HIGH SCHOOL ALGEBRA: THE CYCLE SHOP

UNIT OVERVIEW

This packet contains a curriculum-embedded CCLS aligned task and instructional supports. The task is embedded in a 4-5 week unit on Reasoning with Equations and Inequalities.

TASK DETAILS

Task Name: The Cycle Shop

Grade: High School Algebra (Algebra 1)

Subject: Mathematics

Depth of Knowledge: 3

Task Description: The tasks in the unit access the full range of *Depth of Knowledge* including Recalling and Recognizing, Using Procedures, Explaining and Concluding and Making Connections, Extensions and Justifying.

Standards Assessed:
A-CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
A-CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.
A-REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify the solution method.
A-REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
A-REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Standards for Mathematical Practice:
MP.1 Make sense of problems and persevere in solving them.
MP.3 Construct viable arguments and critique the reasoning of others.
MP.4 Model with mathematics.
MP.7 Look for and make use of structure.

TABLE OF CONTENTS

The task and instructional supports in the following pages are designed to help educators understand and implement tasks that are embedded in Common Core-aligned curricula. While the focus for the 2011-2012 Instructional Expectations is on engaging students in Common Core-aligned culminating tasks, it is imperative that the tasks are embedded in units of study that are also aligned to the new standards. Rather than asking teachers to introduce a task into the semester without context, this work is intended to encourage analysis of student and teacher work to understand what alignment looks like. We have learned through the 2010-2011 Common Core pilots that beginning with rigorous assessments drives significant shifts in curriculum and pedagogy. Universal Design for Learning (UDL) support is included to ensure multiple entry points for all learners, including students with disabilities and English language learners.

Department of Education
Dennis M. Walcott, Chancellor

HIGH SCHOOL ALGEBRA: THE CYCLE SHOP

PERFORMANCE TASK
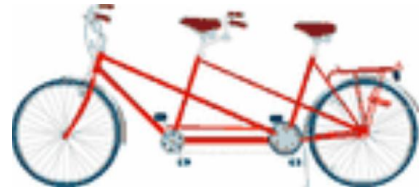
Performance Task
The Cycle Shop

You work for a small business that sells bicycles, tricycles, and tandem bikes. Bicycles have one seat, one handlebar and two wheels. Tricycles have one handlebar, one seat and three wheels. Tandem bikes have two handlebars, two seats and two wheels.

Bicycle   **Tricycle**   **Tandem Bike**

1. On Monday you counted 48 tricycle wheels.

    How many tricycles were in the shop? _____.

    Write an algebraic equation that shows the relationship between the number of wheels (**w**) and the number of tricycles (**t**).

2. On Wednesday there were **no** tandem bikes for sale. There were only bicycles and tricycles in the shop. There are a total of 24 seats and 61 wheels in the shop.

    How many are bicycles and how many are tricycles? _____

    Explain how you figured it out.

Performance Task
The Cycle Shop

3. This month new models have been brought into the shop.   Now, there are a different number of bicycles, tandem bicycles, and tricycles in the shop. There are a total of 135 seats, 118 handle bars, and 269 wheels.

How many bicycles, tandem bicycles and tricycles are there in the Cycle Shop?

HIGH SCHOOL ALGEBRA: THE CYCLE SHOP
RUBRIC

The rubric section contains a scoring guide and performance level descriptions for the Cycle Shop task.

Scoring Guide: The scoring guide is designed specifically to each small performance task. The points highlight each specific piece of student thinking and explanation required of the task and help teachers see common misconceptions (which errors or incorrect explanations) keep happening across several papers. The scoring guide can then be used to refer back to the performance level descriptions.

Performance Level Descriptions: Performance level descriptions help teachers think about the overall qualities of work for each task by providing information about the expected level of performance for students. Performance level descriptions provide score ranges for each level, which are assessed using the scoring guide.

High School Algebra: The Cycle Shop
Scoring Guide

Cycle Shop		Rubric
The core elements of performance required by this task are: • modelling a situation using systems of equations • determining unknowns using multiple constraints • solving equations Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answers **16** tricycles	1	
Writes an equation such as: $t = w/3$	1	2
2. Gives correct answer: **11** bicycles and **13** tricycles	2x1	
Show correct work such as: t: number of tricycles and b: number of bicycles $24 = t + b$ and $61 = 3t + 2b$ so $61 = 3(24 - b) + 2b$ and $b = 11$ and $t = 13$	1	3
3. Gives correct answer: **67** bicycles, **32** tricycle, **45** tandem bikes	1	
Gives an explanation such as: t: number of tricycles b: number of bicycles n: number tandem bikes $144 = t + b + n$ $378 = 2t + 2b + 4n$ $320 = 3t + 2b + 2n$	1	
$(320 = 3t + 2b + 2n) - (288 = 2t + 2b + 2n)$ results in $t = 32$	1	
Substituting for t and using equations 1 & 2 $(112 = b + n) - (314 = 2b + 4n)$ results in n = 45	1	
Substituting for t and n in first equation: $144 = 32 + b + 45$ results in b = 67		3
Total Points		**8**

High School Algebra: The Cycle Shop
Rubric

Performance Level Descriptions and Cut Scores
Performance is reported at four levels: 1 through 4, with 4 as the highest.

Level 1: Demonstrates Minimal Success (0 -2 points)
The student's response shows few of the elements of performance that the tasks demand as defined by the CCLS. The work shows a minimal attempt on the problem and struggles to make a coherent attack on the problem. Communication is limited and shows minimal reasoning. The student responses rarely uses definitions in their explanations. The students struggle to recognize patterns or the structure of the problem situation.

Level 2: Performance Below Standard (3 – 4 points)
The student's response shows some of the elements of performance that the tasks demand and some signs of a coherent attack on the core of some of the problems as defined by the CCSS. However, the shortcomings are substantial and the evidence suggests that the student would not be able to produce high-quality solutions without significant further instruction. The student might ignore or fail to address some of the constraints of the problem. The student may occasionally make sense of quantities in relationships in the problem, but their use of quantity is limited or not fully developed. The student response may not state assumptions, definitions, and previously established results. While the student makes an attack on the problem it is incomplete. The student may recognize some patterns or structures, but has trouble generalizing or using them to solve the problem.

Level 3: Performance at Standard (5 – 6 points)
For most of the task, the student's response shows the main elements of performance that the tasks demand as defined by the CCSS and is organized as a coherent attack on the core of the problem. There are errors or omissions, some of which may be important, but of a kind that the student could well fix, with more time for checking and revision and some limited help. The student explains the problem and identifies constraints. Students make sense of quantities and their relationships in the problem situations. They often use abstractions to represent a problem symbolically or with other mathematical representations. The student response may use assumptions, definitions, and previously established results in constructing arguments. They may make conjectures and build a logical progression of statements to explore the truth of their conjectures. The student might discern patterns or structures and make connections between representations.

