COMMON CORE-ALIGNED TASK
WITH INSTRUCTIONAL SUPPORTS

Mathematics

GRADE 7 MATH: PROPORTIONAL REASONING

UNIT OVERVIEW

This is a 3-4 week unit that focuses on identifying and using unit rates. It also develops students' understanding of proportional relationships represented in equations and graphs. Students use proportional relationships to solve multi-step ratio and percent problems.

TASK DETAILS

Task Name: Proportional Reasoning

Grade: 7

Subject: Math

Task Description: This task consists of five extended-response questions related to proportional reasoning. The short response and extended-response questions require students to write an appropriate response.

Standards Assessed:
7RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks ½ mile in each ¼ hour, compute the unit rate as the complex fraction ½/¼ miles per hour, equivalently 2 miles per hour.
7RP.2 Recognize and represent proportional relationships between quantities.

a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
c. Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.
d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.

7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Standards for Mathematical Practice:
MP.1 Make sense of problems and persevere in solving them.
MP.2 Reason abstractly and quantitatively.
MP.3 Construct viable arguments and critique the reasoning of others.
MP.6 Attend to precision.

Department of Education
Dennis M. Walcott, Chancellor

TABLE OF CONTENTS

The task and instructional supports in the following pages are designed to help educators understand and implement tasks that are embedded in Common Core-aligned curricula. While the focus for the 2011-2012 Instructional Expectations is on engaging students in Common Core-aligned culminating tasks, it is imperative that the tasks are embedded in units of study that are also aligned to the new standards. Rather than asking teachers to introduce a task into the semester without context, this work is intended to encourage analysis of student and teacher work to understand what alignment looks like. We have learned through the 2010-2011 Common Core pilots that beginning with rigorous assessments drives significant shifts in curriculum and pedagogy. Universal Design for Learning (UDL) support is included to ensure multiple entry points for all learners, including students with disabilities and English language learners.

Acknowledgements: The tasks were developed by the 2010-2011 NYC DOE Middle School Performance Based Assessment Pilot Design Studio Writers, in collaboration with the Institute for Learning.

COMMON CORE-ALIGNED TASK
WITH INSTRUCTIONAL SUPPORTS

Mathematics

GRADE 7 MATH: PROPORTIONAL REASONING PERFORMANCE TASK

Department of Education
Dennis M. Walcott, Chancellor

Performance-Based Assessment 1
Proportional Reasoning – Grade 7

Student Name ______________________________

School ______________________________

Date ______________________________

Teacher ______________________________

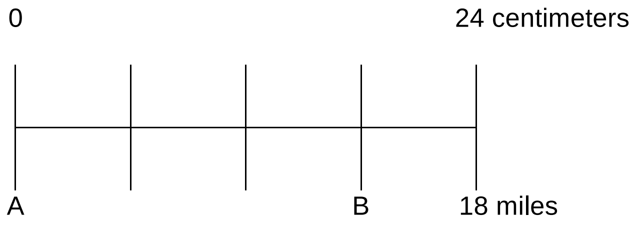
1. Amy and her family were traveling during their vacation. She looked at her watch at Point 1 in the diagram below, and then again at Point 2 in the diagram below. Her mom told her how far they traveled in that time, as noted below.

Point 1 Point 2

a. Based on this information, what is the unit rate of the car? Explain in writing what that unit rate means in the context of the problem.

b. Amy's dad said that the entire trip was 1200 miles. How many hours will it take to complete the trip? Explain how you know.

2. On a map of the United States, 24 centimeters represents 18 miles.



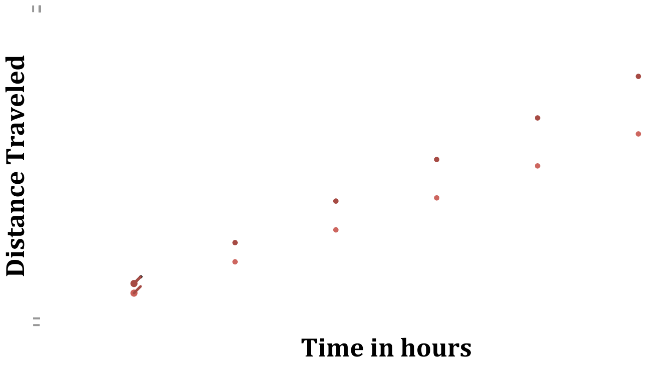
a. How many centimeters represent one mile?

b. How long is the line segment between A and B in centimeters?

c. If A and B represent two cities, what is the actual distance between the two cities?

3. Jack and Jill raced cross-country on motor bikes. Jack drove 325 miles in 5 hours; Jill took 6 ½ hours to travel the same distance as Jack.
   a. Compute the unit rates that describe Jack's average driving speed and Jill's average driving speed. Show how you made your decisions.

   b. A portion of the graph of Jack and Jill's race appears below. Identify which line segment belongs to Jack and which belongs to Jill. Explain in writing how you made your decisions.



4. Reynaldo is planning to drive from New York to San Francisco in his car. Reynaldo started to fill out the table below showing how far in miles he can travel for each gallon of gas he uses.

Gallons	2	4		8	10	12
Miles	56		168	224		

Use the information in Reynaldo's table to answer the questions below.

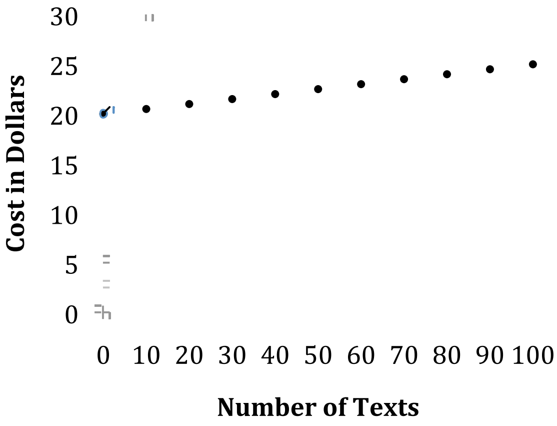
a. Complete the table for Reynaldo. Assume the relationship in the table is proportional.

b. Based on the table, how many miles per gallon did Reynaldo's car get? Explain in writing how you know.

c. Write an equation that Reynaldo can use to find the distance (d) he can drive on any number of gallons of gas (g).

d. When Reynaldo's tank is full, it holds 20 gallons. How far can Reynaldo drive on a full tank of gas?

5. The monthly cost of Jazmine's cell phone plan is graphed on the grid below. Her friend Kiara selected a plan that charges $0.25 per text, with no monthly fee, because she only uses her phone for texting.



a. Write an equation to represent the monthly cost of Kiara's plan for any number of texts.

b. Graph the monthly cost of Kiara's plan on the grid above.

c. Using the graphs above, explain the meaning of the following coordinate pairs:

i. (0, 20): ____

ii. (0, 0): ____

iii. (10, 2.5): ____

iv. (100, 25): ____

d. When one of the girls doubles the number of texts she sends, the cost doubles as well. Who is it? Explain in writing how you know.

COMMON CORE-ALIGNED TASK
WITH INSTRUCTIONAL SUPPORTS

Mathematics

GRADE 7 MATH: PROPORTIONAL REASONING

UNIVERSAL DESIGN FOR LEARNING (UDL) PRINCIPLES

Proportional Reasoning – Math Grade 7
Common Core Learning Standards/
Universal Design for Learning

The goal of using Common Core Learning Standards (CCLS) is to provide the highest academic standards to all of our students. Universal Design for Learning (UDL) is a set of principles that provides teachers with a structure to develop their instruction to meet the needs of a diversity of learners. UDL is a research-based framework that suggests each student learns in a unique manner. A one-size-fits-all approach is not effective to meet the diverse range of learners in our schools. By creating options for how instruction is presented, how students express their ideas, and how teachers can engage students in their learning, instruction can be customized and adjusted to meet individual student needs. In this manner, we can support our students to succeed in the CCLS.

Below are some ideas of how this Common Core Task is aligned with the three principles of UDL; providing options in representation, action/expression, and engagement. As UDL calls for multiple options, the possible list is endless. Please use this as a starting point. Think about your own group of students and assess whether these are options you can use.

REPRESENTATION: *The "what" of learning.* How does the task present information and content in different ways? How do students gather facts and categorize what they see, hear, and read? How are they identifying letters, words, or an author's style?

In this task, teachers can…

- ✓ **Make explicit links between information provided in texts and any accompanying representation of that information in illustrations by** displaying time in analog and digital mode.
- ✓ **Provide text-to-speech access by** reading aloud, creating teacher-made recordings, or employing digital software.
- ✓ **Embed support for unfamiliar references within the text** by defining academic vocabulary, such as "unit rate."

ACTION/EXPRESSION: *The "how" of learning.* How does the task differentiate the ways that students can express what they know? How do they plan and perform tasks? How do students organize and express their ideas?

In this task, teachers can…

- ✓ **Embed support for vocabulary and symbols by** providing access to online tools, such as *The Mathematics Glossary,* which uses multiple means of representation to explain concepts.
- ✓ **Provide multiple examples of novel solutions to authentic problems** by allowing students to see their peers' perspectives on proportional thinking.

ENGAGEMENT: *The "why" of learning.* How does the task stimulate interest and motivation for learning? How do students get engaged? How are they challenged, excited, or interested?

In this task, teachers can…

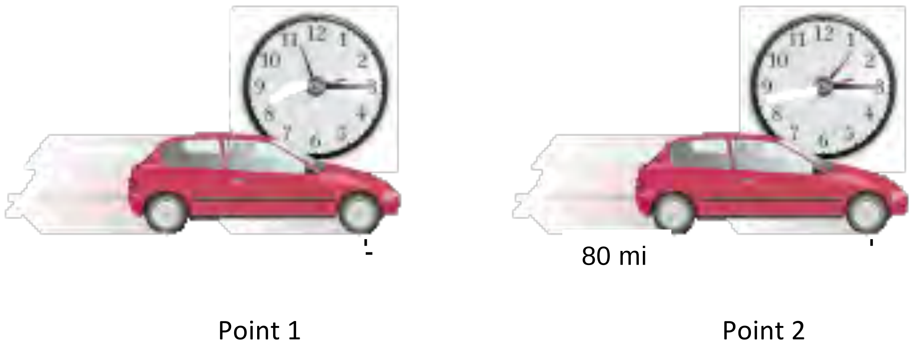
- ✓ **Provide alternatives in the permissible tools and scaffolds to optimize challenge** by providing calculators or designing customized mini-lessons to activate prior knowledge of the concept on proportional thinking.

Visit http://schools.nyc.gov/Academics/CommonCoreLibrary/default.htm to learn more information about UDL.

GRADE 7 MATH: PROPORTIONAL REASONING BENCHMARK PAPERS WITH RUBRICS

This section contains benchmark papers that include student work samples for each of the five tasks in the Proportional Reasoning assessment. Each paper has descriptions of the traits and reasoning for the given score point, including references to the Mathematical Practices.

1. Amy and her family were traveling during their vacation. She looked at her watch at Point 1 in the diagram below, and then again at Point 2 in the diagram below. Her mom told her how far they traveled in that time, as noted below.



   a. Based on this information, what is the unit rate of the car? Explain in words what that unit rate means in the context of the problem.
   b. Amy’s dad said that the entire trip was 1200 miles. How many hours will it take to complete the trip? Explain your reasoning in words.

NYC Grade 7 Assessment 1
Amy's Vacation
Benchmark Papers

3 Points

The response accomplishes the prompted purpose and effectively communicates the student's mathematical understanding. The student's strategy and execution meet the content (including concepts, technique, representations, and connections), thinking processes and qualitative demands of the task. Minor omissions may exist, but do not detract from the correctness of the response.

