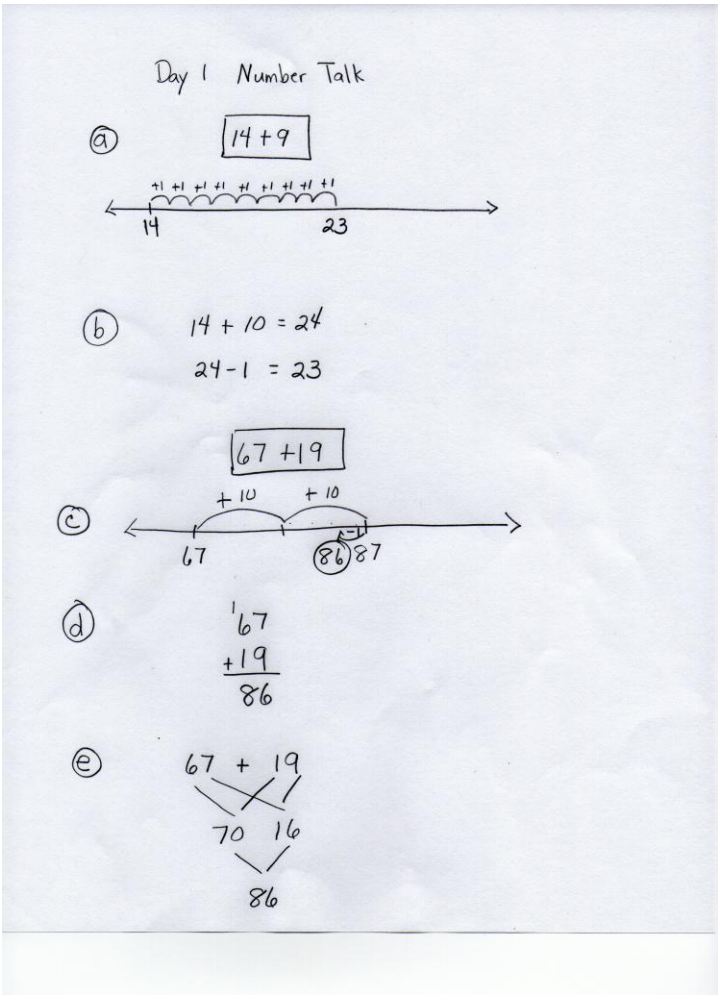
14 + 9

67 + 19

Possible questions about the sample student work below:

a) What strategy does this student use? How can this student be sure s/he has added exactly 9?
b) Why did this student subtract 1 from 24?
c) Why did this student add 10 twice?
d) Students using the standard algorithm mentally can be "pushed" to explain their thinking in depth. For example, if the student says "Seven plus nine is sixteen and carry the one and six and one and one is eight". This may be recorded as 7+9=16 and 1+6+1=8. Students are then "pushed" to explain that the 1+6+1 is representing 10's.
e) Is it ok to start adding the 10's first?

Below are some examples of possible student responses.



Number Talk 2:

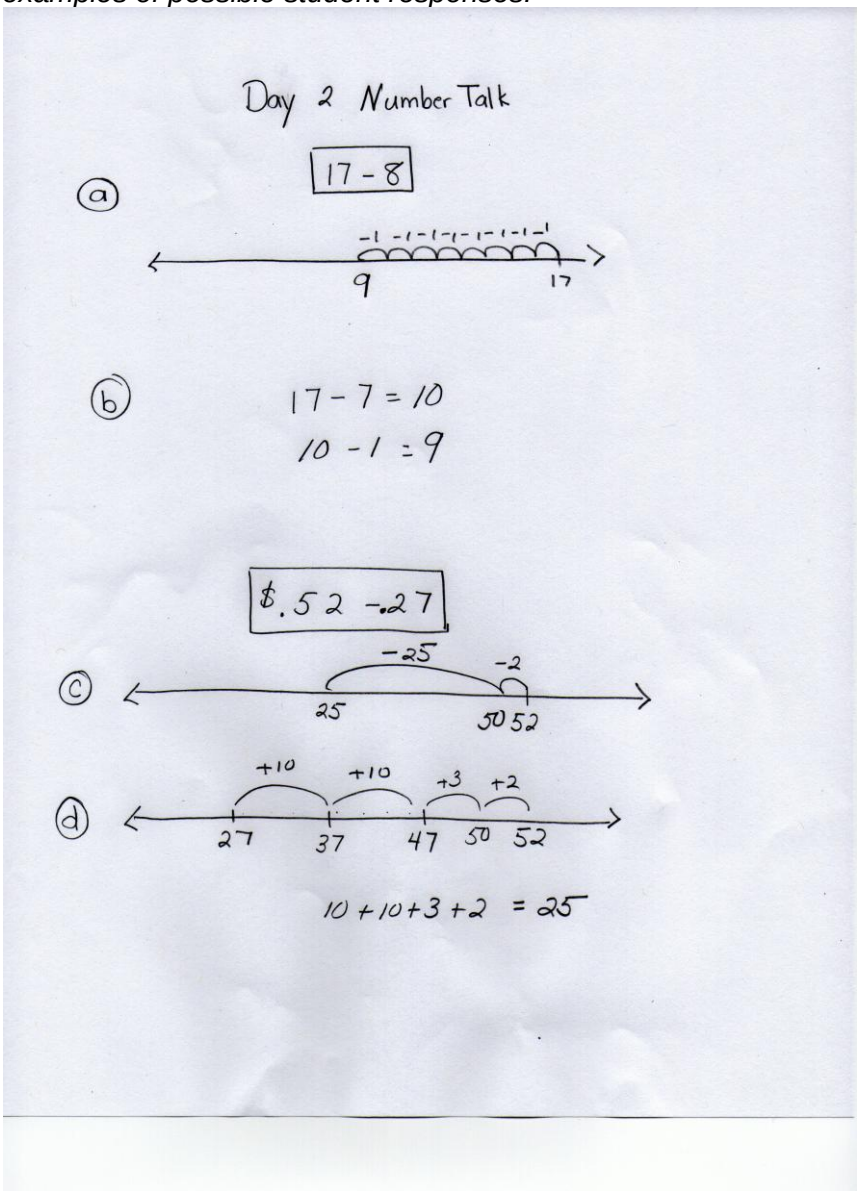
17 - 8

$.52 - .27

Possible questions about student work below:

a) What is this student's strategy? What might have been this student's first step to solving this problem?
b) Why did this student subtract 7 first?
c) Why did this student subtract 2 and then 25?
d) Can you always solve subtraction problems by adding on?

Below are some examples of possible student responses.



Number Talk 3: What did these students do? Why does it work?