Level 4: Achieves Standards at a High Level (7 – 8 points)
The student's response meets the demands of nearly all of the tasks as defined by the CCSS, with few errors. With some more time for checking and revision, excellent solutions would seem likely. The student response shows understanding and use of stated assumptions, definitions and previously established results in construction arguments. The student is able to make conjectures and build a logical progression of statements to explore the truth of their conjecture. The student response routinely interprets their mathematical results in the context of the situation and reflects on whether the results make sense. The communication is precise, using definitions clearly. Students look closely to discern a pattern or structure. The body of work looks at the overall situation of the problem and process, while attending to the details

HIGH SCHOOL ALGEBRA: THE CYCLE SHOP ANNOTATED STUDENT WORK

This section contains annotated student work at a range of score points. The student work shows examples of student understandings and misunderstandings of the task. This section contains annotated student work at a range of score points and implications for instruction for each performance level (excluding the expert level). The student work and annotations are intended to support teachers, showing examples of student understandings and misunderstandings of the task. The annotated student work and implications for instruction can be used to understand how to move students to the next performance level.

Level 4: Achieves Standards at a High Level (Score Range 7 – 8)

The student's response nearly meets the demands of the entire task, with few errors. With more time for checking and revisions, excellent solutions would seem likely. The student response shows understanding and use of stated assumptions, definitions and previously established results in constructing arguments. The student is able to make conjectures and build a logical progression of statements to explore the truth of their conjecture. The student response routinely interprets their mathematical results in the context of the situation and reflects on whether the results make sense. The communication is precise, using definitions clearly. The students look closely to discern a pattern or structure. The body of work looks at the overall situation of the problem and process, while attending to the details.

STUDENT A – Level 4

Student A achieved standards at a high level (4) with a score of 8. The student demonstrates understanding of all parts of the task and provides written work that illustrates and justifies solutions.

You work for a small business that sells bicycles, tricycles, and tandem bikes. Bicycles have one seat, two pedals and two wheels. Tricycles have one seat, two pedals, and three wheels. Tandem bikes have two seats, four pedals and two wheels.

Bicycle   **Tricycle**   **Tandem Bike**

The student was able to find the answer to the number of tricycles and was able to create an equation to represent the situation. A-REI.3 & MP4

1. On Monday you counted 48 tricycle wheels.

How many tricycles were in the shop? _____16_____.

Write an algebraic equation that shows the relationship between the number of wheels (w) and the number of tricycles (t).

$$w = 3t$$

In part two, the student models the situation using two equations. The student labels the variables, show a solution path for solving the system and then checks the solutions. A-REI.1, A-REI.6 MP1, MP3 & MP4

2. On Wednesday there were **no** tandem bikes in the shop. There were only bicycles and tricycles. There are a total of 24 seats and 61 wheels in the shop. How many bicycles and how many triangles are in the shop?

____11 bikes and 13 tricycles____

Show how you figured it out.

T : Tricycles

B : Bicycles

2B + 3T = 61
   B + T = 24

chick :
       11 + 13 = 24
       22 + 39 = 61

2B + 3T = 61
2B + 2T = 48
_____
       T = 13
       B = 11

STUDENT A – Level 4 continued

3. A month later, there are a different number of bicycles, tricycles tandem bikes in the shop. There are a total of 144 front steering handlebars, 378 pedals, and 320 wheels.

How many bicycles, tricycles and tandem bikes are in the shop?

67 Bicycles, 32 tricycles, 45 tandem bicycles

Explain your solution.

B = Bicycles

T = Tricycles

N = Tandem Bikes

EQ1: $B + T + N = 144$

EQ2: $2B + 2T + 4N = 378$

EQ3: $2B + 3T + 2N = 320$

Compare EQ1 + EQ3

$2B + 3T + 2N = 320$

$2B + 2T + 2N = 288$

$T = 32$

Compare EQ1 + EQ2

$2B + 64 + 4N = 378$

$2B + 64 + 2N = 288$

$2N = 90$

$N = 45$

so $B + 32 + 45 = 144$

$B = 67$

The student presents a clear solution to part 3. The variables and values are labeled. The student modeled the situation using a system of three equations. The work showed how the equations were used to find solutions. The values were then checked to verify the solution. The student demonstrated skills in the following standards: A-REI.1, A-REI.6, MP1, MP3, MP4 & MP7.

High School Algebra: The Cycle Shop
Annotated Student Work
Level 3: Performance at Standard (Score Range 5 - 6)

For most of the task, the student's response shows the main elements of performance that the tasks demand and is organized as a coherent attack on the core of the problem. There are errors or omissions, some of which may be important, but of a kind that the student could well fix, with more time for checking, revision and some limited help. The student explains the problem and identifies constraints. The student makes sense of quantities and their relationships in the problem situations. S/he often use abstractions to represent a problem symbolically or with other mathematical representations. The students response may use assumptions, definitions, and previously established results in constructing arguments. They may make conjectures and build a logical progression of statements to explore the truth of their conjectures. The student might discern patterns or structures and make connections between representations.

STUDENT B – Level 3
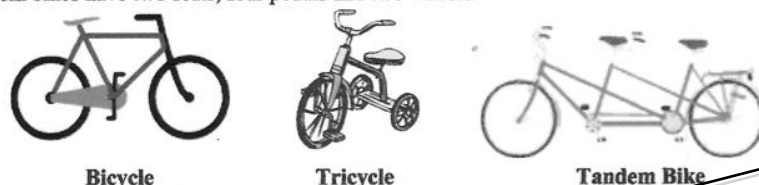
Student B met standard (Level 3) with a score of 5. This student showed understanding of using both constraints to find two unknowns, but the same strategy was more difficult for three constraints and three unknowns.

You work for a small business that sells bicycles, tricycles, and tandem bikes. Bicycles have one seat, two pedals and two wheels. Tricycles have one seat, two pedals, and three wheels. Tandem bikes have two seats, four pedals and two wheels.

Bicycle   Tricycle   Tandem Bike

1. On Monday you counted 48 tricycle wheels.

   How many tricycles were in the shop? __16 Tricycle__

   Write an algebraic equation that shows the relationship between the number of wheels (w) and the number of tricycles (t).

$$3 \times t = W$$

The student was able to find the answer to the number of tricycles and was able to create an equation to represent the situation. A-REI.3 & MP4

2. On Wednesday there were **no** tandem bikes in the shop. There were only bicycles and tricycles. There are a total of 24 seats and 61 wheels in the shop. How many bicycles and how many triangles are in the shop?

   __11 bikes   16 trikes__

   Show how you figured it out.

Bike	trike	total seats	total wheels
	1	2	5
2	2	4	10
5	5	10	25
12	12	24	24 + 36 = 60
13	11	24	26 + 33 = 59
11	13	24	22 + 39 = 61

In part two, the student arrived at the correct answers by using a table and reasoning with guess and check strategy. A-REI.6 MP4, MP7 and MP1.

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STUDENT B – Level 3 continued

3. A month later, there are a different number of bicycles, tricycles tandem bikes in the shop. There are a total of 144 front steering handlebars, 378 pedals, and 320 wheels.

How many bicycles, tricycles and tandem bikes are in the shop?