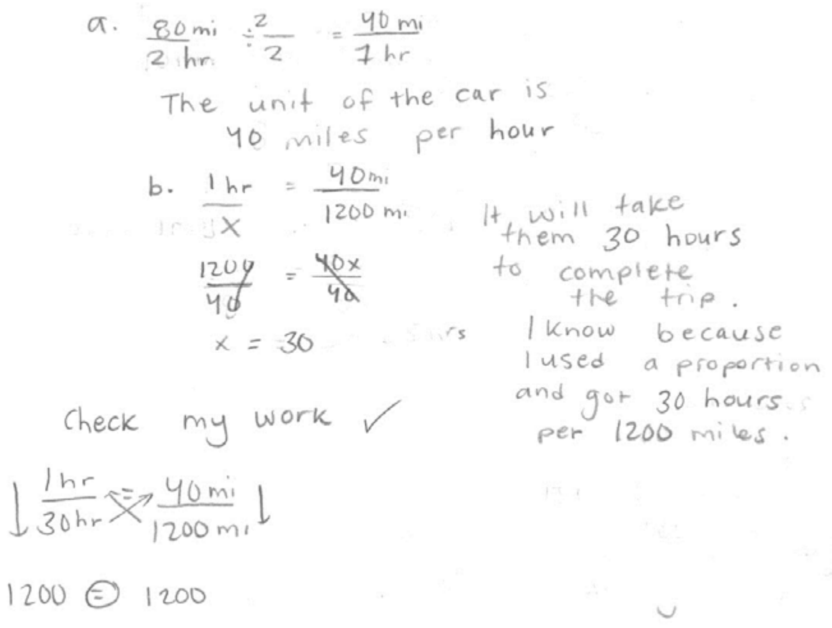
Either verbally or symbolically, the strategy used to solve the problem is stated, as is the work used to find ratios, unit rates, and partial answers to problems. Minor arithmetic errors may be present, but no errors of reasoning appear. Complete explanations are stated, based on work shown.

Accurate reasoning processes demonstrate the Mathematical Practices, (1) Make sense of problems and persevere in solving them and (2) Reason abstractly and quantitatively (since students need to abstract information from the problem, create a mathematical representation of the problem, and correctly work with the ratio and/or proportion). Evidence of the Mathematical Practice, (3) Construct viable arguments and critique the reasoning of others, is demonstrated by complete and accurate explanations. Evidence of the Mathematical Practice, (4) Model with mathematics, is demonstrated by representing the problem with ratios or proportions. Evidence of the Mathematical Practice, (6) Attend to precision, can include proper use of ratio notation and proper labeling of quantities. Evidence of the Mathematical Practice, (7) Look for and make use of structure, may be demonstrated by student recognition, e.g., that the multiples used in a table need not be of equal intervals (see table below) or that part b may be solved with a properly formed proportion, where the variable is equal to the numbers of hours a 1200-mile trip will take.

The reasoning used to solve the parts of the problem may include:

a. Indicating the 80-mile trip took two hours; scaling down the 80 mile : 2 hour rate in tabular or fraction form to 40/1 or 40 miles per hour.
b. Using a proportion or proportional reasoning (e.g., 2 hours is twice 1 hour, so I can half 80 miles (or divide 80 miles by 2) to find the unit rate.
c. Scaling up the 80 mile: 2 hours or 40 mile : 1 hour rate in tabular or fraction form to 1200 miles : 30 hours.
d. Using a proportion or proportional reasoning (e.g., 1200 miles is 40 miles times 30, so I can multiply I hour by 30.)

miles	40	400	1200
hours	1	10	30



a. $\frac{80\text{ mi}}{2\text{ hr}} \div \frac{2}{2} = \frac{40\text{ mi}}{1\text{ hr}}$

The unit of the car is 40 miles per hour

b. $\frac{1\text{ hr}}{x} = \frac{40\text{ mi}}{1200\text{ mi}}$

$\frac{120\cancel{0}}{4\cancel{0}} = \frac{\cancel{40}x}{\cancel{40}}$

$x = 30$

It will take them 30 hours to complete the trip. I know because I used a proportion and got 30 hours per 1200 miles.

Check my work ✓

$\frac{1\text{ hr}}{30\text{ hr}} = \frac{40\text{ mi}}{1200\text{ mi}}$

1200 ⊜ 1200

NYC Grade 7 Assessment 1
Amy's Vacation
Benchmark Papers

2 Points

The response demonstrates adequate evidence of the learning and strategic tools necessary to complete the prompted purpose. It may contain overlooked issues, misleading assumptions, and/or errors in execution. Evidence in the response demonstrates that the student can revise the work to accomplish the task with the help of written feedback or dialogue.

Either verbally or symbolically, the strategy used to solve the problem is stated, as is the work used to find ratios, unit rates, and partial answers to problems. Reasoning may contain incomplete, ambiguous or misrepresentative ideas. Partial explanations are stated, based on work shown. Accurate reasoning processes demonstrate the Mathematical Practices, (1) Make sense of problems and persevere in solving them and (2) Reason abstractly and quantitatively (since students need to abstract information from the problem, create a mathematical representation of the problem, and correctly work with the ratio and/or proportion). Evidence of the Mathematical Practice, (3) Construct viable arguments and critique the reasoning of others, is demonstrated by complete and accurate explanations. Evidence of the Mathematical Practice, (4) Model with mathematics, is demonstrated by representing the problem with ratios or proportions. Evidence of the Mathematical Practice, (6) Attend to precision, can include proper use of ratio notation and proper labeling of quantities. Evidence of the Mathematical Practice, (7) Look for and make use of structure, may be demonstrated by student recognition, e.g., that the multiples used in a table need not be of equal intervals (see table below) or that part b may be solved with a properly formed proportion, where the variable is equal to the numbers of hours a 1200-mile trip will take.

The reasoning used to solve the parts of the problem may include:

a. Indicating the 80-mile trip took some number other than 2 hours; or the distance traveled for the two-hour trip was some number other than 80 miles; scaling down in tabular or fraction form to a unit rate consistent with the number chosen.
b. Correctly using a proportion or proportional reasoning to find the unit rate, but choosing a distance other than 80 miles or a time other than 2 hours.
c. Scaling up the 80 mile : 2 hour or 40 mile : 1 hour rate in tabular or fraction form, but failing to reach or failing to stop at 1200 miles.
d. Using a proportion or proportional reasoning (e.g., 1200 miles is 80 miles times 15), but failing to multiply 2 hours by 15.

miles	40	80	120	160
hours	1	2	3	4

traveled 2 hours

A. unit rate is $\frac{40 \text{ mi}}{1 \text{ hour}}$ the unit rate needs to have a denominater of one. So the 2 was going to be the denominater so I divided $2\overline{)80}$ to get half,

B. Every 2 hour is 80 mi
It will take them 15 hours to get to their destination Because

$$\begin{array}{r} 15 \\ 80\overline{)1200} \\ -80 \\ \hline 400 \\ -400 \\ \hline 0 \end{array}$$

NYC Grade 7 Assessment 1
Amy's Vacation
Benchmark Papers

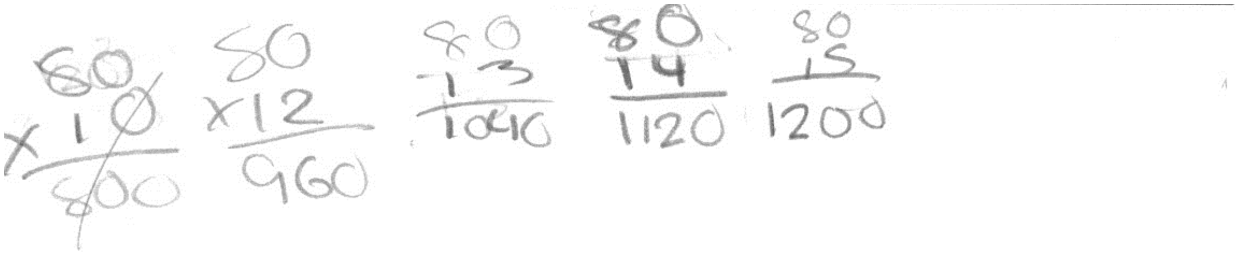
1 Point

The response demonstrates some evidence of mathematical knowledge that is appropriate to the intent of the prompted purpose. An effort was made to accomplish the task, but with little success. Evidence in the response demonstrates that, with instruction, the student can revise the work to accomplish the task.

Some evidence of reasoning is demonstrated either verbally or symbolically, but may be based on misleading assumptions, and/or contain errors in execution. Some work is used to find ratios, or unit rates; or partial answers to portions of the task are evident. Explanations are incorrect, incomplete or not based on work shown. Accurate reasoning processes demonstrate the Mathematical Practices, (1) Make sense of problems and persevere in solving them and (2) Reason abstractly and quantitatively (since students need to abstract information from the problem, create a mathematical representation of the problem, and correctly work with the ratio and/or proportion). Evidence of the Mathematical Practice, (3) Construct viable arguments and critique the reasoning of others, is demonstrated by complete and accurate explanations. Evidence of the Mathematical Practice, (4) Model with mathematics, is demonstrated by representing the problem with ratios or proportions. Evidence of the Mathematical Practice, (6) Attend to precision, can include proper use of ratio notation and proper labeling of quantities. Evidence of the Mathematical Practice, (7) Look for and make use of structure, may be demonstrated by student recognition, e.g., that the multiples used in a table need not be of equal intervals (see table below) or that part b may be solved with a properly formed proportion, where the variable is equal to the numbers of hours a 1200-mile trip will take.

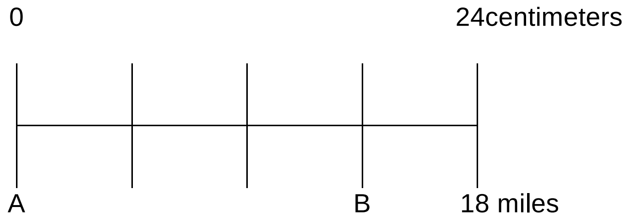
The reasoning used to solve the parts of the problem may include:

a. Failing to find a unit rate.
b. Choosing numbers other than 80 miles and 2 hours to scale down; but scaling down in tabular or fraction form to a unit rate consistent with both incorrect numbers chosen; or correctly using a proportion or proportional reasoning to find the unit rate, but choosing a distance other than 80 miles and a time other than 2 hours.
c. Using a proportion or proportional reasoning, but choosing a distance other than 80 miles and a time other than 2 hours.



$$\begin{array}{r} 80 \\ \times 10 \\ \hline 800 \end{array} \quad \begin{array}{r} 80 \\ \times 12 \\ \hline 960 \end{array} \quad \begin{array}{r} 80 \\ 13 \\ \hline 1040 \end{array} \quad \begin{array}{r} 80 \\ 14 \\ \hline 1120 \end{array} \quad \begin{array}{r} 80 \\ 15 \\ \hline 1200 \end{array}$$

2. On a map of the United States, 24 centimeters represents 18 miles, and the 24 centimeter segment is divided into four equal pieces, as shown in the picture below.



a. How many centimeters represent one mile?

b. How long is the line segment between A and B in centimeters? Use mathematical reasoning to justify your response.

c. If A and B represent two cities, what is the actual distance between the two cities? Use mathematical reasoning to justify your response.

NYC Grade 7 Assessment 1
Map Task
Benchmark Papers

3 Points

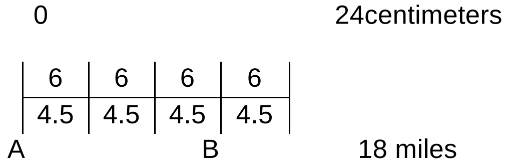
The response accomplishes the prompted purpose and effectively communicates the student's mathematical understanding. The student's strategy and execution meet the content (including concepts, technique, representations, and connections), thinking processes and qualitative demands of the task. Minor omissions may exist, but do not detract from the correctness of the response.

Either verbally or symbolically, the strategy used to solve each part of the problem is stated, as is the work used to find ratios, proportions, and partial answers to problems. Minor arithmetic errors may be present, but no errors of reasoning appear.

Accurate reasoning processes demonstrate the Mathematical Practices, (1) Make sense of problems and persevere in solving them and (2) Reason abstractly and quantitatively (since students need to abstract information from the problem, create a mathematical representation of the problem, and correctly work with unit rate, ratios and/or proportion). Evidence of the Mathematical Practice, (3) Construct viable arguments and critique the reasoning of others, is demonstrated by complete and accurate explanations. Evidence of the Mathematical Practice, (4) Model with mathematics, is demonstrated by representing the problem as pictorially, or with ratios and/or proportions. Evidence of the Mathematical Practice, (6) Attend to precision, can include proper use of ratio notation, proportion, and proper labeling of pictures and quantities. Evidence of the Mathematical Practice, (7) Look for and make use of structure may be demonstrated by student recognition that parts b and c of this problem may successfully be solved with a proportion. Evidence of the Mathematical Practice, (8) Look for and express regularity in repeated reasoning, may be demonstrated by student use of the same reasoning process (repeated calculations) in both parts b and c.