77 + 18 (2 examples - “splitting” and number line)

64 – 45 (2 examples – number line take away and number line add-on?)
Student work in Student Resources to Discuss – Addendum 3

Number Talk 4: Two story problems

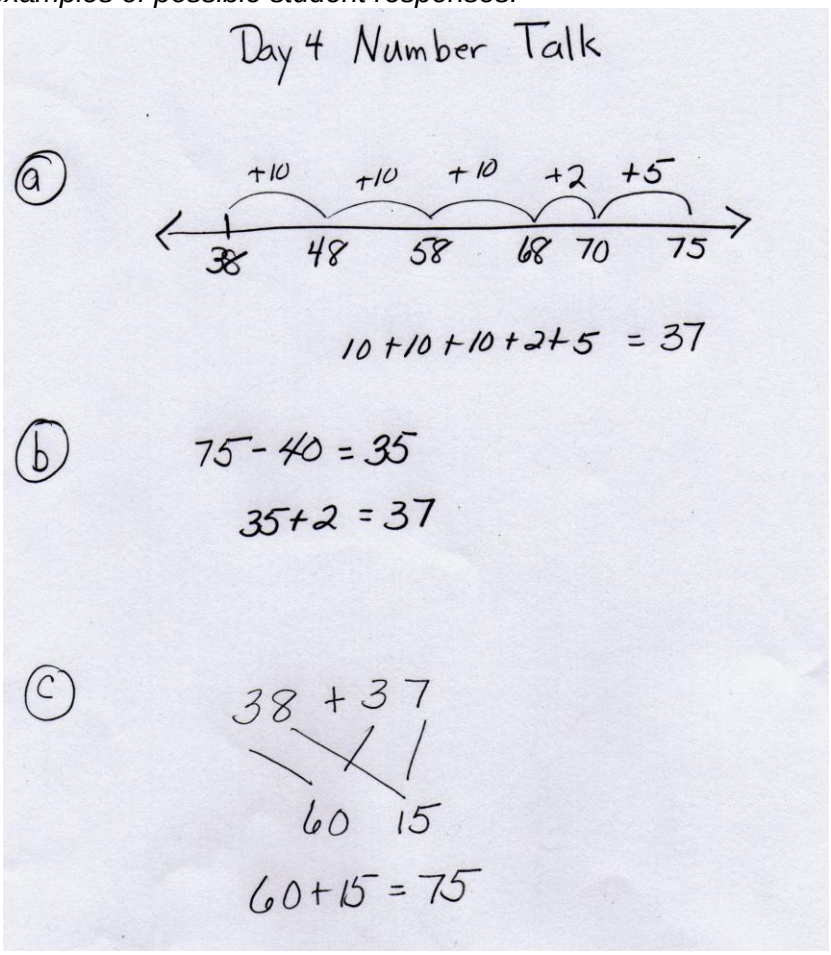
38 Second graders and many Third graders went to the Aquarium. There were 75 students in all. How many Third graders went to the Aquarium ?

Maria had some pennies. There were 38 pennies on the floor of her room. There were 37 pennies in her piggy bank. How many pennies did she have altogether?

Possible questions about student work:

a) How could this student use the number line to solve this problem in a different way?
b) Why did this student subtract 40 from 75?
c) What is a different way to group these numbers to make adding easier?

Below are some examples of possible student responses.



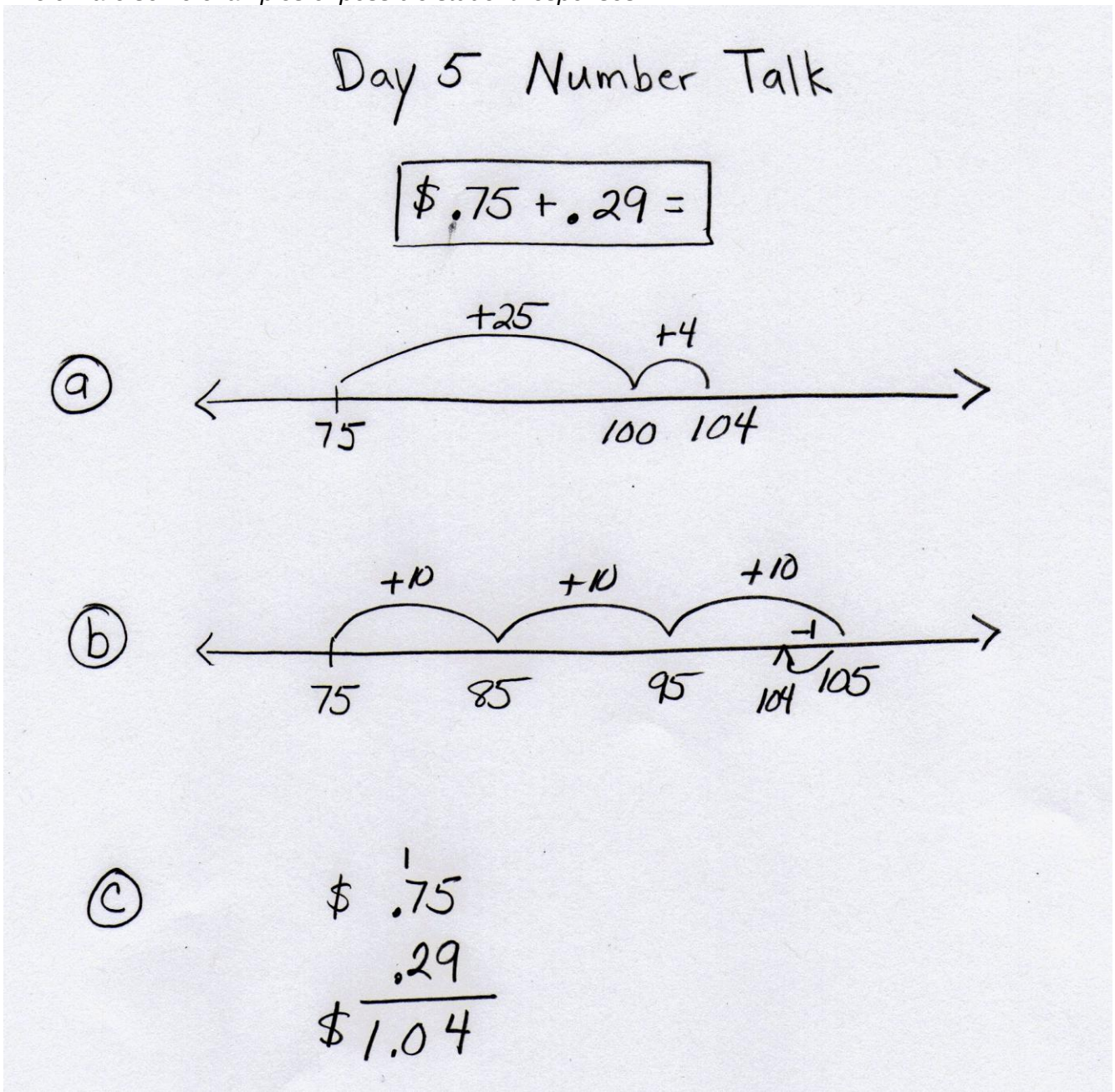
Number Talk 5:

$.75 + .29

Possible questions about student work:

a) Why did this student select 25 to add first?
b) Why did this student need to subtract 1 from 105?
c) Explain why this student could have added $.74+ .30 to get the same answer.

Below are some examples of possible student responses.