48, 39, 55

This student tried to use a similar strategy that was used in part two. It appears the table and data with three variables became more challenging to negotiate. The students did not attempt to use equations to represent situations. The standard A-REI.6 was not met.

Explain your solution.

Bike	Trike	TB	HB	Pedals	Wheels
1	1	1	3	2+2+4 = 8	2+3+2 = 7
10	10	10	30	20+20+40=80	20+30+20=70
40	40	40	120	80+80+160=320	80+120+80= 280
50	50	50	150	100+100+200=400	100+150+100 = 350
48	40	60	148	96+80+240 =	96+120+120 =

Level 3 Implications for Instruction

Students who met standard on the task can still improve their performance. Students must learn to provide complete and clear work. This should involve labeling variables and values to record and keep track of their own thinking. It is equally important that students also check their solutions against all constraints to ensure the solutions are sound.

Representing a system either through a set of equations, a table, a graph and/or diagram is an important and necessary first step. Reasoning a solution using the representation is the second step. If students use a table they must be systematic and complete in determining a solution. If students use equations they must use an equal number of constraints as unknowns in order to ensure a correct solution to the system. They can employ a process like substitution or elimination to isolate variables. They must understand why and how these processes work in order to reconstruct and persevere when faced with a complex set of equations. Students who use a graph to determine solutions for a system must check the coordinates of the intersections to ensure the estimate meets the given conditions. Students should have experiences with all these strategies and understand how the different methods of solving systems relate and connect to one another.

Level 2: Performance below Standard (Score Range 3 - 4)

The student's response shows some of the elements of performance that the tasks demand and some signs of a coherent attack on the core of some of the problems. However, the shortcomings are substantial, and the evidence suggests that the student would not be able to produce high-quality solutions without significant further instruction. The student might ignore or fail to address some of the constraints. The student may occasionally make sense of quantities in relationships to the problem, but their use of quantity is limited or not fully developed. The student response may not state assumptions, definitions, and previously established results. While the student makes an attack on the problem it is incomplete. The student may recognize some patterns or structures, but has trouble generalizing or using them to solve the problem.

STUDENT C – Level 2

Performance Task
The Cycle Shop

You work for a small business that sells bicycles, tricycles, and tandem bikes. Bicycles have one seat, two pedals and two wheels. Tricycles have one seat, two pedals, and three wheels. Tandem bikes have two seats, four pedals and two wheels.

| Bicycle | Tricycle | Tandem Bike |

1. On Monday you counted 48 tricycle wheels.

   How many tricycles were in the shop? __16 trikes__ .

   Write an algebraic equation that shows the relationship between the number of wheels (w) and the number of tricycles (t).

   $$w \div 3 = t,$$

Student C
performed at Level 2 with a score of 4 . The student was able to model the situations with equations, but was unable to use the equations to find solutions.

In part 1, the student correctly found the number of tricycles and formed an equation to represent the relationship in two variables. A-REI.3, A-REI.6 , MP4

2. On Wednesday there were **no** tandem bikes in the shop. There were only bicycles and tricycles. There are a total of 24 seats and 61 wheels in the shop. How many bicycles and how many triangles are in the shop?

   __7 trikes , 21 bikes__

   Show how you figured it out.

The student modeled the situation with two correct equations and two unknowns. MP4

The student was unsuccessful solving the system. The student failed to use the distributive property accurately.

$$2b + 3t = 61 \qquad 1b + 1t = 24 \quad -1t$$
$$1b = 24 + ^-1b$$
$$2(24 + ^-1b) + 3t = 61$$
$$48 + 2t = 61$$
$$-48 \qquad -48$$
$$2t = 13 \qquad t = 13 \div 2$$
$$t = 6\tfrac{1}{2} \cap 7$$

STUDENT C – Level 2 continued

3. A month later, there are a different number of bicycles, tricycles tandem bikes in the shop. There are a total of 144 front steering handlebars, 378 pedals, and 320 wheels.

How many bicycles, tricycles and tandem bikes are in the shop?

48, 48, 48

Explain your solution.

The student was able to represent the situation using three equations and three variables. A-CED.3 MP4

The student was unable to use the equations to determine a solution. A-REI.6 MP7, MP1

$$t + b + d = 144 \qquad 2t + 2b + 4d = 378$$

$$3t + 2b + 2d = 320$$

$$\begin{array}{r} 106 \\ 3\overline{)320} \end{array}$$

$$\begin{array}{r} 48 \\ 3\overline{)144} \end{array}$$

$$\begin{array}{r} 47 \\ 8\overline{)378} \\ 32 \\ \hline 58 \end{array}$$

High School Algebra: The Cycle Shop
Annotated Student Work

Level 2 Implications for Instruction

Part two requires a more complex chain of reasoning than part 1. Students need to sort and reason about two unknowns and two constraints. When students first use variables, they are not precise in what the variable stands for in the situation. If a student use **b** in an equation, does it mean:

- the number of bikes?
- the number of seats on a bike?
- the number of wheels on a bike?
- one bike?

Students are often not sure of what the variable they chose actually represents. Students need to be explicit about their variable assignments. Once a variable is clearly defined by the students, then they must consider coefficients, other variables and constants in making meaning of the situation.

Students need experiences in creating mathematical models for contextual situations. The modeling experience may involve writing number sentences, making numerical tables, creating charts or diagrams, drawing or creating graphs and/or drawing pictures to characterize a situation. Students will benefit from connecting and linking the representations to make sense how the representations model the situation. Students learn to model situations by making sense of the problem and then tinkering with representations through trial and error.

Students need to talk through why a mathematical model makes sense in representing a problem. This can be done in pairs, small groups or by the whole class. More instructional emphasis should be placed on finding and understanding why a representation makes sense, rather than merely finding the answer to problems in context.

Level 1: Demonstrates Minimal Success (Score Range 0 – 2)

The student's response shows few of the elements of performance that the tasks demand. The work shows a minimal attempt on the problem and struggles to make a coherent attack on the problem. Communication is limited and shows minimal reasoning. The student's response rarely uses definitions in their explanations. The student struggles to recognize patterns or the structure of the problem situation.

STUDENT D – Level 1

Performance Task

The Cycle Shop

Student D performed at Level 1, minimal success with a score of 0. The student attempted all the parts of the task, but was not able to achieve a point.

You work for a small business that sells bicycles, tricycles, and tandem bikes. Bicycles have one seat, two pedals and two wheels. Tricycles have one seat, two pedals, and three wheels. Tandem bikes have two seats, four pedals and two wheels.

Bicycle   Tricycle   Tandem Bike

1. On Monday you counted 48 tricycle wheels.

   How many tricycles were in the shop? _____144_____.

   $3 \times 48 = 144$

   Write an algebraic equation that shows the relationship between the number of wheels (w) and the number of tricycles (t).

   $t = 3w$

In part 1, the student makes a classic mistake translating a word problem into an algebraic equation. Since a tricycle has 3 wheels, the student writes the equation t = 3w instead of the correct relationship w = 3t. This is an error in correctly modeling the situation. MP4

2. On Wednesday there were no tandem bikes in the shop. There were only bicycles and tricycles. There are a total of 24 seats and 61 wheels in the shop. How many bicycles and how many triangles are in the shop?