The reasoning used to solve the parts of the problem may include:

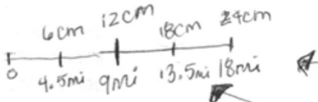
a. Dividing the line segment into equal-sized pieces and reasoning from the picture.
b. Forming the ratio 24 cm : 18 mi. and scaling down to a denominator of 1 or dividing 24 by 18; possibly drawing a picture first and using 6/4.5; possibly using similar reasoning in part c.
c. Noting that the ratio of line segment AB to the 24 cm segment is ¾, and finding ¾(24); using the proportion ¾ = x/24 then scaling 4 up to 24 and 3 up to 18 with tables or multiplication; or solving the proportion as an equation; possibly using similar reasoning in part c.



A. $\frac{24Cm}{18mi} = 1.\overline{3}$cm represents 1 mile

$$\begin{array}{r} 1.3333... \\ 18\overline{)24.00} \\ 18 \\ \hline 60 \\ 54 \\ \hline 60 \end{array}$$

B



the line Segment is 18cm

used the picture

C. the actual distance is 13.5mi

used my picture

NYC Grade 7 Assessment 1
Map Task
Benchmark Papers

2 Points
The response demonstrates adequate evidence of the learning and strategic tools necessary to complete the prompted purpose. It may contain overlooked issues, misleading assumptions, and/or errors in execution. Evidence in the response demonstrates that the student can revise the work to accomplish the task with the help of written feedback or dialogue.

Either verbally or symbolically, the strategy used to solve each part of the problem is stated, as is the work used to find ratios, proportions, and partial answers to problems. Minor arithmetic errors may be present. Reasoning may contain incomplete, ambiguous or misrepresentative ideas. Accurate reasoning processes demonstrate the Mathematical Practices, (1) Make sense of problems and persevere in solving them and (2) Reason abstractly and quantitatively (since students need to abstract information from the problem, create a mathematical representation of the problem, and correctly work with unit rate, ratios and/or proportion). Evidence of the Mathematical Practice, (3) Construct viable arguments and critique the reasoning of others, is demonstrated by complete and accurate explanations. Evidence of the Mathematical Practice, (4) Model with mathematics, is demonstrated by representing the problem as pictorially, or with ratios and/or proportions. Evidence of the Mathematical Practice, (6) Attend to precision, can include proper use of ratio notation, proportion, and proper labeling of pictures and quantities. Evidence of the Mathematical Practice, (7) Look for and make use of structure, may be demonstrated by student recognition that parts b and c of this problem may successfully be solved with a proportion. Evidence of the Mathematical Practice, (8) Look for and express regularity in repeated reasoning, may be demonstrated by student use of the same reasoning process (repeated calculations) in both parts b and c.

The reasoning used to solve the parts of the problem may include:

a. Dividing the line segment into equal-sized pieces and reasoning from the picture, but indicating, e.g., that the 6 cm. segment represents one mile.
b. Forming the ratio 18 cm : 24 mi. or 18 mi. : 24 cm and scaling down to a denominator of 1 or dividing 18 by 24; possibly drawing a picture first and using 4.5/6; possibly using similar reasoning in part c.
c. Incorrectly forming the ratio of line segment AB to the 24 cm segment, but then correctly finding that fraction of 24, or correctly forming the ratio of line segment AB to the 24 cm segment, but then finding that fraction of 18; possibly using scaling or proportions to do so; possibly using similar reasoning in part c.
d. Correctly attempting only two parts of the problem.

0 24centimeters

6	6	6	6
4.5	4.5	4.5	4.5

A B 18 miles

a. $\frac{24 \text{ mi}}{18 \text{ cm}} \div \frac{18}{18} = \frac{1.\overline{3} \text{ cm}}{1 \text{ mi}}$

1.3 cm represent 1 mile

b. The line segment between A and B is 4.5 cm

c. $\frac{1.3 \text{ cm}}{40.5 \text{ cm}} = \frac{1 \text{ mi}}{X}$

$\frac{\cancel{1.3x}}{\cancel{1.3}} = \frac{4.5}{1.3}$

$X = 3.46$

The actual distance between the two cities is 3.46 miles

NYC Grade 7 Assessment 1
Map Task
Benchmark Papers

1 Point

The response demonstrates some evidence of mathematical knowledge that is appropriate to the intent of the prompted purpose. An effort was made to accomplish the task, but with little success. Evidence in the response demonstrates that, with instruction, the student can revise the work to accomplish the task.

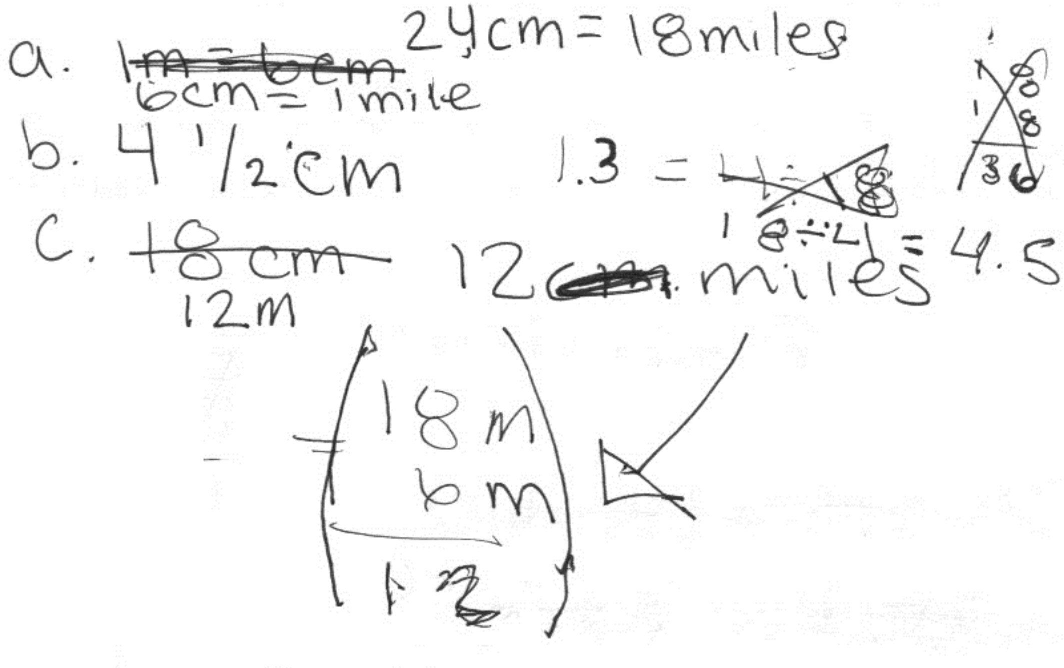
Some evidence of reasoning is demonstrated either verbally or symbolically, is often based on misleading assumptions, and/or contains errors in execution. Some work is used to find ratios or proportion or partial answers to portions of the task are evident. Accurate reasoning processes demonstrate the Mathematical Practices, (1) Make sense of problems and persevere in solving them and (2) Reason abstractly and quantitatively (since students need to abstract information from the problem, create a mathematical representation of the problem, and correctly work with unit rate, ratios and/or proportion). Evidence of the Mathematical Practice, (3) Construct viable arguments and critique the reasoning of others, is demonstrated by complete and accurate explanations. Evidence of the Mathematical Practice, (4) Model with mathematics, is demonstrated by representing the problem as pictorially, or with ratios and/or proportions. Evidence of the Mathematical Practice, (6) Attend to precision, can include proper use of ratio notation, proportion, and proper labeling of pictures and quantities. Evidence of the Mathematical Practice, (7) Look for and make use of structure, may be demonstrated by student recognition that parts b and c of this problem may successfully be solved with a proportion. Evidence of the Mathematical Practice, (8) Look for and express regularity in repeated reasoning, may be demonstrated by student use of the same reasoning process (repeated calculations) in both parts b and c.

The reasoning used to solve the parts of the problem may include:

a. Dividing the line segment into equal-sized pieces and reasoning from the picture, but indicating, e.g., that the 6 cm. or the 4.5 mi. segment represents answers to several parts of the problem.

b. Forming some appropriate ratios, but failing to scale appropriately.

c. Correctly attempting only one part of the problem.



a. ~~1m = 6cm~~ 24cm = 18miles
6cm = 1 mile

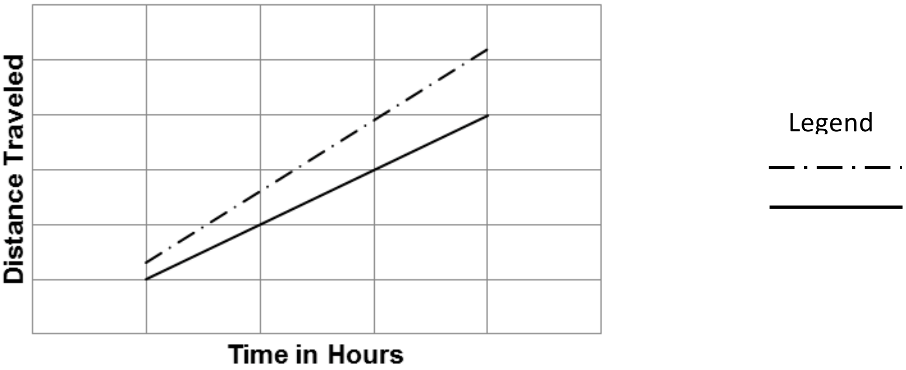
b. 4 1/2 cm   1.3 = ~~4÷18~~ 18÷4 = 4.5

c. ~~18 cm~~ 12 ~~cm~~ miles
12m

3. Jack and Jill raced cross-country on motor bikes. Jack drove 325 miles in 5 hours; Jill took 6 ½ hours to travel the same distance as Jack.

   a. Compute the unit rates that describe Jack's average driving speed and Jill's average driving speed. Show how you made your decisions.

   b. A portion of the graph of Jack and Jill's race appears below. Identify which line segment belongs to Jack and which belongs to Jill. Explain in words how you decided which line segment belongs to Jack and which belongs to Jill.



NYC Grade 7 Assessment 1
Jack and Jill’s Race Task
Benchmark Papers

3 Points

The response accomplishes the prompted purpose and effectively communicates the student's mathematical understanding. The student's strategy and execution meet the content (including concepts, technique, representations, and connections), thinking processes and qualitative demands of the task. Minor omissions may exist, but do not detract from the correctness of the response.

Either verbally or symbolically, the strategy used to solve the problem is stated, as is the work used to find ratios and unit rates. Minor arithmetic errors may be present, but no errors of reasoning appear. Complete explanations are stated, based on work shown.

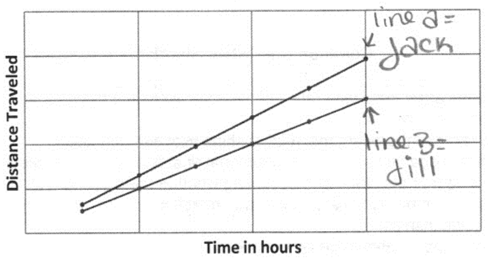
Accurate reasoning processes demonstrate the Mathematical Practices, (1) Make sense of problems and persevere in solving them and (2) Reason abstractly and quantitatively (since students need to abstract information from the problem, create a mathematical representation of the problem, and correctly work with unit rate/constant of proportionality.) Evidence of the Mathematical Practice, (3) Construct viable arguments and critique the reasoning of others, is demonstrated by complete and accurate explanations. Evidence of the Mathematical Practice, (4) Model with mathematics, is demonstrated by representing the problem with ratios and/or proportions. Evidence of the Mathematical Practice, (6) Attend to precision, can include proper use of ratio notation and proper labeling of quantities.

The reasoning used to solve part b of the problem may include:

a. Recognizing that the line rising more quickly must represent the faster biker.

b. Noting that, for any one or all points on the graph, those appearing on the upper line show a larger distance traveled in the same amount of time as those points on the lower line; may or may not reference that the x-coordinate represents amount of time traveled while the y-coordinate represents distance traveled.

c. Building a table of values for each biker and matching the table to the graph, possibly by choosing scales for the axes.



a. $\frac{325m}{5hrs}$ $\frac{65 miles}{1hr}$

325÷5÷65

Jack rode 65 miles Per hour.

a2. $\frac{325m}{6^{1/2}hrs} = \frac{x=50}{1}$

325÷65=50

Jill rode 50 miles Per hour

I think that line A
shows Jack's Progress
because it is moving upward
at a faster Pace than line B.
This is Because Jack rode faster than Jill.

NYC Grade 7 Assessment 1
Jack and Jill's Race Task
Benchmark Papers

2 Points

The response demonstrates adequate evidence of the learning and strategic tools necessary to complete the prompted purpose. It may contain overlooked issues, misleading assumptions, and/or errors in execution. Evidence in the response demonstrates that the student can revise the work to accomplish the task with the help of written feedback or dialogue.