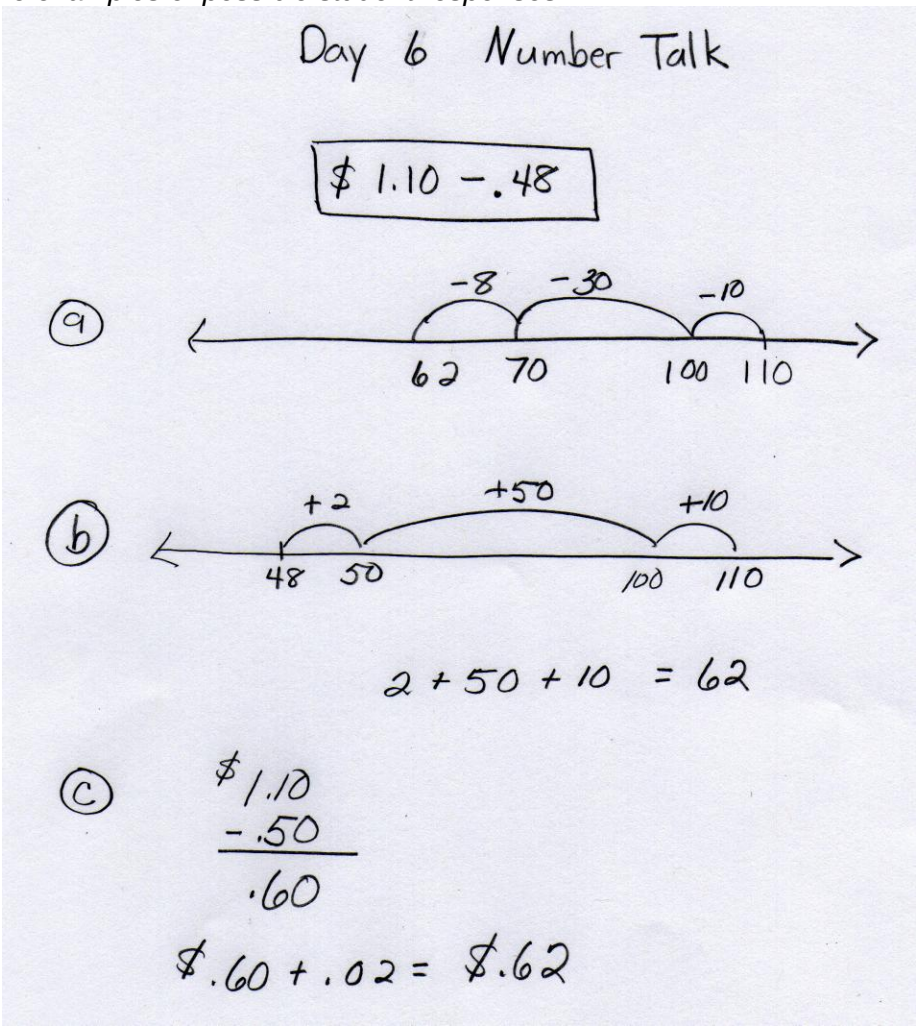
Number Talk 6:

$1.10 - .48

Possible questions about student work:

a) Why did this student subtract 10 first?
b) Was 50 easy to add in this problem? Why?
c) Why did this student add $.02 to $.60?

Below are some examples of possible student responses.



Number Talk 7: What did these students do? Why does it work?

74 + 18

$1.26 - .49
Student work in Student Resources to Discuss – Addendum 3

I**ndividual post-assessment work (15 - 20 minutes)**
Finally, reissue the Assessment task, Grandmother's Birthday, and ask students to have another go at it. It is helpful if this is done in a different color, so that you can see what they have learned. This will help you monitor what has been gained from the lesson.

Grandmother's Birthday

1. Jack's grandmother is having a birthday next month. 42 guests will come to the party. So far, Jack and his mother have written 17 invitations. How many more invitations will they need to write?
   Show your answer and all of your work. _____ invitations

2. Grandmother is turning 60 years old. Jack found 26 candles for her birthday cake. How many more candles will he need?
   Show your answer and all of your work. _____ candles

Student Scored Work a.

Grandmother's Birthday

1. Jack's Grandmother Mimi is having a birthday next month. 42 guests will come to the party. So far, Jack and his mother have written 17 invitations. How many more invitations will they need to write?
   Show your answer and all of your work. 30 × invitations

42
-17
30 ×

2. Grandmother Mimi is turning 60 years old. Jack found 26 candles for the birthday cake. How many more candles will he need?

   Show your answer and all of your work. 46 × candles

60
-26 ×
46

0

Student Scored Work b.

Grandmother's Birthday

1. Jack's Grandmother Mimi is having a birthday next month. 42 guests will come to the party. So far, Jack and his mother have written 17 invitations. How many more invitations will they need to write?
   Show your answer and all of your work. 25 ✓ invitations 1

17, 27, 37, 40, 42 ✓ 1

2. Grandmother Mimi is turning 60 years old. Jack found 26 candles for the birthday cake. How many more candles will he need?

   Show your answer and all of your work. 34 ✓ candles 1

26, 36, 46, 56, 60 ✓ 1

(4)

Student Scored Work C.

Grandmother's Birthday

1. Jack's Grandmother Mimi is having a birthday next month. 42 guests will come to the party. So far, Jack and his mother have written 17 invitations. How many more invitations will they need to write?
   Show your answer and all of your work. 25 ✓ invitations

15, 18, 19, 20, 21, 22, 23, 24, 25, 26, 25
28, 29, 30, 31, 32, 33, 34, 35, 36 35,
38 39, 40, 41, 42

1 ✓ 1

2. Grandmother Mimi is turning 60 years old. Jack found 26 candles for the birthday cake. How many more candles will he need?
   Show your answer and all of your work. 35 ✗ candles

60 59, 58, 57, 56, 55, 54, 53, 52, 51, 50,
49, 48, 47, 46, 45, 44, 43, 42, 41, 40, 39,
38, 37, 36, 35, 34, 33, 32, 31, 30, 29, 28, 25
26

0 ✓ 1

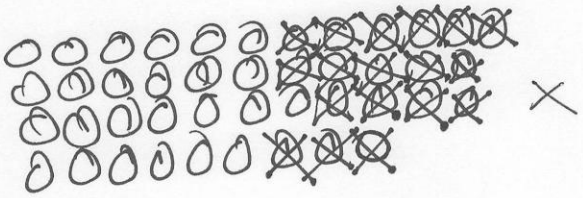
3

Student Scored Work

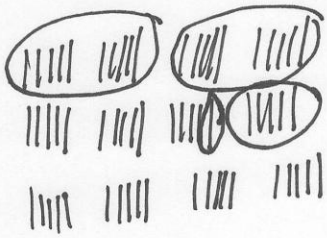
d.

Grandmother's Birthday

1. Jack's Grandmother Mimi is having a birthday next month. 42 guests will come to the party. So far, Jack and his mother have written 17 invitations. How many more invitations will they need to write?
   Show your answer and all of your work. 25 invitations



2. Grandmother Mimi is turning 60 years old. Jack found 26 candles for the birthday cake. How many more candles will he need?
   Show your answer and all of your work. 44 candles

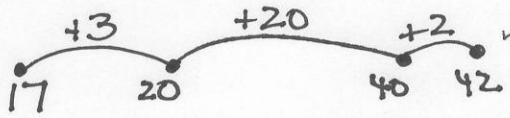


Student Scored Work

e.