   _____12 bikes_____ and ___12 trike___

   Show how you figured it out.

Bikes	seats	Wheels
1	1	2
12	12	24
12	12	

   bike 12
   Trike 12
   _____
            24

In part 2, the student starts to make a table, but is content to merely look at just one of the two constraints (total of 24 seats). The student only checks equal values for the two cycles. The student lacks the ability to reason abstractly and quantitatively.

STUDENT D – Level 1 continued

> In part 3, the student appears to try and simplify the problem. This occasionally happens when students are unsuccessful creating a model of the situation. The student finds numbers from the stem of the problem and performs one-step operations to calculate an answer. MP4

3. A month later, there are a different number of bicycles, tricycles tandem bikes in the shop. There are a total of 144 front steering handlebars, 378 pedals, and 320 wheels.

How many bicycles, tricycles and tandem bikes are in the shop?

48

Explain your solution.

48
3⟌144

12
24

53
6⟌320
30
20
18
2

STUDENT E – Level 1

Performance Task
The Cycle Shop

You work for a small business that sells bicycles, tricycles, and tandem bikes. Bicycles have one seat, two pedals and two wheels. Tricycles have one seat, two pedals, and three wheels. Tandem bikes have two seats, four pedals and two wheels.

Bicycle **Tricycle** **Tandem Bike**

Student E performs at Level 1 with a score of 2. The student shows some success modeling the situations but there are errors in successfully using the structures to find solutions.

1. On Monday you counted 48 tricycle wheels.

How many tricycles were in the shop? ___16___.

Write an algebraic equation that shows the relationship between the number of wheels (w) and the number of tricycles (t).

$48 = 3(16)$ $T = 3W$

In part 1, the student arrives at the correct answer and shows work, but writes an incorrect equation reversing the variables. This could be an issue of accuracy or misrepresenting equations. A-REI.1

2. On Wednesday there were **no** tandem bikes in the shop. There were only bicycles and tricycles. There are a total of 24 seats and 61 wheels in the shop. How many bicycles and how many triangles are in the shop?

___17 BIKES , 7 TRIKES___

Show how you figured it out.

17 BIKES
$\frac{61}{24} \quad \frac{24}{85}$
$2(17)=34$

B	W
1	2
2	4
3	6
7	14
12	24
17	34

T	S	W
1	1	3
2	2	6
3	3	9
4	4	12
5	5	15
6	6	18
7	7	21

$\frac{61}{21} \quad 40$ $\frac{24}{-7} \quad 17$

Performance Task   The Cycle Shop   P 1

In part 2, the students creates a t-table and uses a guess and check strategy. The student seems to only focus on one constraint (tricycles) and finds numbers of seats and wheels without checking the second constraints. The student's work shows lack of reasoning.

STUDENT E – Level 1 continued

In part 3, the student represents only one of the three equations correctly. It is not clear the student understands what the variables represent. This student needs more experience representing situations with equations. A-REI.6, MP4

3. A month later, there are a different number of bicycles, tricycles tandem bikes in the shop. There are a total of 144 front steering handlebars, 378 pedals, and 320 wheels.

How many bicycles, tricycles and tandem bikes are in the shop?

_____

Explain your solution.

$B + T + W = 144$
$B + T + W = 378$
$B + T + W = 320$

$$\begin{array}{r} 320 \\ -144 \\ \hline 176 \end{array}$$

Level 1 Implications for Instruction

Students need support in problem solving and looking for structure in mathematical situations. They need experiences in identifying the unknowns in a situation and determining the relationship between the quantities. Having students engage in conversations with other students about what is unknown and the relationship between quantities is an important step in developing understanding of a problem situation.

Representing the relationship of two quantities in an equation is an important step. This can first be learned by writing number sentences for specific values and their relationships. Only after representing a specific case can students understand an equation that involves unknown quantities. Students need multiple experiences at modeling situations by using equations. Understanding why an equation is incorrect for a situation allows students to make sense of the process in a different manner.

HIGH SCHOOL ALGEBRA: THE CYCLE SHOP
INSTRUCTIONAL SUPPORTS

The instructional supports on the following pages include a unit outline with formative assessments and suggested learning activities. Teachers may use this unit outline as it is described, integrate parts of it into a currently existing curriculum unit, or use it as a model or checklist for a currently existing unit on a different topic.

Department of Education
Dennis M. Walcott, Chancellor

Unit Outline

Introduction: This unit outline provides an example of how teachers may integrate performance tasks into a unit. *Teachers may (a) use this unit outline as it is described below; (b) integrate parts of it into a currently existing curriculum unit; or (c) use it as a model or checklist for a currently existing unit on a different topic.*

Grade 9 Algebra 1: Solving Systems of Equations

UNIT TOPIC AND LENGTH:

➢ The length of the unit includes suggested time spent on the classroom instruction of lessons and administration of assessments. Please note that this outline does not include individual lessons. The units should run between 20 and 25 standard periods of instruction. Five of the periods will involve the pre-assessment (0.5 periods), introducing and supporting problem solving on the long lesson (2 periods), teaching the formative assessment lesson (2 periods) and the final assessment (0.5 periods).

COMMON CORE LEARNING STANDARDS:

➢ **A-CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
➢ **A-CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*
➢ **A-REI.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify the solution method.
➢ **A-REI.3** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
➢ **A-REI.6** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
➢ **MP.1** Make sense of problems and persevere in solving them.
➢ **MP.3** Construct viable arguments and critique the reasoning of others.
➢ **MP.4** Model with mathematics.
➢ **MP.7** Look for and make use of structure.

BIG IDEAS/ENDURING UNDERSTANDINGS:
Student will understand:
➢ How to represent a system with tables, graphs and equations.
➢ How to solve a system of equations.
➢ How to use the unknown, constraints and their relationships to model a system.

ESSENTIAL QUESTIONS:

➢ How can situations be modeled as a system of linear equations and how to find solutions using all constraints?

CONTENT:	**SKILLS:**
- The big idea of the unit is modeling situations using algebraic tools such as tables, graphs, equations and use to solve systems. - Students determine relationships of objects and operations needed to develop a system. - Students apply the new knowledge to solving problems in context.	- Student will use graphs, tables and equations to model simultaneous linear systems. - Students will use algebraic techniques to find solutions to two variable linear systems. - Students will compare and contrast graphs of their systems.

ASSESSMENT EVIDENCE AND ACTIVITIES:

INITIAL ASSESSMENT :
The unit begins with the touchstone performance task *Fencing*. The task is designed to measure what students bring to the unit in regards to their knowledge of solving a basic system of two linear equations with two unknowns. Please reference *Fencing* for full details.

FORMATIVE ASSESSMENT:

The Formative Assessment Lesson is entitled *Solving Linear Equations in Two Variables*. A different pre-assessment task should be administered prior to the two-day lesson, either in class or for homework. Student should spend no more than 10 minutes on the task. Teachers review the student work prior to teaching the lesson. The FAL comes with complete teacher notes and the student pages. Please reference *Solving Linear Equations in Two Variables* for full details.