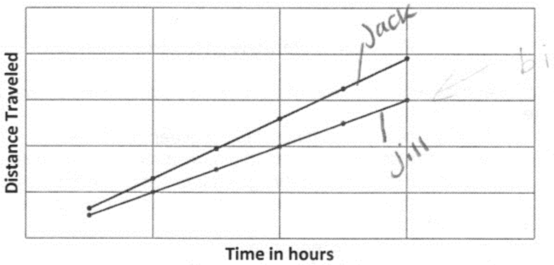
Either verbally or symbolically, the strategy used to solve the problem is stated, as is the work used to find ratios and unit rates. Reasoning may contain incomplete, ambiguous or misrepresentative ideas. Partial explanations are stated, based on work shown. Accurate reasoning processes demonstrate the Mathematical Practices, (1) Make sense of problems and persevere in solving them and (2) Reason abstractly and quantitatively (since students need to abstract information from the problem, create a mathematical representation of the problem, and correctly work with unit rate/constant of proportionality). Evidence of the Mathematical Practice, (3) Construct viable arguments and critique the reasoning of others, is demonstrated by complete and accurate explanations. Evidence of the Mathematical Practice, (4) Model with mathematics, is demonstrated by representing the problem with ratios and/or proportions. Evidence of the Mathematical Practice, (6) Attend to precision, can include proper use of ratio notation and proper labeling of quantities.

The reasoning used to solve part b of the problem may include:

a. Recognizing that the top line must represent the faster biker, but failing to note that the top line is rising more quickly.

b. Generally noting that the points appearing on the upper line show a larger distance traveled without noting that the larger distance is occurring in the same amount of time.

c. Building a table of values for each biker and matching the table to the graph, possibly by choosing scales for the axes; scales may be inappropriately chosen or tables of values may be arbitrary.



a. i $\frac{325\text{ mi}}{5\text{ hr}} \div \frac{5}{5} = \frac{65}{1\text{ hr}}$

Jack's average speed is 65 mi/hr

ii. $\frac{325\text{ mi}}{6.5\text{ hr}} \div \frac{6.5}{\div 6.5} = \frac{50}{1\text{ hr}}$

Jill's average driving speed is 50 mi/hr

b. ii I made my decision because Jack drives at a higher rate per hour

NYC Grade 7 Assessment 1
Jack and Jill's Race Task
Benchmark Papers

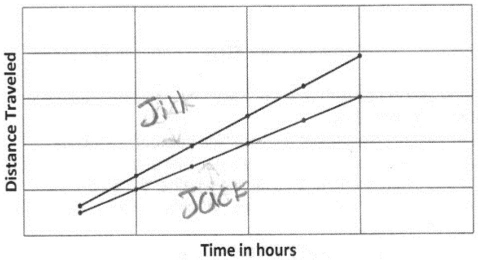
1 Point

The response demonstrates some evidence of mathematical knowledge that is appropriate to the intent of the prompted purpose. An effort was made to accomplish the task, but with little success. Evidence in the response demonstrates that, with instruction, the student can revise the work to accomplish the task.

Some evidence of reasoning is demonstrated either verbally or symbolically, is often based on misleading assumptions, and/or contains errors in execution. Some work is used to find ratios and unit rates or partial answers to portions of the task are evident. Explanations are incorrect, incomplete or not based on work shown. Accurate reasoning processes demonstrate the Mathematical Practices, (1) Make sense of problems and persevere in solving them and (2) Reason abstractly and quantitatively (since students need to abstract information from the problem, create a mathematical representation of the problem, and correctly work with unit rate/constant of proportionality). Evidence of the Mathematical Practice, (3) Construct viable arguments and critique the reasoning of others, is demonstrated by complete and accurate explanations. Evidence of the Mathematical Practice, (4) Model with mathematics, is demonstrated by representing the problem with ratios and/or proportions. Evidence of the Mathematical Practice, (6) Attend to precision, can include proper use of ratio notation and proper labeling of quantities.

The reasoning used to solve the parts of the problem may include:

a. Assuming proportionality rather than trying to find unit rate.
b. Reference to the length of the segments rather than distance and time.
c. Assuming the upper line suggests more time rather than a larger distance when compared to time.



a $\frac{5 \text{ hours}}{325 \text{ miles}} = \frac{6\frac{1}{2} \text{ hours}}{x}$

325

$\frac{5x}{5} =$ $325 \cdot 6\frac{1}{2} = 2112.5$

$x = 422.5$

b= I think the line at the bottom is Jack and the top is Jill because it took Jill $6\frac{1}{2}$ hours just to go as far as Jack while it took Jack 5 hours to go 325 miles

42.25
$\sqrt{2112.5}$
-841
-10
12
10
25
25
00

4. Reynaldo is planning to drive from New York to San Francisco in his car. Reynaldo started to fill out the table below showing how far in miles he can travel for each gallon of gas he uses.

Gallons	2	4		8	10	12
Miles	56		168	224		

Use the information in Reynaldo's table to answer the questions below.

a. Complete the table for Reynaldo. Assume the relationship in the table is proportional.

b. Based on the table, how many miles per gallon did Reynaldo's car get? Explain your reasoning in words.

c. Write an equation that Reynaldo can use to find the distance (d) he can drive on any number of gallons of gas (g).

d. When Reynaldo's tank is full, it holds 20 gallons. How far can Reynaldo drive on a full tank of gas?

3 Points

The response accomplishes the prompted purpose and effectively communicates the student's mathematical understanding. The student's strategy and execution meet the content (including concepts, technique, representations, and connections), thinking processes and qualitative demands of the task. Minor omissions may exist, but do not detract from the correctness of the response.

Either verbally or symbolically, the strategy used to solve the problem is stated, as is the work used to find ratio, unit rates, and equation. Minor arithmetic errors may be present, but no errors of reasoning appear. Complete explanations are stated, based on work shown.

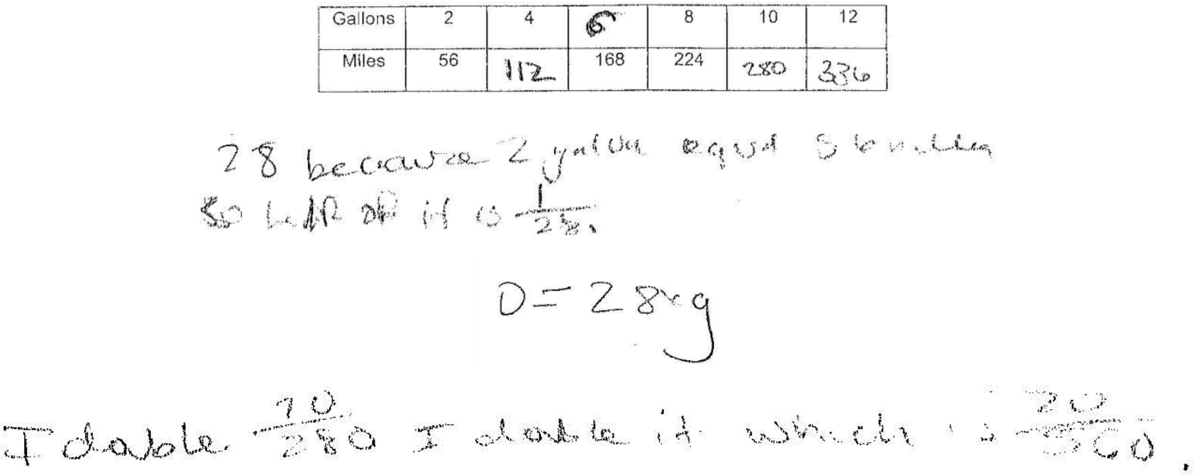
Accurate reasoning processes demonstrate the Mathematical Practices (1) Make sense of problems and persevere in solving them, and (2) Reason abstractly and quantitatively (since students need to abstract information from the problem, create a mathematical representation of the problem, and correctly work both with unit rate/constant of proportionality). Evidence of the Mathematical Practice, (3) Construct viable arguments and critique the reasoning of others, is demonstrated by complete and accurate explanations. Evidence of the Mathematical Practice, (4) Model with mathematics, is demonstrated by the linear equation shown in part c. Evidence of the Mathematical Practice, (6) Attend to precision, can include proper use of ratio and/or proportion notation, proper symbolism and proper labeling.

The reasoning used to solve the parts of the problem may include:

a. Indicating that the unit rate can be multiplied by the number of gallons to find the distance, and using that process.

b. Using a proportion or proportional reasoning (e.g., 2 times 10 equals 20, so I can multiply 56 by 10 to find distance).

c. Extending the table of values.



Gallons	2	4	6	8	10	12
Miles	56	112	168	224	280	336

28 because 2 galon equal 56 milles so half of it is $\frac{1}{28}$.

D = 28xg

I dable $\frac{10}{280}$ I dable it which is $\frac{20}{560}$.

NYC Grade 7 Assessment 1
Reynaldo's Drive Task
Benchmark Papers

2 Points

The response demonstrates adequate evidence of the learning and strategic tools necessary to complete the prompted purpose. It may contain overlooked issues, misleading assumptions, and/or errors in execution. Evidence in the response demonstrates that the student can revise the work to accomplish the task with the help of written feedback or dialogue.

Either verbally or symbolically, the strategy used to solve the problem is stated, as is the work used to find ratio, unit rates, proportionality and equation. Reasoning may contain incomplete, ambiguous or misrepresentative ideas. Partial explanations are stated, based on work shown. Accurate reasoning processes demonstrate the Mathematical Practices (1) Make sense of problems and persevere in solving them, and (2) Reason abstractly and quantitatively (since students need to abstract information from the problem, create a mathematical representation of the problem, and correctly work both with unit rate/constant of proportionality). Evidence of the Mathematical Practice, (3) Construct viable arguments and critique the reasoning of others, is demonstrated by complete and accurate explanations. Evidence of the Mathematical Practice, (4) Model with mathematics, is demonstrated by the linear equation shown in part c. Evidence of the Mathematical Practice, (6) Attend to precision, can include proper use of ratio and/or proportion notation, proper symbolism and proper labeling.

The reasoning used to solve the parts of the problem may include:

a. Indicating that the unit rate can be multiplied by the number of gallons to find the distance, and using that process, but using an incorrect unit rate or number of gallons.

b. Using a proportion or proportional reasoning incorrectly (e.g., 20 gallons is 2 times 10, so I can multiply 10 by 10 to find distance).

c. Extending the table of values, but failing to recognize the need to stop at 20 gallons.

Gallons	2	4	6	8	10	12
Miles	56	112	168	224	280	336

b. $\frac{56}{2} \div \frac{2}{2} = \frac{28\text{ mi}}{1\text{ gal}}$

Reynaldo used 28 miles per gallon

c. a constant rate of 28 miles per gallon

d= 28g

d. $\frac{1\text{gal}}{20\text{gal}} = \frac{28\text{ mi}}{x}$

Reynaldo can drive 560 miles on a full tank

$\frac{\cancel{1}x}{1} = \frac{560}{1}$

x = 560

NYC Grade 7 Assessment 1
Reynaldo's Drive Task
Benchmark Papers

1 Point

The response demonstrates some evidence of mathematical knowledge that is appropriate to the intent of the prompted purpose. An effort was made to accomplish the task, but with little success. Evidence in the response demonstrates that, with instruction, the student can revise the work to accomplish the task.

Some evidence of reasoning is demonstrated either verbally or symbolically, is often based on misleading assumptions, and/or contains errors in execution. Some work is used to find ratio, unit rates, proportionality and equation or partial answers to portions of the task are evident. Explanations are incorrect, incomplete or not based on work shown. Accurate reasoning processes demonstrate the Mathematical Practices (1) Make sense of problems and persevere in solving them, and (2) Reason abstractly and quantitatively (since students need to abstract information from the problem, create a mathematical representation of the problem, and correctly work both with unit rate/constant of proportionality). Evidence of the Mathematical Practice, (3) Construct viable arguments and critique the reasoning of others, is demonstrated by complete and accurate explanations. Evidence of the Mathematical Practice, (4) Model with mathematics, is demonstrated by the linear equation shown in part c. Evidence of the Mathematical Practice, (6) Attend to precision, can include proper use of ratio and/or proportion notation, proper symbolism and proper labeling.

The reasoning used to solve the parts of the problem may include:

a. No attempt at mathematical reasoning to respond to part b.
b. Some attempt to scale, but failure to maintain the ratio, typically by reverting to addition.
c. Failure to attempt at least two parts of the problem.