Grandmother's Birthday

1. Jack's Grandmother Mimi is having a birthday next month. 42 guests will come to the party. So far, Jack and his mother have written 17 invitations. How many more invitations will they need to write?
   Show your answer and all of your work. 25 ✓ invitations



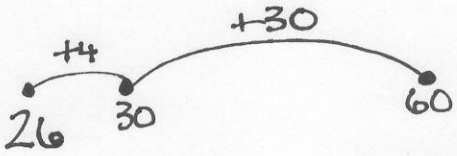
20
2
+ 3
25

1

1

2. Grandmother Mimi is turning 60 years old. Jack found 26 candles for the birthday cake. How many more candles will he need?

   Show your answer and all of your work. 34 ✓ candles



1

1

4

Addendum 1

Sample Number Talk A- Using Properties of Operations

Teacher introduces a problem on the board, such as:

26 + 19 =

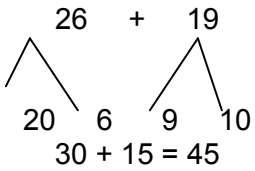
Students are given individual think time to find a solution. Students may show "quiet fingers" on their chest to show when they have a strategy. Students are encouraged to continue thinking of new strategies, indicated by more "quiet fingers", while other students process and find their first solution.

After the think time, have students share their strategies with a neighbor or partner to maximize the amount of discussion in the class and to hold all students accountable for thinking of a strategy.

The teacher may ask for students to give solutions that they have found and then ask students to share their strategies.

For example, a student might add the tens together and then the ones and then combine the answers.

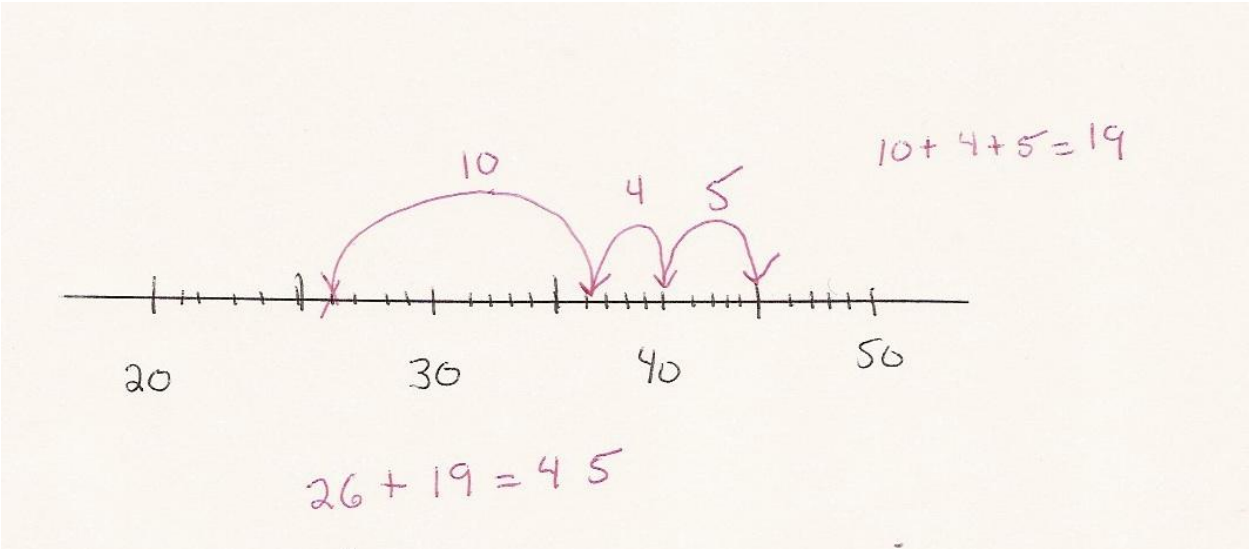
"*I added 20 and 10 to get 30. Then I added 6 + 9 to get 15."* The teacher might ask clarifying questions, "How did you think about adding the 6 and 9? How does this help you get a solution? Where does the 20 come from?" This helps students to clarify their explanations and solidify their understanding of place value. *"The 2 in twenty six represents 20. Next I added 30 and 15 to get 45."*



A second student might say*, "I know that 19 is 1 away from 20 so I take one from the 26 to make 15 and add the one to 19 to make 20. 25 + 20 = 45*

A third student might use a counting up strategy*, "I started at 26 and added ten to get 36. Then I added four from the 9 to get 40. When I broke the 9 apart to get 4, I had 5 left, so 40 + 5 is 45."*

This strategy might be good to illustrate using a number line.



The teacher might then follow this by asking students to use this strategy on their mini-white boards to show how the student would solve:

43 + 58

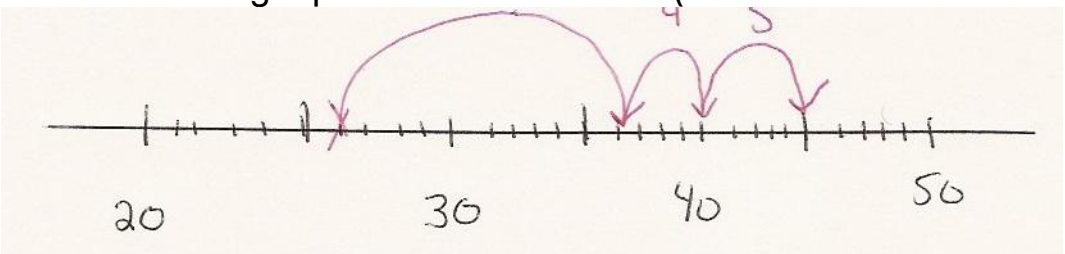
Using the mini-white boards allows the teacher to see how students are thinking about the number line and the strategy. Are students picking logical numbers for their jumps? How are they decomposing the numbers? This allows the teacher to control the conversation by picking which students to choose to share their work. If available document readers make it very easy to share student work.

If most students seem to be able to make sense of the strategy, the teacher might ask, "*Can we use this strategy to do subtraction?"*

62 – 17

Again students can use the mini-white boards to draw their number lines.

If the number line strategy doesn't come up naturally in the class discussion. The teacher might put this on the board. *(without numbers showing).*



I saw a student in another class doing this to solve our problem. What do you think the student is thinking? Allow some private think time. Then have students share ideas with a partner. Finally have a whole class discussion about the strategy.

Sharing Strategies and Solution Paths Around Number, Operations and Place Value

The purpose of this formative assessment lesson is to have students practice mathematical reasoning around number, operations and place value that will further their understanding of these concepts. The focus is on student ideas. In order for children to learn, understand, and remember, they need time interacting with ideas, thinking about where these ideas fit in relation to what they already know, uncovering the logic, and then applying it to their thinking around these ideas. Explaining their reasoning helps to solidify and extend their understanding.