FINAL PERFORMANCE TASK:

The final performance assessment is entitled *The Cycle Shop*. It should be administer during a class period. Most students will complete the task in about 20 – 25 minutes, although time should not be a factor. The teacher should provide a reasonable amount of time for all students to finish. The students should be allowed to use any tools or materials they normally use in their classroom. The task can be read to the students and all accommodations delineated in IEP should be followed. Please reference *The Cycle Shop* for full details.

LEARNING PLAN & ACTIVITIES:
- The unit is designed with a pre-assessment task, an expert investigation, a formative assessment lesson and a final assessment. The mathematics of the unit involves writing equations from context and describing relationships using two variables. Students will

learn how to model situations by deriving simultaneous equations and solving the systems through tabular, graphical or algebraic means. This unit is designed to accompany the curriculum a teacher currently uses to teach the topics listed. That includes instruction in modeling linear systems in two variables using equations, graphs and tables. It also includes developing techniques in solving systems through intersections of lines, or using methods to solve equations. The elements in the unit will provide activities to foster formative assessment practices, conceptual understanding and non-routine problem solving.

➢ The expert investigation is entitled **The Wheel Shop**. It contains four separate but mathematically related problems labeled Part B, Part C, Part D, and Part E. All students should start with the Part B task and then proceed at their own speed to Part C and perhaps Part E. It is more important for the student to work deeply on a part and complete a write up than to merely work through and find answers. It is the student's responsibility to be reflective and thorough in their explanations, findings and justifications. The investigation comes with administration and teacher notes, the expert investigation, report guidelines and rubric.

Re-engagement: The unit begins with a pre-assessment called sponsored walk. After the teacher analyzes the student performances decision of how to proceed through the lesson is important. Instead of going back to re-teach skill and concepts lacking, it is much more powerful for the teacher to use the work students have already done on a contextual problem to help them build upon their understanding from previous thinking.   That process is called "re-engagement". It is powerful to use student work because students become very engaged in the process of figuring out what someone else is thinking. This process of analyzing and contrasting student thinking raises the cognitive demand for students and supports them to be more reflective about their own thinking. The re-engagement lesson will depend upon the results form the students in each individual class. Thus, each lesson will look very different from class to class. Students have already done the task on their own and now the important ideas need to be brought out and examined. In the process, students must have the opportunity to confront and understand the error in the logic of their misconceptions. Often, as teachers, we try to prevent errors by giving frequent reminders, such as "line up the decimal point". But actually errors provide great opportunities for learning for all students. Students don't let go of misconceptions until they understand why they don't make sense. For the student, there is underlying logic to their misconceptions.

Re-engagement – Confronting misconceptions, providing feedback on thinking, going deeper into the mathematics.
1. Start with a foundational problem to bring all the students along; this allows students to clarify and articulate important mathematics in order to better understand the entirety of the task.
2. Share different student approaches and ask all students to make sense of each strategy. Have all students compare the strategies to look for the mathematical connections and relationships.
3. Have students analyze misconceptions and discuss why they don't make sense. In the process students can let go of misconceptions and clarify their thinking about big mathematical ideas.
4. Have students determine how a strategy could be modified to get the correct solution. Have students look for the seeds of mathematical thinking in the selected student work.

Think/Write/Pair/Share is a high leverage strategy that respects individual time to process and organize ideas before engaging in peer-to-peer discussions. This process can be used throughout the unit as a vehicle for students to self reflect, construct new meaning by building on the ideas of others, and strengthen their arguments.

Journal Entries for Reflection: Using a prompt such as, *"How has my thinking changed as a result of what I have discussed with my peers?" or "How can I improve my argument or explanation using evidence and content vocabulary?"* can provide valuable opportunities for students to tweak their own solutions, during class or for homework, and subsequently, deepen their understanding of content.

Purposeful Questioning and Feedback are instructional supports that can help refocus students' attention on specific aspects of their work. Some suggestions based on misunderstandings and misconceptions are included in the Formative Assessment Lesson ***Solving Linear Equations in Two Variables***. Although these error patterns/questions relate to this specific assessment task they can be easily modified to address similar misconceptions that are revealed from any other problems or tasks used.

RESOURCES:

> ➢ Normal materials used in math class include manipulatives such as cards for matching activity, square tiles, counters, and cm graph paper.
> ➢ All the materials referenced in the assessments, formative assessment lesson and expert investigation are included. Most supplementary materials are located in the appendix, including the established scored benchmark papers and some student work examples.
> ➢ What isn't included in print materials can be found on a TBA websites.

HIGH SCHOOL ALGEBRA: THE CYCLE SHOP

INITIAL TASK: FENCING

Fencing

This problem gives you the chance to:
- interpret given information
- choose and use an appropriate method to solve a problem

Jon buys fencing for his yard.

He pays $122 for 5 fence posts and 4 fence panels.
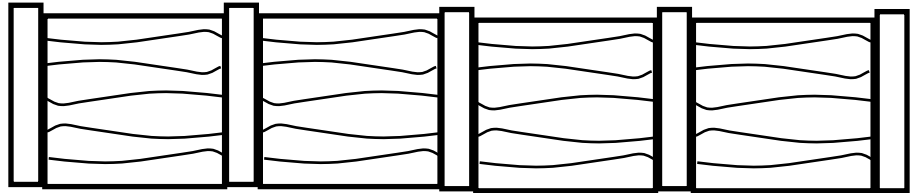
He pays $570 for 21 fence posts and 20 fence panels.

How much does he pay for 4 fence posts and 3 fence panels? $_____

Show how you figured it out.
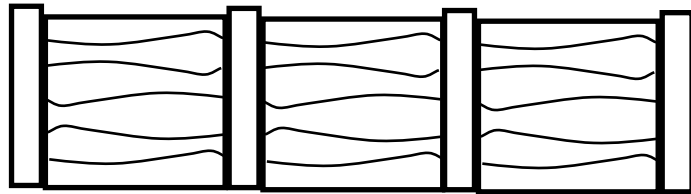
HIGH SCHOOL ALGEBRA: THE CYCLE SHOP

FORMATIVE ASSESSMENT LESSON: SOLVING LINEAR EQUATIONS IN TWO VARIABLES

Mathematics Assessment Project

Formative Assessment Lesson Materials

Solving Linear Equations in Two Variables

MARS Shell Center
University of Nottingham & UC Berkeley
Beta Version

If you encounter errors or other issues in this version, please send details to the MAP team
c/o map.feedback@mathshell.org.

Solving Linear Equations in Two Variables

Mathematical goals

This lesson unit is intended to help you assess how well students are able to formulate and solve problems using algebra and, in particular, to identify and help students who have the following difficulties:

- Solving a problem using two linear equations with two variables.
- Interpreting the meaning of algebraic expressions.

Common Core State Standards

This lesson involves *mathematical content* in the standards from across the grades, with emphasis on:

A-CED: Create equations that describe numbers or relationships.
A-REI: Solve systems of equations.