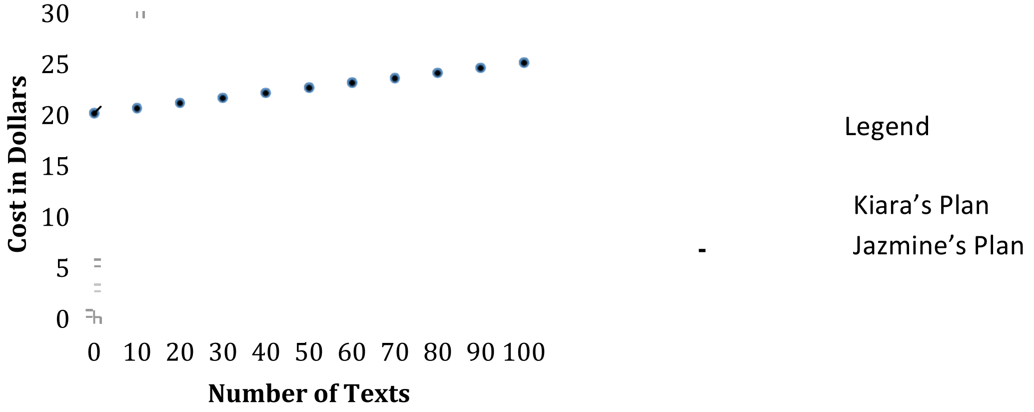
ⓐ

gallons	2	4	6	8	10	12
Miles	56	112	168	224	280	336

ⓑ He used the number 56.

ⓒ Keep adding 2 for the gallons and Keep adding 56 for the mites

5. The monthly cost of Jazmine's cell phone plan is graphed on the grid below.



Her friend Kiara selected a plan that charges 25¢ per text, with no monthly fee, because she only uses her phone for texting.

a. Write an equation to represent the monthly cost of Kiara's plan for any number of texts.

b. Graph the monthly cost of Kiara's plan on the grid above.

c. Using the graphs above, explain the meaning of the following coordinate pairs:

   i. (0, 20): ______________________________

   ii. (0, 0): ______________________________

   iii. (10, 2.5): ______________________________

   iv. (100, 25): ______________________________

d. When one of the girls doubles the number of texts she sends, the cost doubles as well. Who is it? Explain your reasoning in words.

3 Points

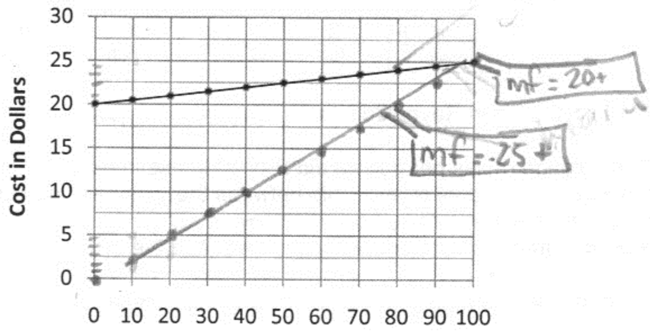
The response accomplishes the prompted purpose and effectively communicates the student's mathematical understanding. The student's strategy and execution meet the content (including concepts, technique, representations, and connections), thinking processes and qualitative demands of the task. Minor omissions may exist, but do not detract from the correctness of the response.

Either verbally or symbolically, the strategy used to solve the problem is stated, as is the work used to find equation and graph. Appropriately identify to which girl each coordinate pair is associated and refer to the x-coordinate as number of texts and the y-coordinate as cost in dollars. Minor arithmetic errors may be present, but no errors of reasoning appear. Complete explanations are stated, based on work shown.

Accurate reasoning processes demonstrate the Mathematical Practices, (1) Make sense of problems and persevere in solving them, and (2) Reason abstractly and quantitatively (since students need to abstract information from the problem, create a mathematical representation of the problem, and correctly work with the rations, proportions or proportional reasoning, and/or equations). Evidence of the Mathematical Practice, (3) Construct viable arguments and critique the reasoning of others, is demonstrated by complete and accurate explanations. Evidence of the Mathematical Practice, (4) Model with mathematics, is demonstrated by representing Kiara's plan as an equation, correctly representing her information graphically, and using proportions, tables and/or multiplication statements to determine the answer to part c. Evidence of the Mathematical Practice, (6) Attend to precision, can include proper use of ratio and or proportion notation and proper labeling of quantities. Evidence of the Mathematical Practices, (7) Look for and make use of structure, and (8) Look for and express regularity in repeated reasoning, may be demonstrated by student recognition from accurate responses to parts iii. and iv. -- that Kiara's plan is the only one that describes a proportional relationship OR from comparison of the two linear equations, and noting that Kiara's plan is the only one where $b = 0$, the indicator of a proportional relationship.

The reasoning used to solve the parts of the problem may include:

a. For part a, indicating that the unit rate 25¢ can be multiplied by the number of texts to find the cost.
b. For part d, correctly identifying Kiara as the girl whose cost doubles. Possible arguments for this result include:
    i. using a proportion or proportional reasoning if appropriate (e.g., for Kiara, 10 texts cost \$2.50 while 20 cost \$5. So, Kiara's cost doubles. For Jazmine, 10 texts cost \$20 while 20 cost a little more than \$20, not double \$20, or \$40.
    ii. using parts iii. and iv. as evidence, if correctly identified in part c.
    iii. building a table of values for each girl and observing the results.
    iv. noting only Kiara's line passes through (0, 0); citing as evidence that a line representing a proportional relationship must pass through (0, 0) and that, if a line doesn't pass through (0, 0), the relationship is not proportional. Explaining why that signifies a proportional relationship.
    v. finding the equation for Jazmine's plan and noting as evidence that, in the equation describing Jazmine's plan, $b \neq 0$ while in the equation describing Kiara's plan, $b = 0$; explaining why that signifies a proportional relationship.



a. Monthly fee = .25 t

i. (0, 20): 20 dollars per 0 text
ii. (0, 0): 0 dollars per 0 text
iii. (10, 2.5): 2.5 dollars per 10 texts
iv. (100, 25): 25 dollars per 100 texts

texts, t	0	10	20	30	40	50	60	70	80	90	100
Costs, c	20	20.5	21	21.5	22	22.5	23	23.5	24	24.5	25

d. Kiara is the girl because she is charged at a constant rate of .25 texts per hour Starting at 0,0.

2 Points

The response demonstrates adequate evidence of the learning and strategic tools necessary to complete the prompted purpose. It may contain overlooked issues, misleading assumptions, and/or errors in execution. Evidence in the response demonstrates that the student can revise the work to accomplish the task with the help of written feedback or dialogue.

Either verbally or symbolically, the strategy used to solve the problem is stated, as is the work used to find ratio, unit rates, proportionality and equation. Reasoning may contain incomplete, ambiguous or misrepresentative ideas. Partial explanations are stated, based on work shown.

Accurate reasoning processes demonstrate the Mathematical Practices, (1) Make sense of problems and persevere in solving them, and (2) Reason abstractly and quantitatively (since students need to abstract information from the problem, create a mathematical representation of the problem, and correctly work with the rations, proportions or proportional reasoning, and/or equations). Evidence of the Mathematical Practice, (3) Construct viable arguments and critique the reasoning of others, is demonstrated by complete and accurate explanations. Evidence of the Mathematical Practice, (4) Model with mathematics, is demonstrated by representing Kiara's plan as an equation, correctly representing her information graphically, and using proportions, tables and/or multiplication statements to determine the answer to part c. Evidence of the Mathematical Practice, (6) Attend to precision, can include proper use of ratio and or proportion notation and proper labeling of quantities. Evidence of the Mathematical Practices, (7) Look for and make use of structure, and (8) Look for and express regularity in repeated reasoning, may be demonstrated by student recognition from accurate responses to parts iii. and iv. -- that Kiara's plan is the only one that describes a proportional relationship OR from comparison of the two linear equations, and noting that Kiara's plan is the only one where $b = 0$, the indicator of a proportional relationship.

The reasoning used to solve the parts of the problem may include:

a. Possibly indicating that the unit rate 25¢ can be added repeatedly to find the cost.

b. For part c, reversing the meaning of the x- and y-variables in the explanation.

c. For part d, correctly identifying Kiara as the girl whose cost doubles, but failing to provide an adequate explanation as to why. Possible arguments for this result include:

    i. Inaccurately or inadequately using a proportion or proportional reasoning (e.g., Kiara's cost doubles because she pays 25¢ per text.)

    ii. Attempting to use parts iii. and iv. from part c as evidence, but failing to correctly identify information there.

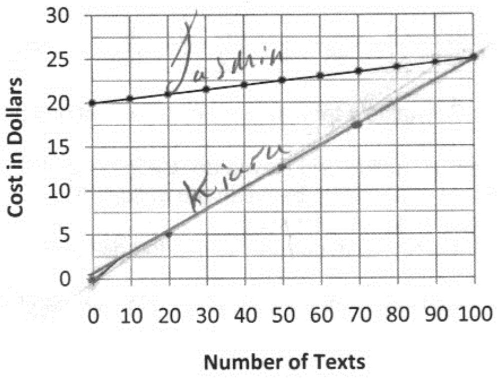
    iii. building a table of values for each girl and observing the results, but failing to cite evidence from the table in the explanation.

    iv. noting only Kiara's line passes through (0, 0); citing as evidence that a line representing a proportional relationship must pass through (0, 0) and that, if a line doesn't pass through (0, 0), the relationship is not proportional. Failing to explain why that signifies a proportional relationship.

    v. finding the equation for Jazmine's plan and noting as evidence that, in the equation describing Jazmine's plan, $b \neq 0$ while in the equation describing Kiara's plan, $b = 0$; failing to explain why that signifies a proportional relationship.

d. Failing to correctly answer at least three of the four parts of the problem.

NYC Grade 7 Assessment 1
Jazmine's Cell Phone Plan Task
Benchmark Papers



Kiara Pay = $0.25 x text

i. (0, 20): On Jasmin graph she is sending no text messages

ii. (0, 0): On Jasmin no text no bill.

iii. (10, 2.5): Ten text's on Jasmin

iv. (100, 25): 100 on Jasmin graph.

A.

Month	0	1	2	3	4
Bill	0	25$	50$	75$	100

D. Kiara because she pay $0.25 for every text so if she double the number or text's the bill doubles too.

NYC Grade 7 Assessment 1
Jazmine’s Cell Phone Plan Task
Benchmark Papers

1 Point

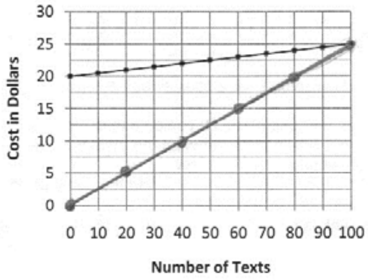
The response demonstrates some evidence of mathematical knowledge that is appropriate to the intent of the prompted purpose. An effort was made to accomplish the task, but with little success. Evidence in the response demonstrates that, with instruction, the student can revise the work to accomplish the task.

Some evidence of reasoning is demonstrated either verbally or symbolically, is often based on misleading assumptions, and/or contains errors in execution. Some work is used to find ratio, unit rates, proportionality and equation or partial answers to portions of the task are evident. Explanations are incorrect, incomplete or not based on work shown.

Accurate reasoning processes demonstrate the Mathematical Practices, (1) Make sense of problems and persevere in solving them, and (2) Reason abstractly and quantitatively (since students need to abstract information from the problem, create a mathematical representation of the problem, and correctly work with the rations, proportions or proportional reasoning, and/or equations). Evidence of the Mathematical Practice, (3) Construct viable arguments and critique the reasoning of others, is demonstrated by complete and accurate explanations. Evidence of the Mathematical Practice, (4) Model with mathematics, is demonstrated by representing Kiara’s plan as an equation, correctly representing her information graphically, and using proportions, tables and/or multiplication statements to determine the answer to part c. Evidence of the Mathematical Practice, (6) Attend to precision, can include proper use of ratio and or proportion notation and proper labeling of quantities. Evidence of the Mathematical Practices, (7) Look for and make use of structure, and (8) Look for and express regularity in repeated reasoning, may be demonstrated by student recognition from accurate responses to parts iii. and iv. -- that Kiara’s plan is the only one that describes a proportional relationship OR from comparison of the two linear equations, and noting that Kiara’s plan is the only one where b = 0, the indicator of a proportional relationship.

The reasoning used to solve the parts of the problem may include:

a. Possibly indicating that the unit rate 25¢ can be divided by the number of texts to find the cost.
b. For part c, reversing the meaning of the x- and y-variables in the explanation.
c. For part d, incorrectly identifying Jazmine as the girl whose cost doubles, but failing to provide an adequate explanation as to why.
e. Failing to correctly answer at least two of the four parts of the problem.