As such, correct and incorrect ideas should be accepted during the discussion of strategies and solution paths. Students and teachers need to respectfully accept correct and incorrect responses during mathematical discussions. It is important to establish a classroom atmosphere where students feel safe to share their ideas. Students should be guided to understand that learning can occur when a response is incorrect. Questions are posed by the teacher and by students that will move all towards the underlying mathematics that determines the correctness of answers.

When learners are first introduced to activities such as “Number Talks”, it is important to explain their purpose and to describe how they should work during these discussion times. The emphasis is on understanding. We need to think and talk about problems to solidify our learning. In order to benefit from these discussions we need to remember these things:

- We share ideas and listen to others.
- We ask “why does this work” until we understand
- We respect one another's opinions
- We know that we learn from mistakes as well as from correct answers.
- Our goal is for the students, the teacher and the mathematics to agree in the end!

The table that follows includes some common issues confronted when sharing student thinking around number, operations and place value. The suggested questions and prompts are a beginning list that will grow as you work with students to understand and make sense of the mathematics. Teachers who are new to the practice of “Number Talks” may want to consider focusing on one or two types of questions and one or two recording strategies at a time until these become integrated into their teaching styles. You will find examples of questions and recording styles in the sample problem section of this addendum. Using the number line to record strategies will be a beneficial tool for students to use throughout their mathematics courses.

Sample problem
75 - 39

Common Issues	Suggested questions and prompts
Difficulty getting started on a solution path. Ex: blank faces in the classroom audience	• What is this problem asking us to find out? • What do you know? • Without giving the answer, can someone explain how they are thinking about solving this problem? MP1 Make sense of problems and persevere in solving them.
Limited number of participants or responses. Ex: only one solution path presented	• After quiet think time suggest: "turn to one other person and share your answer and how you thought about it." Then prompt: "let's list your answers." Then, "let's share our solution strategies." • Who thought of it in a similar way? • Who thought of it in a different way? • Does anyone have the same answer but a different way to explain it? MP3 Construct viable arguments and critique the reasoning of others.
Few or no questions asked around solution strategies	• Does anyone agree or disagree with ___'s work? • Who can explain ___'s thinking? MP3 Construct viable arguments and critique the reasoning of others.
Connections not made and/or reasoning not deep	• Would someone like to add to this? • Will that work with a similar problem? Let's try 84 -38. • How is this problem similar to yesterday's?
Error in solution path Ex: ____ presents the solution as "I took away forty from the seventy-five and that made *forty-five* and I added one to get 46."	• Do you agree or disagree with this? Why? • Who can repeat what ___ said? • Can we make a model or a drawing for that? What might that look like? • Could both/all of these answers be correct? MP7 Look for and make use of structure.

Procedural explanation unconnected to place value. Ex: Student says, "I lined up the problem with the 39 under the 75. I crossed out the 7 and made it a 6 and added a one to the 5. Then the 5 becomes 15.Then, I subtracted and got 36." Ex: Student says, "15 – 9 = 6 and 6 – 3 = 3, so the answer is 36."	• Can you explain more about your procedure to us? Why should I cross out the seven and write 6? Remember we are talking about 2-digit numbers here. What does the "75" look like in base ten blocks? • You said I should cross out the seven in seventy-five and write six and then add the one to the five becomes fifteen. Where did the "1" come from? Can you either show me why this makes sense? • Who else can explain why this works? • Who can explain why the 5 becomes 15 ones? • Record exactly what the student says: 15 – 9 = 6 and 6 – 3 = 3. The teacher then asks how this becomes 36. Often this will guide the student to realize the importance of the 6 and 3 representing tens (60 and 30). MP5 Use appropriate tools strategically.
Procedural explanation unconnected to the definition of subtraction as a missing addend approach. Ex: Student says, "I started with 39 and added 1 to make 40. I knew that 40 + 35 is equal to 75. So, I added the 1 and the 35 and got 36."	• You used addition to solve this subtraction problem. Why is that possible? • Will that always work? Why? • How could we use this strategy to solve a similar problem – say, 83 – 67. MP2 Reason abstractly and quantitatively.
No connection made to previous problems. Ex: previous problems were 43 – 10, 43 – 9, 58 – 20, 58 – 19.	• What ideas have you learned before that might be useful in solving 75 – 39? • Have we ever solved a problem like 75 – 39 before? • How does today's problem relate to yesterday's problems? (58-30, 58-19) MP7 Look for and make use of structure.
No connections made to one another's strategies. Ex: Melissa's strategy – 75 – 40 = 35, 35 + 1 = 36, John's strategy – 75 – 35 = 40, 40 – 4 = 36 *When recording these or any strategies, be careful not to use "run on" equations e.g. 75 - 40 = 35 + 1 = 36*	• How is Melissa's strategy similar or different from the way John solved it? MP1 Make sense of problems and persevere in solving them. MP6 Attend to precision.

Correct Solution	Can you solve this same problem using a different method? How are the two methods different? Is this new method more efficient than your first way? Why? MP3 Construct viable arguments and critique the reasoning of others.

Student Responses to Discuss

Describe the solution paths and strategies used by each student one at a time.

You might, for example:

Ask students to make sense of the student work.

Discuss the computational procedure and/or the notations used.

Help the students to focus on the properties of operations or the properties of numbers that help to make these strategies efficient.

Discuss what the student might do to complete his or her solution or to make it more efficient.

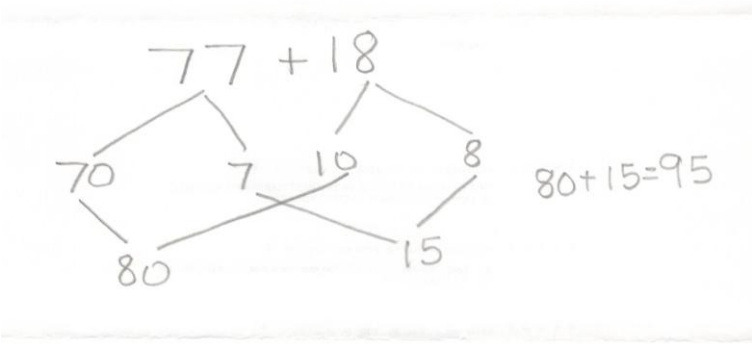
It is better to spend quality time on each solution path you share than to quickly share all the work included here

Sample Responses to Discuss

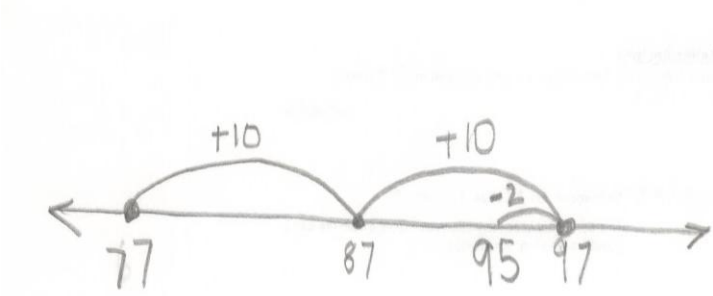
Day 3 Number Talk

Here is the work of three students around 77 + 18**.**
Share one at a time. For each piece of work:
Ask students to make sense of the student work. *For example, "where is the 18 in these examples?*
Discuss the computational procedure and/or the notations used. *For example, "why did Rose break 18 into 10 + 3 + 5?"*
Help the students to focus on the properties of operations or the properties of numbers that help to make these strategies efficient. *For example, "when you are adding numbers does it matter which one you start with?" (commutative property over addition)*
Explain what the student needs to do to complete his or her solution. *For example, "what equation would represent Amy's solution to the problem?"*