This lesson involves a range of *mathematical practices*, with emphasis on:

2.   Reason abstractly and quantitatively.
3.   Construct viable arguments and critique the reasoning of others.

Introduction

This lesson is structured in the following way:

- Before the lesson, students work individually on the assessment task *Notebooks and Pens*. You then review their work and create questions for students to answer in order to improve their solutions.
- During the lesson, students work individually on a task that requires them to interpret and solve two equations in two variables. Students then compare and discuss their solutions in small groups.
- In the same small groups, students evaluate some sample solutions of the same task.
- In a whole-class discussion, students explain and compare the alternative solution strategies they have seen and used.
- Finally, students use what they have learned to revise their work on *Notebooks and Pens*.

Materials required

- Each individual student will need two copies of the assessment task *Notebooks and Pens,* and a copy of the lesson task *Cash Registers*.
- Each small group of students will need a blank sheet of paper, and copies of the four sheets *Sample Student Work.*
- Graph paper should be kept in reserve and used only when requested.
- Projector resources are provided to support the whole-class discussion.

Time needed

Approximately fifteen minutes before the lesson, a one-hour lesson, and ten minutes in a follow-up lesson (or for homework). Timings given are only approximate. Exact timings will depend on the needs of the class.

Before the lesson

Assessment task: *Notebooks and Pens* (15 minutes)

Have the students do this task in class or for homework a day or more before the formative assessment lesson. This will give you an opportunity to assess the work and to find out the kinds of difficulties students have with it. You will then be able to target your help more effectively in the follow-up lesson.

Give each student a copy of *Notebooks and Pens.*

Introduce the task briefly and help the class to understand the problem and its context.

> *Read through the questions, and try to answer them as carefully as you can.*
>
> *Show all your work, so that I can understand your reasoning.*

It is important that students are allowed to answer the questions without assistance, as far as possible.

Students should not worry too much if they cannot understand or do everything, because there will be a lesson using a similar task, which should help them. Explain to students that by the end of the next lesson, they should expect to answer questions such as these confidently. This is their goal.

Notebooks and Pens

A store sells pens at $2 and notebooks at $5.
n = number of notebooks sold.
p = number of pens sold.
The following equations are true:
$4n = p$
$5n + 2p = 39$
Here is what Dan and Emma think the equations mean:

I think the first equation means that the store sells four times as many notebooks as pens.

I think the second equation means that the store sold 5 notebooks and 2 pens.

Dan   **Emma**

Is Dan correct?
If you think Dan is wrong, explain the mistake and explain what *you* think $4n = p$ means.

Is Emma correct?
If you think Emma is wrong, explain the mistake and explain what *you* think $5n + 2p = 39$ means.

Figure out for yourself the number of pens and the number of notebooks sold in the store.
Number of pens sold = ... Number of notebooks sold = ...

Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding. The purpose of doing this is to forewarn you of issues that will arise during the lesson itself, so that you may prepare carefully.

We suggest that you do not score students' work. The research shows that this will be counterproductive, as it encourages students to compare their scores and distracts their attention from what they can do to improve their mathematics.

Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some suggestions for these are given on the next page. These have been drawn from common difficulties observed in trials of this lesson unit.

We suggest that you write a list of your own questions, based on your students' work, using the ideas below. You may choose to write questions on each student's work. If you do not have time to do this, select a few questions that will be of help to the majority of students. These can be written on the board at the end of the lesson.
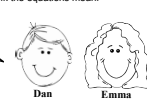
Common issues:	Suggested questions and prompts:
Student assumes that the letter stands for an object not a number For example: The student says that the statements are correct. Or: The student realizes the equations are incorrect, but is unable to explain why.	• *What does the letter p represent?* • *Write the equation as a sentence. Does your sentence match what Dan/Emma said?* • *If n = 3, what would p equal in the first equation? Which is greater: n or p?* • *Are there more notebooks than pens? How do you know?*
Student only uses one equation For example: The student finds a value or values for *n* and *p* that fits one equation but not the other, such as *n* = 1 and *p* = 4 for the first equation.	• *For this equation, is there another pair of values for n and p? And another? How do you know which value is correct?* • *How can you check that your values for n and p work for both equations?*
Student produces unsystematic guess and check work For example: The student works out three or four seemingly unconnected combinations of values for *n* and *p*.	• *What is a sensible value to try for n (or p)? Why?* • *Can you organize your work in a table?*
Student provides poor explanation For example: The student presents the work as a series of unexplained numbers and/or calculations.	• *Would someone unfamiliar with your type of solution easily understand your work?* • *Have you explained how you arrived at your answer?*
Student makes algebraic mistakes For example: The student makes a mistake when manipulating the algebra in the equations.	• *How can you check that your answer is correct?*
Student solves the two equations correctly Student needs an extension task.	• *Can you now use a different method, for example, a table, a graph, or algebra?* • *Is this method better than your original one? Why?*

Suggested lesson outline

Individual work: *Cash Registers* (10 minutes)

Give each student the task sheet *Cash Registers*. Help students to understand the problem, and explain the context of the task briefly.

> *Spend ten minutes on your own answering these questions.*
>
> *What does "simultaneously" mean?*
>
> *Show all your work on the sheet.*

Cash Registers

The drawer of a cash register contains some quarters and some dollar bills.

x = the number of quarter coins in the cash register.
y = the number of dollar bills in the cash register.

The following two equations are true:

$$3x = y$$
$$4x + y = 70$$

1. Explain in words the meaning of each equation.

2. Find two pairs of values for x and y that satisfy the first equation.

3. Find two pairs of values for x and y that satisfy the second equation.

4. Find pairs of values for x and y that satisfy both equations *simultaneously*.

Students who sit together often produce similar answers and, when they come to compare their work, they have little to discuss.

For this reason we suggest that, when students do this task individually, you ask them to move to different seats. Then, for the collaborative task, allow them to return to their usual places. Experience has shown that this produces more profitable discussions.

Collaborative small-group work: *Cash Registers* (10 minutes)

Organize the class into small groups of two or three students and hand out a fresh sheet of paper to each group. Students should now have another go at the task, but this time they will combine their ideas.

> *I want you now to work together in your groups.*
>
> *Your task is to produce an answer together that is better than your individual ones.*

Throughout this activity, encourage students to articulate their reasoning, justify their choices mathematically, and question the choices put forward by others.

As students work you have two tasks, to note student approaches to their work, and to support their thinking.

Note student approaches to their work

How do students choose to tackle this task? Notice the variety in approaches. Notice any common errors. You can use this information to focus your questioning in the whole-class discussion towards the end of the lesson.

Support student thinking

Try not to make suggestions that prompt students towards a particular answer. Instead, ask questions to help students clarify their thinking.

You may find that some students interpret the letters as "quarters" and "dollars" rather than the number of quarters and number of dollars. For example, they may say things like:

> "$3x = y$ means three times as many quarters as dollars."
> "$4x + y = 70$ means 4 quarters plus dollars equals 70."
> "There is \$70 in the till."