ⓐ C=0.25x T C=Cost, T=text

i. (0, 20): A Monthly plan For Jazmine's Plan
ii. (0, 0): 0 number OF text with No Charge
iii. (10, 2.5): The Number of text with Kiara
iv. (100, 25): The amount oF text From Jazmine and Kiara.

ⓑ

Dollar	25$	50$	75$	100$
texts	100	200	300	400

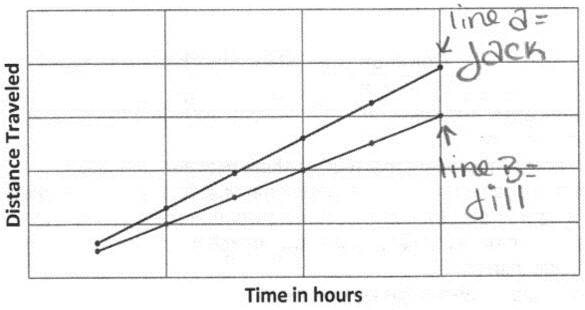
ⓒ No because the value OF one text
ⓓ IS not the Same as a dollar.

Seventh-Grade Assessment #1
Annotations of Student Work

Jack and Jill Task

3. Jack and Jill raced cross-country on motor bikes. Jack drove 325 miles in 5 hours; Jill took 6 ½ hours to travel the same distance as Jack.
   a. Compute the unit rates that describe Jack's average driving speed and Jill's average driving speed. Show how you made your decisions.
   b. A portion of the graph of Jack and Jill's race appears below. Identify which line segment belongs to Jack and which belongs to Jill. Explain in words how you decided which line segment belongs to Jack and which belongs to Jill.

Annotation of Student Work at a Score of 3



a. $\frac{325m}{5hrs}$ $\frac{65 miles}{1hr}$

$325 \div 5 = 65$

Jack rode 65 miles per hour.

a2. $\frac{325m}{6^{1/2}hrs} = \frac{x=50}{1}$

~~[illegible]~~ $325 \div 6.5 = 50$

Jill rode 50 miles per hour

I think that line A shows Jack's Progress because it is moving upward at a faster pace than line B. This is Because Jack rode faster than Jill.

Criterion & Score Point	Evidence	Instructional next steps. To meet CCSS Standards, this student needs to:
Mathematical Content		
The student has earned a three because the work shows proficiency with the entire intended content and mathematical practices identified for the task.		
7.RP.1 Compute unit rates associated with rates of fractions.	The student has found both Jack and Jill's average driving speed or unit rate by dividing the total number of miles by the total number of hours.	
7.RP.2 Recognize and represent proportional relationships between quantities.	The student sets up correct ratios and proportions and determines the unit rate, or miles per hour, for both Jack and Jill.	
7.RP.2b Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions.	The student determines the unit rate for Jack and Jill's trip from the verbal description in the problem. In Part b, the student associates the larger unit rate with the line having a steeper slope.	This student would benefit from experiences with a set of graphs that show axes labeled in the opposite way (i.e., horizontal axis is distance and vertical axis is time), in which the steeper slope would not be associated with the larger unit rate. This presentation would encourage the student to "grapple with" the interpretation of slope in different situations.
Mathematical Practices	Mathematical Practices Used	Next Steps With the Student
(1) Make sense of problems and persevere in solving them	The student makes sense of Jack and Jill's trips. The student has a pathway for solving the problem and arrives at the solution.	
(2) Reason abstractly and quantitatively	The student abstracts information from the context of the problem, represents it numerically by writing a ratio and a proportion, then correctly refers back to the context of the problem and accurately associates the steeper line with the faster speed.	
(3) Construct viable arguments and critique the reasoning of others	The student constructs an argument when he accurately claims that Jack has gone faster, but he does not deliberately defend his claim by referring to the information in part a. A more convincing argument might sound like, "I think that line A shows Jack's progress because the vertical change (distance traveled) in line A is greater than what is shown in line B."	The student could benefit from seeing models of writing in which a student has made a claim and provides evidence and reasoning from their solution path. For example, show a paper in which the student claims, "I think that line A shows Jack's progress because the horizontal change (elapsed time) is the same for both line segments but the vertical change (distance traveled) in line A is greater than what is shown in line B." Engage the student in a discussion of the difference between this response and the response he has given.
(4) Model with mathematics	The student models the context with ratios (in this case, unit rates), comparing distance to time for both Jack and Jill and with correct division statements to determine the unit rate. The student indicates that the ratios are equal, forming a proportion for Jill's situation. The student accurately makes connections between the ratios, the context and the information on the graph.	
(6) Attend to precision	The student accurately sets up a ratio and proportional relationship with quantities labeled for both Jack and Jill's trips. The student divides accurately, and labels unit rate correctly for the context of the problem.	

Jack and Jill Task

4. Jack and Jill raced cross-country on motor bikes. Jack drove 325 miles in 5 hours; Jill took 6 ½ hours to travel the same distance as Jack.
    a. Compute the unit rates that describe Jack's average driving speed and Jill's average driving speed. Show how you made your decisions.
    b. A portion of the graph of Jack and Jill's race appears below. Identify which line segment belongs to Jack and which belongs to Jill. Explain in words how you decided which line segment belongs to Jack and which belongs to Jill.

Annotation of Student Work at a Score of 2

$\frac{5}{325}$ $\frac{6.5}{325}$

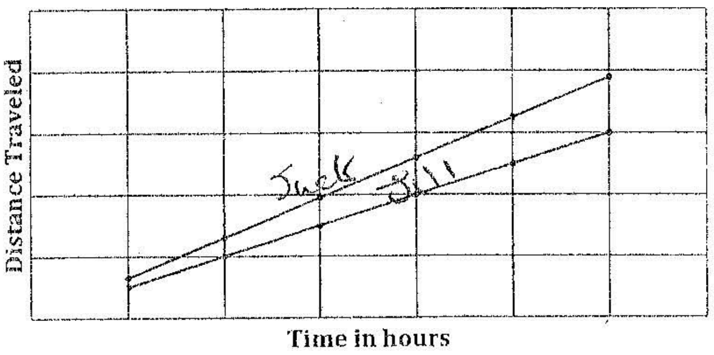
Jack = $\frac{1}{65}$ Jill $\frac{1}{50}$

Jack 65 miles per hour

Jill 50 miles every hour

325 ÷ 5 = 65

325 ÷ 6.5 = 50



Jack made it fatler in less time so he is on top Jill still took more time. Jill went less of a distance each hour

Criterion & Score Point	Evidence	Instructional next steps. To meet CCSS Standards, this student needs to:
Mathematical Content		
The student has earned a two because he has not demonstrated several of the content standards accurately. He has also not made use of several of the required mathematical practices.		
7.RP.1 Compute unit rates associated with ratios of fractions.	The student has determined both Jack and Jill's average driving speed or unit rate by dividing the total number of miles by the total number of hours.	
7.RP.2 Recognize and represent proportional relationships between quantities.	The student has set up ratios that permit him to solve the problem. Although the student correctly determines both Jack and Jill's average speed in miles per hour, the ratios the student has written compare time to distance rather than distance to time. The student's conclusion doesn't follow directly from the ratios they have written because, e.g., 1/50 does not equal 50 miles per hour.	The student's claim that Jack = 1/65 and Jill = 1/50 is counter to his argument that Jack travels 65 miles per hour. Challenging the student to explain what his ratios represent contextually can lead to the realization that 325/5 and 325/6.5 will better represent the context.
7.RP.2b Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions.	The student determines the unit rate for Jack and Jill's trip from the verbal description in the problem. In part (b), the student makes both accurate and inaccurate statements. e.g., "Jack made it faster in less time so he is on top." This is incorrect; Jack actually went the same distance as Jill in less time. The student is comparing time and time rather than distance and time. e.g., "Jill still took more time. Jill went less of a distance each hour." This is an accurate statement since the student is correctly comparing distance and time. It is the use of both accurate and inaccurate statements that do not permit us to determine what the student understands.	Display the student's response to prompt a class discussion (without identifying the student's name). Pose the questions, "What does it mean to say: "Jack made it faster in less time, so he is on top." "Jill still took more time. Jill went less of a distance each hour." Engage the class in a discussion and a revision of the statements. Following the discussion, "Step Back" and ask students how the original statements compare to the revised statements and what is the added value of the revisions. (Distance versus time comparison is needed in order to talk about average speed; simply saying, "more time" does not imply faster or slower; read and interpret the graph to know that Jill went a lesser distance each hour.)
Mathematical Practices	Mathematical Practices Used	Next Steps With the Student
(1) Make sense of problems and persevere in solving them	The student makes sense of Jack and Jill's trips. The student has a pathway for solving the problem and arrives at a correct solution to part a, but an ambiguous response to part b.	
(2) Reason abstractly and quantitatively	The student abstracts information from the context of a problem, represents it numerically, and competently works with unit rate because his paper shows a unit rate. He does not reason as well from a graphical representation. While the graph is labeled correctly, the explanation contains inaccuracies, which call into question exactly what the student understands.	
(3) Construct viable arguments and critique the reasoning of others	The student fails to support his arguments because he makes both accurate and inaccurate claims	See suggestion **7.RP.2b** above for ways to lead a whole-class discussion of the mathematical reasoning needed to explain the characteristics of the line segment that represents the faster average speed, and to support a claim related to Jack's and/or Jill's graph.

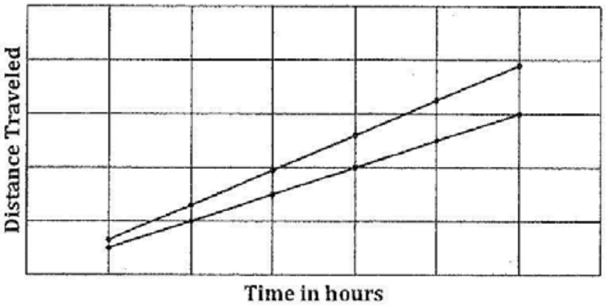
Mathematical Practices	Mathematical Practices Used	Next Steps With the Student
(4) Model with mathematics	The student models the context with inverted ratios, comparing time to distance for both Jack and Jill and with correct division statements to determine the unit rate. The student accurately makes connections between the ratios, the context and the information on the graph.	While the student accurately implies that 5/325 = 1/50 and 6.5/325 = 1/50 with the statement Jack = 1/65 and Jill = 1/50, the latter two statements are incorrect. He also recognizes with his labels that unit rate, in this instance, is indicated in miles per hour. As a result, the student should be asked to examine his model for finding unit rate and explain what his ratios represent; if necessary, to label his quantities, and consider whether hours/mile can be the same as miles per hour (i.e., whether 5/325 6.5/325, Jack = 1/65 and Jill = 1/50 properly model the context.)
(6) Attend to precision	The student accurately divides to determine unit rates and labels the average speed for both Jack and Jill's trips in miles per hour. He inaccurately states that Jill = 1/50 and Jack = 1/65.	See above. The student can be challenged to discuss how labeling his ratios can help him relate his work to the context of the problem.

Jack and Jill Task

5. Jack and Jill raced cross-country on motor bikes. Jack drove 325 miles in 5 hours; Jill took 6 ½ hours to travel the same distance as Jack.
   a. Compute the unit rates that describe Jack's average driving speed and Jill's average driving speed. Show how you made your decisions.
   b. A portion of the graph of Jack and Jill's race appears below. Identify which line segment belongs to Jack and which belongs to Jill. Explain in words how you decided which line segment belongs to Jack and which belongs to Jill.