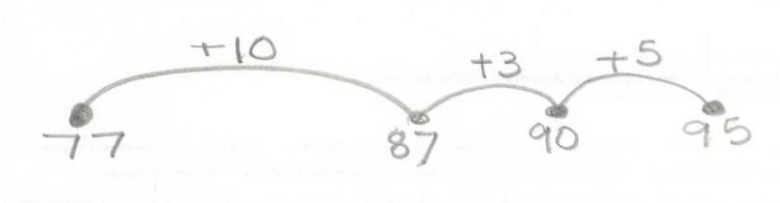
Micheal



Amy



Rosy



Day 3 Number Talk

Here is the work of three students around 64 – 25.

Share one at a time. For each piece of work:

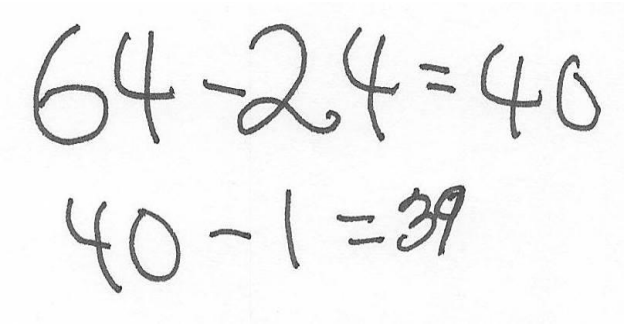
Ask students to make sense of the student work. *For example, "where did the 24 come from in Jon's work?"*

Discuss the computational procedure and/or the notations used. *For example, "how did Lindy get an answer of 41?" (Rather than spot the error for students)*

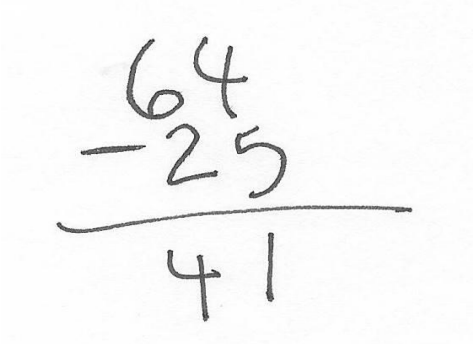
Help the students to focus on the properties of operations or the properties of numbers that help to make these strategies efficient. *For example, "why did Huen subtract 20 then 4 and then 1?"*

Explain what the student needs to do to complete his or her solution. *For example, "what does Lindy need to understand about subtraction?" (the commutative property does not hold over subtraction).*

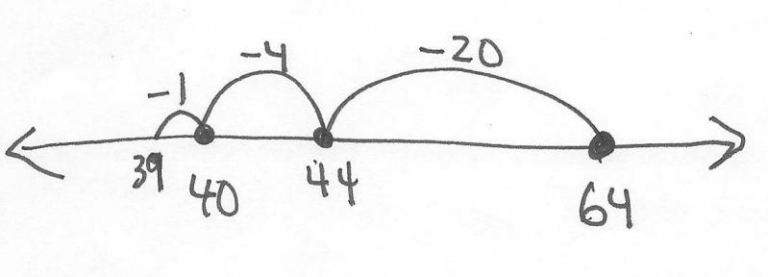
Jon



Lindy



Huen



Day 7 Number Talk

Here is the work of three students around 74 + 18.

Share one at a time. For each piece of work:

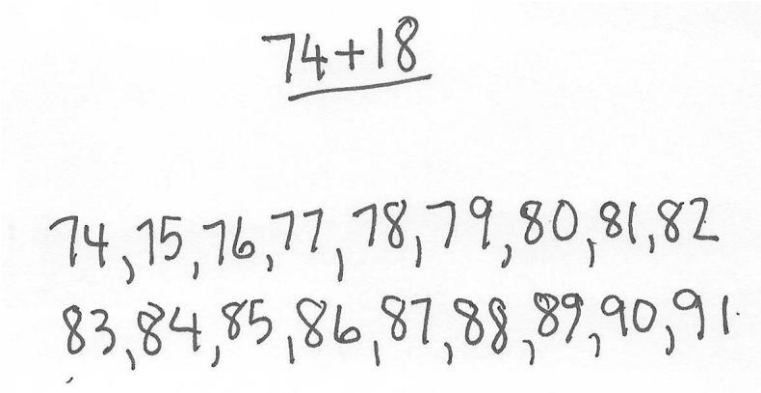
Ask students to make sense of the student work. *For example, "why did Jorge add 20 to begin?"*

Discuss the computational procedure and/or the notations used. *For example, "explain how Jenny broke the numbers apart. Why did she select these numbers?"*

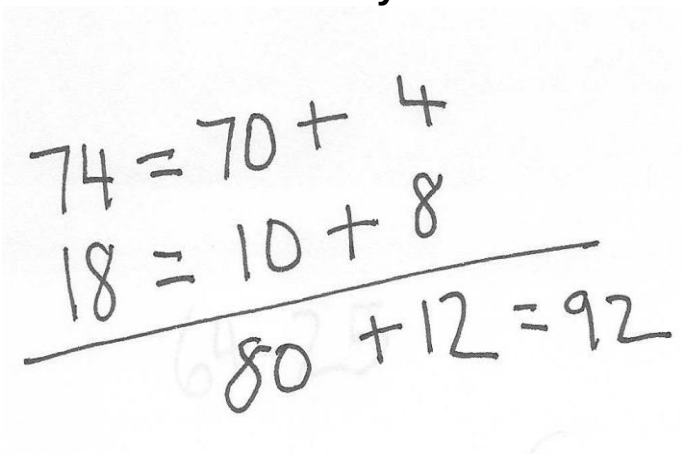
Help the students to focus on the properties of operations or the properties of numbers that help to make these strategies efficient. *For example, "would Dan get the correct solution if he had started at 18 and counted on 74? Would this be an efficient strategy?"*

Explain what the student needs to do to complete his or her solution. *For example, "would Dan's strategy be efficient for 74 + 123?"*

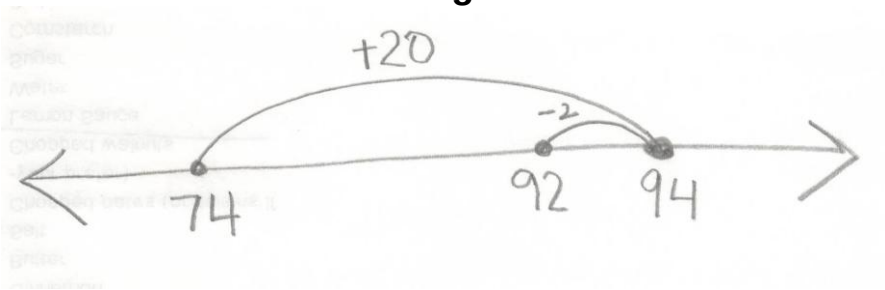
Dan



Jenny



Jorge



Day 7 Number Talk

Here is the work of three students around $1.26 - .49.