The following questions and prompts may be helpful for both students struggling with the task and those making quick progress:

> *What do the letters x and y represent?*
>
> *Replace x and y in this equation by words and now say what the equation means.*
>
> *Are there more dollar bills or more quarters in the cash register? How do you know?*
>
> *Do you have any values for x and y that work for the first equation? How can you check to see if they also work for the second one? If these don't fit, what other values for x and y can you use?*
>
> *Why have you chosen these values for x and y?*

        4

Suppose there are 5 quarters in the drawers of the cash register, so x = 5. From the first equation, how many dollar bills are there? [15.] From the second equation, how many dollar bills are there? [50.] There cannot be both 15 and 50 dollar bills!

Can you find a value for x that will give the same answer in both cases?

How can you check that your answer is right?

Can you use these equations to calculate the amount of money in the cash register?

If the whole class is struggling on the same issue, you may want to write a couple of questions on the board and organize a brief whole-class discussion. You could also ask students who performed well in the assessment to help struggling students.

Collaborative analysis of *Sample Student Work* (15 minutes)

When all groups have made a reasonable attempt, ask them to put their work to one side. Give each group copies of the *Sample Student Work.* This task will give students the opportunity to discuss and evaluate possible approaches to the task, without providing a complete solution strategy.

Ideally, all groups will review all four pieces of work. However, if you are running out of time, choose just two solutions for all groups to analyze, using what you have learned during the lesson about what students find most difficult.

Encourage students to think more deeply using the following questions. (These are reproduced on the projector resource *Assessing Sample Student Work*.)

You are the teacher and have to assess this work.

Correct the work and write comments on the accuracy and organization of each response.

- *What do you like about the work?*
- *What method did the student use? Is it clear? Is it accurate? Is it efficient?*
- *What errors did the student make?*
- *How might the work be improved?*

During this small-group work, support the students as before. Also, check to see which of the explanations students find more difficult to understand. Note similarities and differences between the sample approaches and those the students used in the group work.

Plenary whole-class discussion: comparing different approaches (15 minutes)

Hold a whole-class discussion to consider the different approaches used in the sample work. Focus the discussion on those parts of the task that the students found difficult. Ask representatives from each group to explain and critique one student's method from the *Sample Student Work*. During the discussion you may find it helpful to use the projector resources, which are slides showing the different sample solutions.

Which approach did you like best? Why?

Which approach did you find most difficult to understand? Why?

Next lesson: Improve individual responses to *Notebooks and Pens* (10 minutes)

Have students do this task at the beginning of the next lesson if you do not have time during the lesson itself. Some teachers like to set this task for homework.

Return the students' individual work on the assessment task *Notebooks and Pens* along with a second blank copy of the task sheet.

Look at your original responses and think about what you have learned this lesson.

Using what you have learned, try to improve your work.

5

If you have not written questions on individual pieces of work then write your list of questions on the board. Students are to select from this list only the questions appropriate to their own work.

Solutions

Assessment task: *Notebooks and Pens*

I think the first equation means that the store sells four times as many notebooks as pens.

Dan

Emma

I think the second equation means that the store sold 5 notebooks and 2 pens.
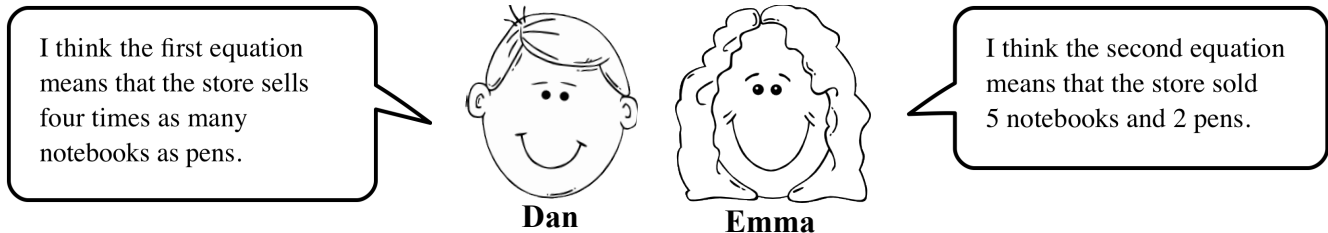
Dan is incorrect:

Dan has misinterpreted n to mean, "notebooks sold" rather than "*the number of* notebooks sold."

So he has read the equation "$4n = p$" as "there are four notebooks sold for every single pen sold."

The equation actually means, "4 times the number of notebooks sold equals the number of pens sold," or "the store sells four times more pens than notebooks."

Emma is incorrect:

Emma has also misinterpreted n to mean "notebooks" rather than "*the number of* notebooks."

In the second statement, $5n$ does not mean, "there are 5 notebooks." It means "5 times the number of notebooks."

Since each notebook costs $5, $5n$ gives you the amount of money taken from selling notebooks, and since each pen costs $2, $2p$ gives you the amount of money taken from selling pens. So $5n + 2p = 39$ means that $39 was taken altogether from selling notebooks and pens at these prices. However, the equation does not, in isolation, tell you how many notebooks or pens were sold.

Using the first equation to substitute $4n$ for p in the second equation gives $n = 3$ and $p = 12$.

3 notebooks and 12 pens were sold.

Cash Registers

1. The number of dollar bills is three times the number of quarters.
   Four times the number of quarters plus the number of dollar bills totals 70.

2. Possible values: (4, 12) or (8, 24).

3. Possible values: (12, 22) or (7, 42).

4. $x = 10, y = 30$

Comments on *Sample Student Work*

Ava used "guess and check" with both equations

Strengths: Her work is systematic and easy to follow.

Weaknesses: Her method is inefficient and, although it is systematic, she has not reflected on each answer to determine the next set of values to check.

Her lack of progress leads to her abandoning the task.

Ava could add an explanation about her solution method.

Ethan used an elimination method

Strengths: This method can work if equations are manipulated carefully.

Weaknesses: Ethan makes a mistake when rearranging the first equation. Consequently, when the two equations are added together, a variable is not eliminated, but instead Ethan has created an equation with two variables.

Ethan briefly used guess and check. This gives many solutions. Ethan has simply opted to figure out two solutions. Both answers are incorrect. Ethan has not explained his working or why he was happy with the second set of values.

If the first equation had been $3x + y = 0$, what would still be wrong with Ethan's method?

Would this method ever obtain just one solution?

Joe used a substitution method

Strengths: This is an efficient method.

Weaknesses: Joe failed to multiply all the terms on the left-hand side of the equation by three, so he obtained an incorrect answer.

If Joe had substituted $3x$ for y into the second equation the solution would have been very straightforward.

Mia used a graphical approach

Strengths: This method can work.

Weaknesses: In this case a graphical approach is not a very efficient strategy.

Mia has made an error in her second table: $y = 66$ not 56.

Mia could have used the co-ordinates $(20, -10)$ to help plot the second line. There are no labels on either axis. The scale of Mia's graph means that the lines are not plotted accurately.

Was Mia right to abandon $(20, -10)$ as a point to be used to plot the second line?

Notebooks and Pens

A store sells pens at $2 and notebooks at $5.

n = number of notebooks sold.

p = number of pens sold.