Annotation of Student Work at a Score of 1

Jack $\begin{array}{r} 65 \\ 5\overline{)32^{2}5} \end{array}$ Jill $\begin{array}{r} 54.1 \\ 6\overline{)32^{2}5} \end{array}$



Jace race aprears longer then jill because his unit rate was higher then Jill

Criterion & Score Point	Evidence	Instructional next steps. To meet CCSS Standards, this student needs to:
Mathematical Content		
The student has earned a one because he has not demonstrated that he understands content standards and he has made use of very few of the mathematical practices.		
7.RP.1 Compute unit rates associated with ratios of fractions.	Although the student does not write ratios, he does select the appropriate operation of division to calculate the unit rate. There is an error in the second division problem, since the student uses 6 instead of 6.5 as the divisor, thus missing the target of this content standard.	Ask the student to describe the situation. Ask the student what the numbers his division problems represent in the context of the problem, and if he can use this to label his answer. Ask the student what he means when he says Jack's unit rate is higher than Jill's.
7.RP.2 Recognize and represent proportional relationships between quantities.	The student does not demonstrate that he recognizes a proportional relationship.	
7.RP.2b Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions.	The student uses the verbal description in the problem to write the two division problems in part a), one of which is appropriate to calculate the unit rate. In part b), he may be referring to these results as unit rates, although it is not clear that he has associated these quantities with the graph. The student associates "longer" with unit rate, thus missing the target of identifying unit rate in graphs.	The teacher might ask the student to write down everything he notices about the graph and what this tells us about the context of the problem. If many students share these misunderstandings, the students may benefit from a graphical exploration, both with paper and pencil and with graphing technology, to make observations about what an increasing unit rate does to the appearance of a graph., and why that might be the case.
Mathematical Practices	Mathematical Practices Used	Next Steps With the Student
(1) Make sense of problems and persevere in solving them	The student knows to divide the total number of miles by the hours. Although he has ignored the decimal in Jill's distance, he has used the correct operation to calculate the miles per hour.	The student would benefit from talking with peers about what is known in the problem and what he is to figure out. The student should be grouped with peers who are able to make sense of problems. Student teams should be taught methods of prompting their peers to interpret the information in problems.
(2) Reason abstractly and quantitatively	The student works with the correct distance and time for Jack's trip; however, not for Jill's trip. In part b), the student says Jack's unit rate is higher than Jill's, so he appears to know which amount is greater. The student makes an incorrect claim when he states that, "Jack's race appears longer."	The student would benefit from talking about what he knows in the problem, what he is attempting to figure out and how he might approach the problem. Once the student has attempted to solve the problem, the teacher should ask the student to refer to the meaning of his work within the context of the problem.
(3) Construct viable arguments and critique the reasoning of others	The student does not construct a viable argument. There is no evidence in his response that would indicate he is making sense of the graphical representation.	Because the student in not making sense of the problem, it is unlikely that he will share his reasoning related to the mathematical ideas. The student can benefit, however, from listening to others' reasoning and then explaining back how the reasoning of those students supports their claim. The student should also be asked to add on to his peers' reasoning. .
(4) Model with mathematics	The student models the context with division appropriately in this situation; however, his response does not indicate that he knows how to interpret his division results in the context of the problem, or how it might be useful in answering the question about interpreting the graph in part b.	Consistent, patterned ways of working with this student is needed so that the student internalizes a set of reflective questions, meant to guide him as he thinks through a task. Maintaining a written record of his thinking will also help the student move from his written description to formalizing and abstracting the mathematical ideas.
(6) Attend to precision	The student does not accurately read and select the numbers in the problem. He uses 6 instead of 6.5 as the divisor in part b. We do not know if the student understands the meaning of the quotients in the context of the problem because he used no units as labels. He has not labeled his graph, and he suggests that Jack traveled longer rather than faster than Jill.	The teacher should ask the student what his division problems mean in the context of the situation.

COMMON CORE-ALIGNED TASK
WITH INSTRUCTIONAL SUPPORTS

Mathematics

GRADE 7 MATH: PROPORTIONAL REASONING INSTRUCTIONAL SUPPORTS

These instructional supports include three arcs of related lessons: a sequence of high-level instructional tasks that address the set of targeted Common Core State Standards for Mathematical Content and Common Core State Standards Standards for Mathematical Practice assessed in the first performance-based assessment related to Ratios and Proportional Relationships. The tasks are designed to support student learning in preparation for Assessment #1. Each of the high-level instructional tasks are accompanied by a lesson guide.

The lesson guides (which begin on page 51) provide teachers with the mathematical goals of the lesson, as well as possible student solution paths, errors and misconceptions, and talk moves for engaging students in rigorous teaching and learning. Teachers may choose to use them to support their planning and instruction while teaching the arcs of lessons.

7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.

7.RP.2 Recognize and represent proportional relationships between quantities.

- (a) Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
- (b) Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
- (c) Represent proportional relationships by equations.
- (d) Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation.

7 RP.3 Use proportional relationships to solve multistep ratio and percent problems.

Essential Understandings of Ratios, Proportions & Proportional Reasoning (NCTM)

1. Reasoning with ratios involves attending to and coordinating two quantities.
2. A ratio is a multiplicative comparison of two quantities, or it is a joining of two quantities in a composed unit.
3. Forming a ratio as a measure of a real-world attribute involves isolating that attribute from other attributes and understanding the effect of changing each quantity on the attribute of interest.
4. A number of mathematical connections link rations and fractions:
    a. Ratios are often expressed in fraction notation, although ratios and fractions do not have identical meaning.
    b. Ratios are often used to make “part-part” comparisons, but fractions are not.
    c. Ratios and fractions can be thought of as overlapping sets.
    d. Ratios can often be meaningfully reinterpreted as fractions.
5. Ratios can be meaningfully reinterpreted as quotients.
6. A proportion is a relationship of equality between two ratios. In a proportion, the ratio of two quantities remains constant as the corresponding values of the quantities change.
7. Proportional reasoning is complex and involves understanding that:
    a. Equivalent ratios can be created by iterating and/or partitioning a composed unit;
    b. If one quantity in a ratio is multiplied or divided by a particular factor, then the other quantity must be multiplied or divided by the same factor to maintain the proportional relationship; and
    c. The two types of ratios – composed units and multiplicative comparisons – are related.
8. A rate is a set of infinitely many equivalent ratios.
9. Several ways of reasoning, all grounded in sense making, can be generalized into algorithms for solving proportion problems.
10. Superficial cues present in the context of a problem do not provide sufficient evidence of proportional relationships between quantities.

Flowers
7 RP.2a, b, c maybe

EU #1, 2, 3, 4, 6, 8

The table below shows the price for buying bunches of mix-and-match flowers at a local supermarket.

Number of Bunches	3	6	9	12	15
Price	9	18	27	36	45

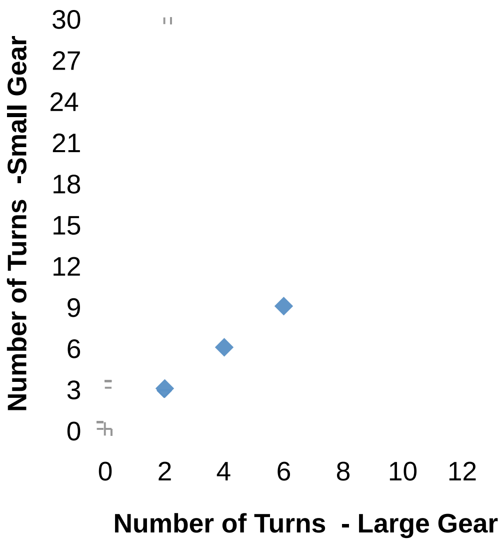
1. Generate several ratios of price to number of bunches. Describe any patterns you see.
2. Predict the price for buying 100 bunches. Explain how you made your decisions.
3. Predict how many bunches you can but with $63. Explain how you made your decisions.

(Use to define proportional relationships)

Gears
7 RP.1 maybe
7 RP.2a, b, d

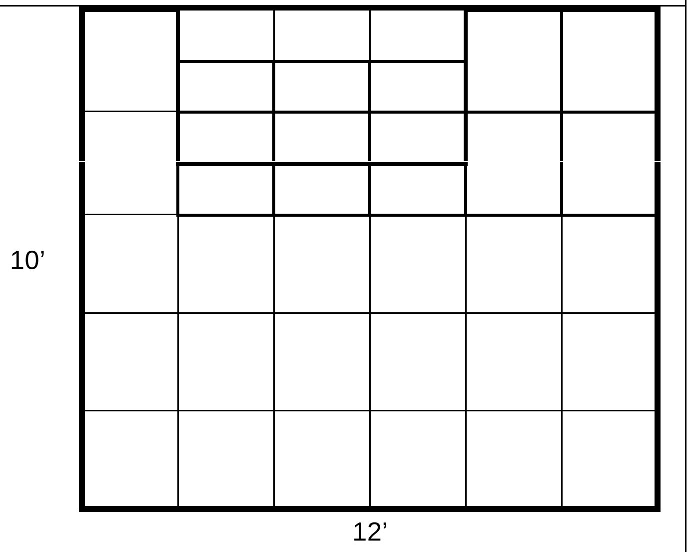
EU #1, 2, 3, 4, 6, 8

Aaron collected data on the number of times a pair of interlocking gears like those shown above turned each minute. The graph below shows his data.



1. Generate several ratios comparing the number of times the large gear turned compared to number of times the small gear turned. Describe any patterns you see.
2. Write the multiplicative inverse of any ratio from #1 and explain what it tells you.
3. Add three points to the graph that the gears will generate and explain how you know they belong on the graph.

(Use to test for proportional relationships)

Bicycle Shop 7 RP.2, 3 EU #1, 2, 3, 4, 6, 8, 9	Two bicycle shops build custom-made bicycles. Bicycle City charges \$160 plus \$80 for each day that it takes to build the bicycle. Bike Town charges \$120 for each day that it takes to build the bicycle. For what number of days will the charge be the same at each store? *(Use to distinguish proportional relationships from those that are not.)*
Bedroom 7 RP.1 7 RP.2 b 7 RP.3 EU #1, 2, 3, 5, 6, 7b	The picture to the right represents a 10’ by 12’ rectangular bedroom. a. A rectangle representing Shantia’s bed is located at the top of the picture. Determine the actual size of her bed and explain how you made your decisions. b. The rectangle representing the bedroom measures 3” in width and 2 ½ inches in height. What is the actual length of the line segments representing Shantia’s bed? c. Locate a rectangular dresser in Shantia’s room. Describe how to determine the length of the line segments representing the dresser and the actual length and width of the dresser. 

7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.

7.RP.2 Recognize and represent proportional relationships between quantities.

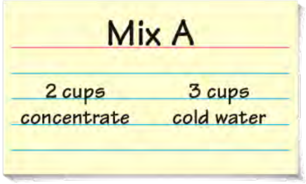
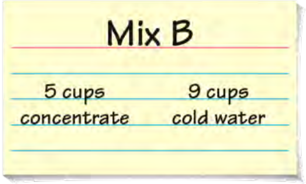
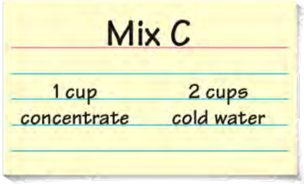
- (a) Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
- (b) Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
- (c) Represent proportional relationships by equations.
- (d) Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation.

7 RP.3 Use proportional relationships to solve multistep ratio and percent problems.

Essential Understandings of Ratios, Proportions & Proportional Reasoning (NCTM)

1. Reasoning with ratios involves attending to and coordinating two quantities.
2. A ratio is a multiplicative comparison of two quantities, or it is a joining of two quantities in a composed unit.
3. Forming a ratio as a measure of a real-world attribute involves isolating that attribute from other attributes and understanding the effect of changing each quantity on the attribute of interest.
4. A number of mathematical connections link rations and fractions:
    a. Ratios are often expressed in fraction notation, although ratios and fractions do not have identical meaning.
    b. Ratios are often used to make “part-part” comparisons, but fractions are not.
    c. Ratios and fractions can be thought of as overlapping sets.
    d. Ratios can often be meaningfully reinterpreted as fractions.
5. Ratios can be meaningfully reinterpreted as quotients.
6. A proportion is a relationship of equality between two ratios. In a proportion, the ratio of two quantities remains constant as the corresponding values of the quantities change.
7. Proportional reasoning is complex and involves understanding that:
    a. Equivalent ratios can be created by iterating and/or partitioning a composed unit;
    b. If one quantity in a ratio is multiplied or divided by a particular factor, then the other quantity must be multiplied or divided by the same factor to maintain the proportional relationship; and
    c. The two types of ratios – composed units and multiplicative comparisons – are related.
8. A rate is a set of infinitely many equivalent ratios.
9. Several ways of reasoning, all grounded in sense making, can be generalized into algorithms for solving proportion problems.
10. Superficial cues present in the context of a problem do not provide sufficient evidence of proportional relationships between quantities.