Share one at a time. For each piece of work:

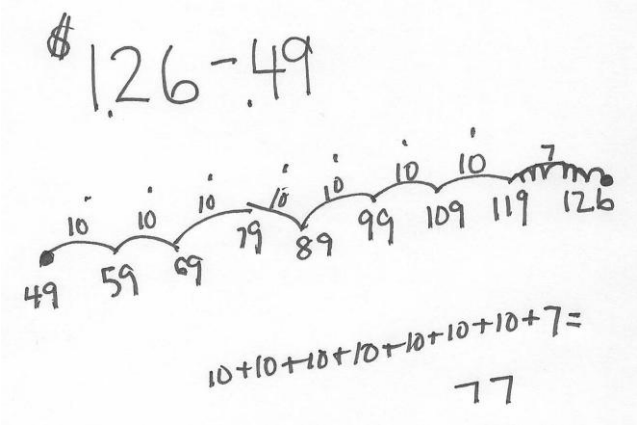
Ask students to make sense of the student work. *For example, "why did Dennis add $.01 to $.76?"*

Discuss the computational procedure and/or the notations used. *For example, "in Maya's work, where is the $.49?"*

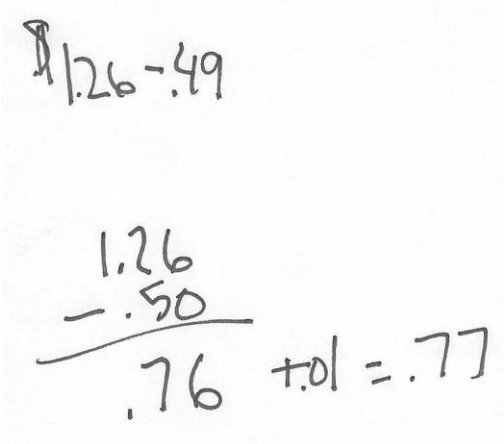
Help the students to focus on the properties of operations or the properties of numbers that help to make these strategies efficient. *For example, "why did Karen add on to solve a subtraction problem? Does this always work?"*

Explain what the student needs to do to complete his or her solution. *For example, "how could Maya solve the problem in fewer steps?"*

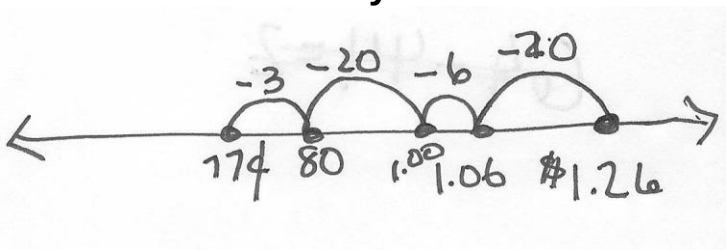
Karen

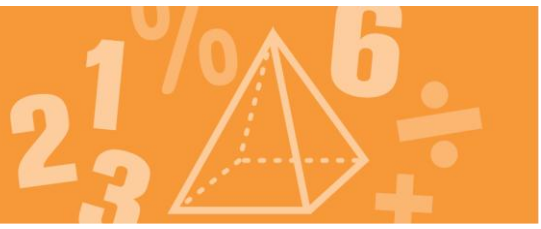


Dennis



Maya





GRADE 2 MATH: CAROL'S NUMBERS
INVESTIGATION: GOT YOUR NUMBER

Got Your Number

Primary Version Level A

Materials: The deck of cards (1-9) for each pair.

Discussion on the rug: (Teacher starts a discussion about the number 10) "Why is the number 10 an important number?" (The teacher invites ideas from the class). "We are going to play a fun game today. It is called Make Ten." (Teacher demonstrates how to play the game with two players). "We play in pair with a deck of number cards. Each player picks 7 cards from the deck. Look at your cards and find two cards that add-up (count up) to 10. For example: 8 and 2 makes 10. If you can make ten, then put those two cards together in a 10-pairs pile and then pick two more cards. If you can't make ten, then say 'pass' and pick a new card. Switch turns with each other until all the cards on the deck are picked and all the pairs that make ten are found. We will examine our 10-pairs pile all together."

In small groups: (Each group plays the game until the deck is used and all pairs of ten made. Have them look over and count up how many 10-pairs they made. After the games, the teacher asks the class to list the 10-pairs that the students made.) (Teacher asks the following questions) "Suppose you are playing this game with a new friend. Explain to your friend how you play the game and which cards you need to put together to make 10-pairs" (At the end of the investigation have students either discuss or dictate a response to these summary questions above)

Got Your Number

Level A:

Carol and Melissa are playing a game. They have a deck of 36 cards with just the numbers 1 through 9. After they mix up the cards, they put them into a pile. Below are the rules:

- Deal five number cards to each player.
- Use any three of your cards.
- Pick three numbers that add to a number near 20.
- Write a number sentence with your three cards and the total that is near 20.
- Find your score. Your score is the difference between your total and 20.
- For example you picked the cards 6, 9, 7. 6 + 9 + 7 = 22. So your total is 22. To find your score, subtract 20 from 22. 22 – 20 = 2.
- Shuffle the cards and replay another round.

Play the game seven times. At the end of the game, sum all seven scores for each player. The player with the lowest total is the winner.