The following equations are true:

$4n = p$

$5n + 2p = 39$

Here is what Dan and Emma think the equations mean:

> I think the first equation means that the store sells four times as many notebooks as pens.

Dan   **Emma**

> I think the second equation means that the store sold 5 notebooks and 2 pens.

Are Dan and Emma correct?

If you think Dan is wrong, explain the mistake and explain what *you* think the equation means.

If you think Emma is wrong, explain the mistake and explain what *you* think the equation means.

Figure out for yourself the number of pens and the number of notebooks sold in the store.

Cash Registers

The drawer of a cash register contains some quarters and some dollar bills.

x = the number of quarter coins in the cash register.
y = the number of dollar bills in the cash register.

The following two equations are true:

$3x = y$
$4x + y = 70$

1. Explain in words the meaning of each equation.

----

2. Find two pairs of values for x and y that satisfy the first equation.

----

3. Find two pairs of values for x and y that satisfy the second equation.

----

4. Find pairs of values for x and y that satisfy both equations *simultaneously*.

S-2

Sample Student Work: Ava

$$3 \quad x = y \qquad y = 3x$$

$$4x + y = 10$$
$$4x + y = 7 \quad \times$$

$$\text{Try} \quad x = 1 \qquad y = 3$$

$$\text{Try} \quad x = 2 \qquad y = 6$$

$$8 + 6 = 14 \quad \times$$

$$\text{Try} \quad x = 3 \quad y - 9$$

$$12 + 9 = 21 \quad \times$$

$$\text{Try} \quad x = 4 \quad y = 12$$

$$16 + 12 = 28 \quad \times$$

You are the teacher and have to assess this work.

- What do you like about Ava's work?
- What method did she use? Is it clear? Is it accurate? Is it efficient?
- What errors did Ava make?
- How might her work be improved?

Sample Student Work: Ethan

$$3x = y$$
$$4x + y = 70$$

$$3x + y = 0$$
$$\underline{4x + y = 70} \ +$$
$$7x + 2y = 70$$

$$x = 5\overset{3}{} \quad 7 \times 5 = 35$$
$$35 + 2y = 70$$
$$2y = 35$$
$$y = 17 \cdot 5 - \text{must be a whole number}$$

$$\boxed{x = 6}$$

$$7 \times 6 = 42$$
$$42 + 2y = 70$$
$$2y = 28$$
$$y = 14$$

You are the teacher and have to assess this work.

- What do you like about Ethan's work?
- What method did he use? Is it clear? Is it accurate? Is it efficient?
- What errors did Ethan make?
- How might his work be improved?

....

....

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     S-4

Sample Student Work: Joe

$$3x = y \qquad\qquad\qquad x = \frac{y}{3}$$

$$4x + y = 70$$

$$4 \times \frac{y}{3} + y = 70$$

$$\times 3 \qquad 4y + y = 210$$

$$5y = 210$$

$$\boxed{y = 42} \qquad x = \frac{42}{3} = \boxed{14}$$

You are the teacher and have to assess this work.

- What do you like about Joe's work?
- What method did Joe use? Is it clear? Is it accurate? Is it efficient?
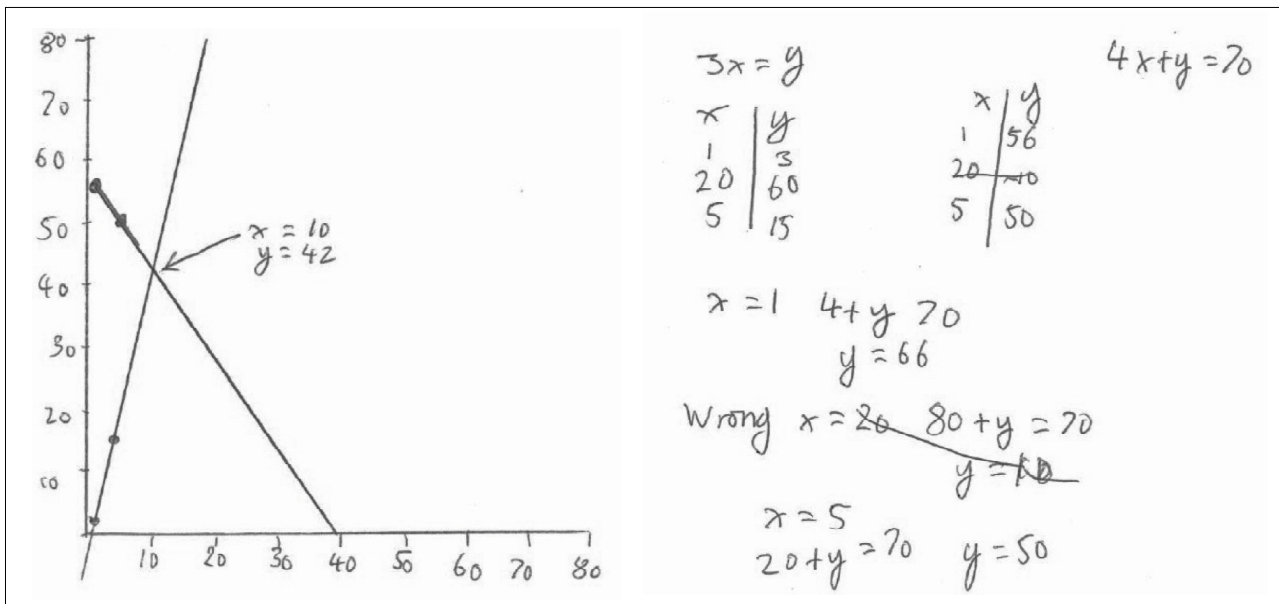- What errors did Joe make?
- How might the work be improved?

Sample Student Work: Mia

You are the teacher and have to assess this work.

- What do you like about Mia's work?
- What method did she use? Is it clear? Is it accurate? Is it efficient?
- What errors did Mia make?
- How might her work be improved?

　　　S-6

HIGH SCHOOL ALGEBRA: THE CYCLE SHOP

INVESTIGATION: THE WHEEL SHOP

Department of
Education
Dennis M. Walcott, Chancellor

EXPERT INVESTIGATION

The Wheel Shop

Level B:

The Wheel Shop sells other kinds of vehicles.   There are bicycles and go-carts in a different room of the shop.   Each bicycle has only one seat and each go-cart has only one seat. There are a total of 21 seats and 54 wheels in that room.

How many bicycles and how many go-carts are in the Wheel Shop?

Explain how you figured it out.

EXPERT INVESTIGATION

The Wheel Shop

Level C:

Three months later some vehicles have sold and new models have been brought into the Wheel Shop. Now, there are a different number of bicycles, tandem bicycles, and tricycles in the shop. There are a total of 135 seats, 118 front handle bars (that steer the bike), and 269 wheels.

How many are bicycles, tandem bicycles and tricycles are there in the Wheel Shop?

The Wheel Shop

Level D:

In the back stockroom at the Wheel Shop, the number of seats and horns equaled the number of wheels. The number seats and handle bars equaled the number of horns. Twice the number of wheels is equal to three times number of handle bars. Determine the relationship of horns to seats.