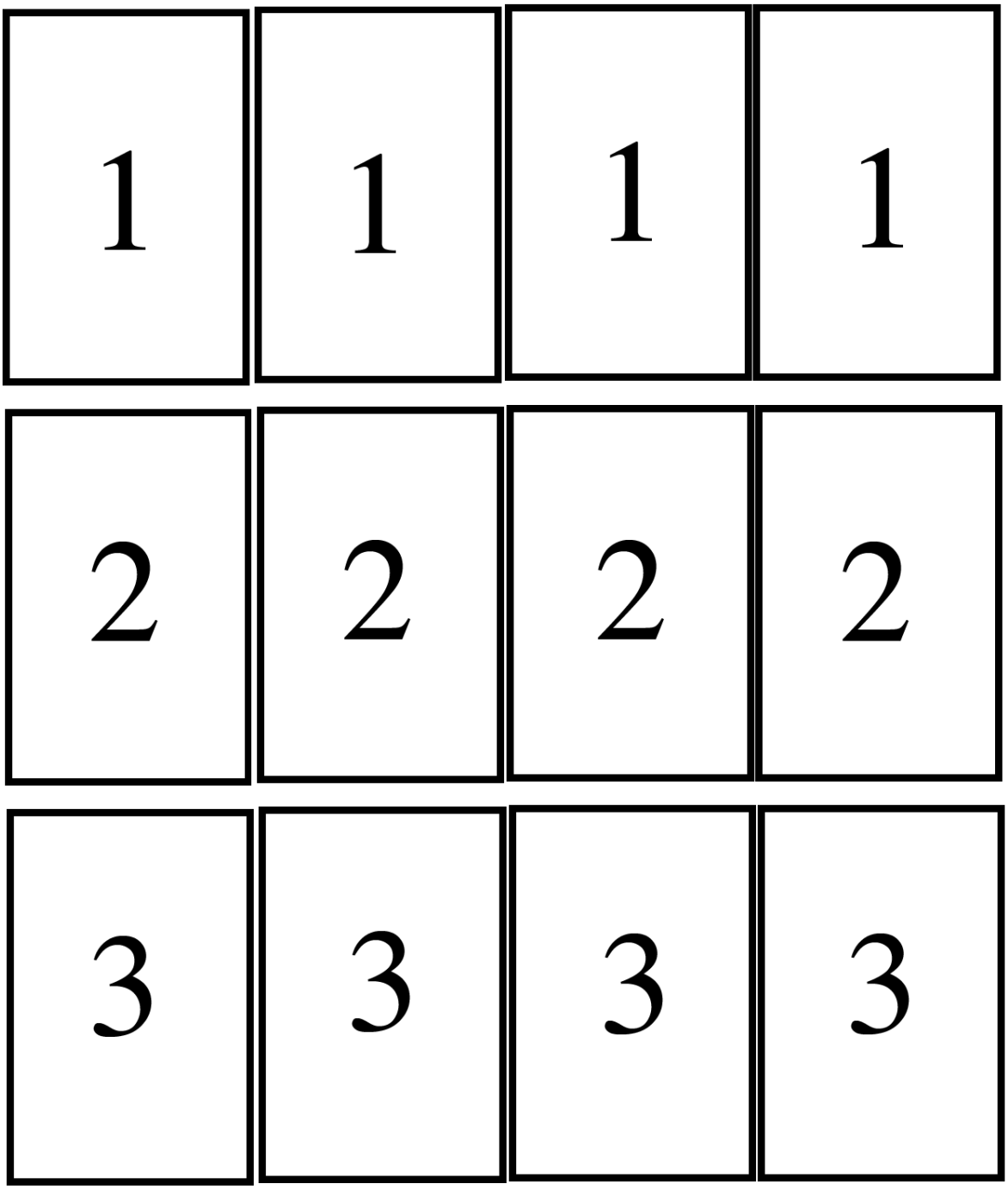
Got Your Number

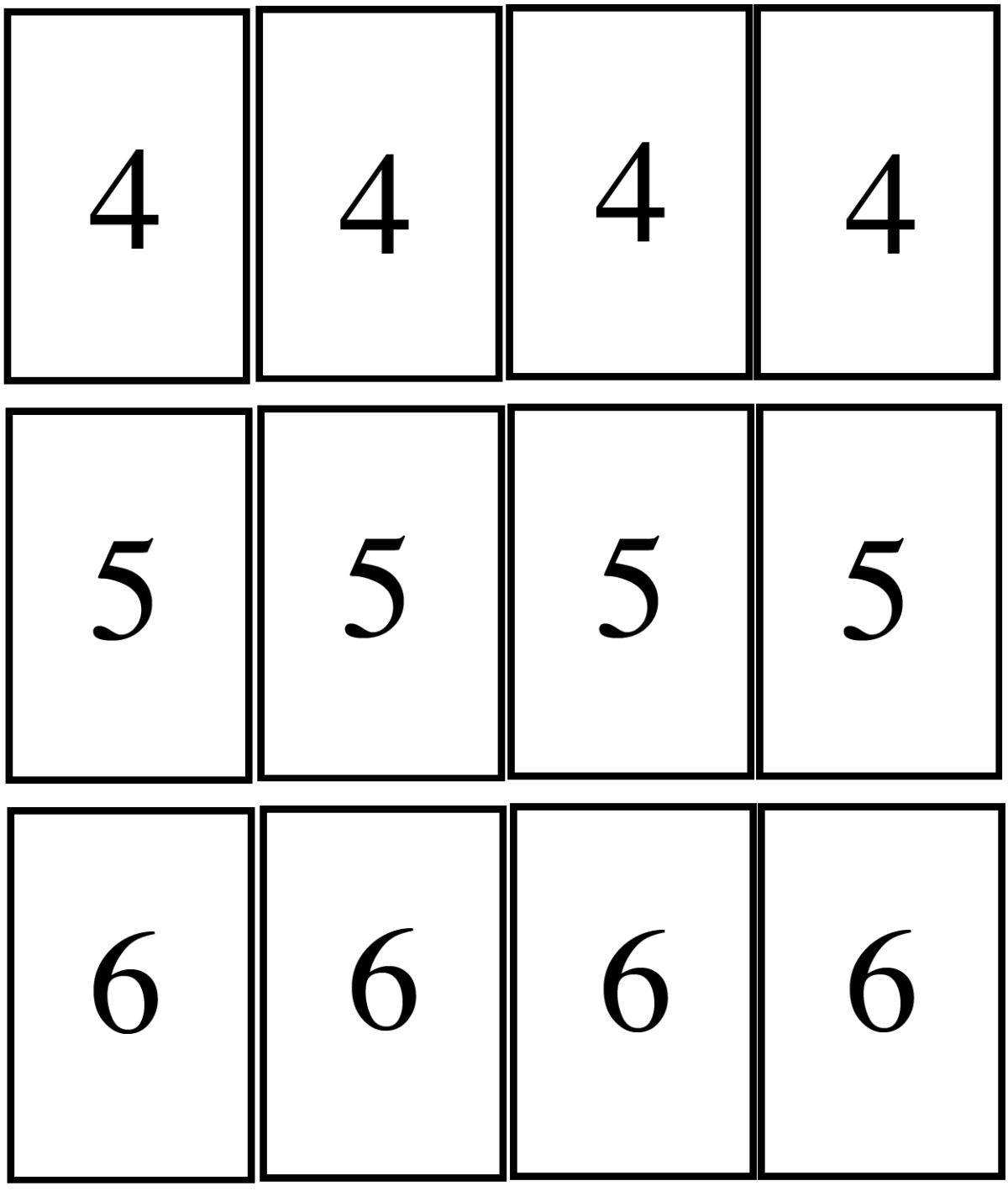
Level B:

Sandy and Sally are playing a game. They have a deck of 36 cards with just the numbers 1 through 9. After they mix up the cards, they put them into a pile. Below are the rules:

- Deal six digit cards to each player.
- Select any four of your cards to make 2 numbers. Each number would be a two-digit number.
- Arrange the numbers and then add them to get a sum as near 100 as possible.
- Once you have selected the two numbers and found the sum, write out the equations.
- Determine your score by finding the difference (distance) between your number and 100.
- Shuffle the cards and replay another round.
- Play the game seven times. At the end of the game, sum all seven scores for each player. The player with the lowest total is the winner.

Explain the strategy you used to try and win the games. Explain why you chose that strategy.


1 1 1 1
2 2 2 2
3 3 3 3


4 4 4 4
5 5 5 5
6 6 6 6