<table>
<tr>
<td>Weight on the Moon
7 RP.1
7 RP.2 c
7 RP.3
EU #1, 2, 3, 4, 8</td>
<td>Physics tells us that weights of objects on the moon are proportional to their weights on Earth. Suppose a 180-pound man weighs 30 pounds on the moon. What will a 60-pound boy weigh on the moon?</td>
</tr>
<tr>
<td>Ounces of Coffee
7 RP.1
7 RP.2 a, b
EU #1, 2, 3, 4, 8, 9</td>
<td>Julia made observations about the selling price of a new coffee that sold in three different-sized bags. She recorded those observations in the following table:
<table>
<tr><td>Ounces of Coffee</td><td>6</td><td>8</td><td>16</td></tr>
<tr><td>Price in Dollars</td><td>2.40</td><td>3.20</td><td>6.40</td></tr>
</table>
a) Is there a proportional relationship between the amount of coffee and the price? Why or why not?
b) Find the unit rates associated with this problem.
c) Explain in writing what the unit rates mean in the context of this problem.
d) Explain in writing why it is helpful for Julia to determine if the relationship between the amount of coffee and the price is proportional before she buys a bag of the new coffee.</td>
</tr>
</table>

Mixing Juice 7 RP.1 7 RP.3 EU #1, 2, 3, 4, 6, 7, 8	Julia and Mariah attend summer camp. Everyone at the camp helps with the cooking and cleanup at meal times. One morning, Julia and Mariah make orange juice for all the campers. They plan to make the juice by mixing water and frozen orange juice concentrate. To find the mix that tastes best, they decide to test some mixes.     **Developing Comparison Strategies** A. Which mix will make juice that is the most “orangey”? Explain. B. Which mix will make juice that is the least “orangey”? Explain. C. Which comparison statement is correct? Explain. a. $\frac{5}{9}$ of Mix B is concentrate $\frac{5}{14}$ of Mix B is concentrate D. Assume that each camper will get $\frac{1}{2}$ cup of juice. 1. For each mix, how many batches are needed to make juice for 240 campers? 2. For each mix, how much concentrate and how much water are needed to make juice for 240 campers?
Investing Money 7 RP.2 c 7 RP.3 EU #5, 6, 7, 9	Ray and Crystal buy and sell bicycle parts in their neighborhood. Because they invested money in this small business in a ratio of 2:3, they will split the profit in a ratio of 2:3. If the profit from the business is $1000, how much money will each person receive? Show how to use a model to solve the problem.
Light Bulbs 7 RP.1 7 RP.2 c 7 RP.3 EU #5, 6, 7, 9	Alazar Electric Company sells light bulbs to major outlets like Home Depot, Sears, Walmart and other big chains. They sample their bulbs for defects routinely. a. A sample of 96 light bulbs consisted of 4 defective ones. Assume that today’s batch of 6,000 light bulbs has the same proportion of defective bulbs as the sample. Determine the total number of defective bulbs made today. b. The big businesses they sell to accept no larger than a 4% rate of defective bulbs. Does today’s batch meet that expectation? Explain how you made your decisions.

7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.

7.RP.2 Recognize and represent proportional relationships between quantities.

- (a) Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
- (b) Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
- (c) Represent proportional relationships by equations.
- (d) Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation.

7 RP.3 Use proportional relationships to solve multistep ratio and percent problems.

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4. A number of mathematical connections link rations and fractions:
   a. Ratios are often expressed in fraction notation, although ratios and fractions do not have identical meaning.
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   c. Ratios and fractions can be thought of as overlapping sets.
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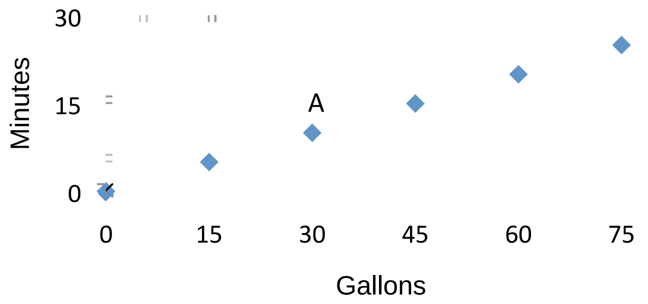
   a. Equivalent ratios can be created by iterating and/or partitioning a composed unit;
   b. If one quantity in a ratio is multiplied or divided by a particular factor, then the other quantity must be multiplied or divided by the same factor to maintain the proportional relationship; and
   c. The two types of ratios – composed units and multiplicative comparisons – are related.
8. A rate is a set of infinitely many equivalent ratios.
9. Several ways of reasoning, all grounded in sense making, can be generalized into algorithms for solving proportion problems.
10. Superficial cues present in the context of a problem do not provide sufficient evidence of proportional relationships between quantities.

Trading Cards 7 RP.2 7 RP.3 EU #1, 2, 3, 4, 6, 7, 8	If the cost of trading cards is two packs for $6, how much will it cost to buy 10 packages? *Teaching with Curriculum Focal Points, Focus in Grade 7*, NCTM. See book for lesson suggestions.
Melinda and Akira's Walk 7 RP.1 7 RP.2 c 7 RP.3 EU #1, 2, 3, 4, 6, 7, 8, 9	Melinda and her sister Akira are walking around the track at school. Melinda and Akira walk at a steady rate and Melinda walks 5 feet in the same time that Akira walks 2 feet. a) Set up a table and draw a graph to represent this situation. Let the x-axis represent the number of feet that Melinda walks and the y-axis represent the number of feet that Akira walks. b) When Melinda walks 45 feet, how far will Akira walk? Explain in writing or show how you found your answer.
Taking a Shower 7 RP.1 7 RP.2 a c d 7 RP.3 EU #1, 2, 3, 4, 5, 6, 7, 8, 9	The graph below shows the amount of time a person can shower with a certain amount of water.  a) Does the graph represent a proportional relationship? Why or why not? b) How long can a person shower with 15 gallons of water? With 60 gallons of water? c) What information does point A on the graph tell us about the amount of time a person can shower with a certain amount of water? d) Explain how to find a unit rate associated with this graph. e) Write the equation of the line relating the time in minutes, m, a person can shower with a given amount of water, w.

BICYCLE SHOP
SEVENTH GRADE LESSON GUIDE

LESSON OVERVIEW:

The Bicycle Shop task asks students to identify the constant of proportionality and identify graphically, in a table or algebraically the solution to a system of linear relationships.

Note: Expand -- what is the purpose of the task – i.e. what mathematical ideas will students grapple with via engaging in the task?

COMMON CORE STATE STANDARDS:

- **7.RP.2** Recognize and represent proportional relationships between quantities.
  - a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
  - b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
  - c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn*.
    1. Explain what a point *(x, y)* on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r)* where *r* is the unit rate.
- **7.RP.3** Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

NCTM ESSENTIAL UNDERSTANDINGS[1]:

1. Reasoning with ratios involves attending to and coordinating two quantities.
2. A ratio is a multiplicative comparison of two quantities, or it is a joining of two quantities in a composed unit.
3. Forming a ratio as a measure of a real-world attribute involves isolating that attribute from other attributes and understanding the effect of changing each quantity on the attribute of interest.
4. A number of mathematical connections link rations and fractions:
   a. Ratios are often expressed in fraction notation, although ratios and fractions do not have identical meaning.
   b. Ratios are often used to make “part-part” comparisons, but fractions are not.
   c. Ratios and fractions can be thought of as overlapping sets.
   d. Ratios can often be meaningfully reinterpreted as fractions.
6. A proportion is a relationship of equality between two ratios. In a proportion, the ratio of two quantities remains constant as the corresponding values of the quantities change.
8. A rate is a set of infinitely many equivalent ratios.

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[1] NCTM (2010) Developing Essential Understandings of Ratios, Proportions & Proportional Reasoning: Grades 6 -8.

DRIVING QUESTIONS:

- How can we decide if two quantities are in a proportional relationship using a context, table, graph and equation?
- How can we find the solution to a system of linear equations using a table or a graph?
- What does the solution to a system of linear equation mean in the context of a problem?

SKILLS DEVELOPED:

Students will be able to:.

- *determine whether or not two quantities are in a proportional relationship using a variety of representations*
- *identify the constant of proportionality in a table, context, graph and equation.*
- *find the solution to a system of linear equations using a table or a graph.*
- *Interpret the meaning of the solution to a system of linear equations within the context of a problem.*

MATERIALS:
“Bicycle Shop” Task, Document Projector or Chart Paper

GROUPING:
Students will begin their work individually, but will then work in pairs or triads.

SET-UP

Instructions to Students:

Using either a document reader or overhead projector present the task to the class. Have one student read the question that follow the graph and tabular representations.

Ask the students: “What do you know?” “What is the question asking you?”

Inform the students that there are several ways to get the answers to the questions asked. First each individual must work alone for at least 5 minutes after which they will share their initial findings with their group. Then they will continue to work out a common solution.

Expectations that all students must adhere to: explain their thinking and reasoning, use correct mathematical language and symbols in their explanations or solutions, justify their solutions, make sense of other students’ explanations; seek help from the teacher or students when they do not understand.

EXPLORE PHASE: Supporting Students’ Exploration of the Mathematical Ideas

Private Think Time: Allow students to work individually for 3-5 minutes without intervening, though you might want to circulate quickly to get an idea of the strategies that they are using.

Small-Group Work: After 3-5 minutes, ask students to work with their partner or in their small groups. As students are working, circulate around the room. Be persistent in:

- asking questions related to the mathematical ideas, problem-solving strategies, and connections between representations.
- asking students to explain their thinking and reasoning.
- asking students to explain in their own words, and build onto, what other students have said.

As you circulate, identify solution paths that you will have groups share during the Share, Discuss, Analyze Phase, and decide on the sequence that you would like for them to be shared. Give groups a “heads up” that you will be asking them to come to the front of the room. If a document reader is not available, give selected groups an OVH transparency or chart paper to write their solution on.

Possible Solution Paths

1. Making a table

Number of Days	Bike City	Bike Town
0	160	0
1	240	120
2	320	240
3	400	360
4	480	480
5	560	600
6	640	720

Possible Assessing and Advancing Questions

Assessing Questions

- What do the numbers represent in your table?
- How did you determine the numbers in your table?
- How does the table help you to solve the problem?

Advancing Questions

- Will there be another day at which two stores will charge the same amount? How do you know?
- Is there a proportional relationship between the number of days and the charge for either of the bike stores? How do you know?
- How do we see the daily rate for each of the bike shops in the table?

2. Drawing a graph

Assessing Questions

- What do the two lines represent in your graph?
- How does the graph help you to solve the problem?

Advancing Questions

- What do the points (0, 0) and (0, 160) mean the context of the problem?
- Is there a proportional relationship between the number of days and the charge for either of the bike stores? How do you know?
- What does the point (4,480) mean in the context of the problem?
- How do we see the daily rate for each of the bike shops in the graph?
- Will there be another day at which two stores will charge the same amount? How do you know?

3. Algebraic Solution (IF NO GROUPS ATTEMPT AN ALGEBRAIC SOLUTION, IT IS NOT NECESSARY TO PRESS FOR IT AT THIS TIME) Bike City: $80x + 160 = y$ Bike Town: $120x = y$ $80x + 160 = 120x$ $\underline{-80x \qquad - 80x}$ *Step 1: Subtract 80x from both sides of the equal sign* $\frac{160}{40} = \frac{40x}{40}$ *Step 2: Divide both sides by 40* $4 = x$ *It will cost the same at Day 4.* **NOTE:** The algebraic solution will not be discussed during the SDA phase since this was not a standard identified for this task. Explain to the students that you will just ask them to share, and explain, their equations. Return to the algebraic solution for this task when you move to solving systems of linear equations.	**Assessing Questions** • What do the two equations mean in the context of the problem? What does x represent? y? What do the 80, 160, and 120 represent? • Why did you make the equations equal? • What does the solution 4 = x mean in the context of the problem? **Advancing Questions** • Do either of these equations represent a proportional relationship? How do you know? • How can we find the rate of change in the equations? • Are either of the rates of change also a constant of proportionality? How do you know?
Possible Errors and Misconceptions	**Possible Questions to Address Errors and Misconceptions**
Graphing Errors: - Inconsistent intervals - plotting coordinates incorrectly Mistaking Bike City as proportional since it has a constant rate of change. Thinking that any rate of change is also a constant of proportionality Incorrect equations	**Assessing Questions** • What have you done so far? **Advancing Questions** • What elements do graphs need to have in place in order to be accurate? • Why does it help to plot this data on the same graph? • Is there another way you can show the relationship between the x and y values for Bike City? Bike Town? • Can you describe some similarities and differences between the two graphs? • What's the rate of change (constant of proportionality)? What is value of y when x equals 0? • We've looked at graphs that are proportional. How is Bike City's graph different from these? (Have graphs ready that show proportionality)
SHARE DISCUSS ANALYZE PHASE: Deepening and Connecting Mathematical Understanding	

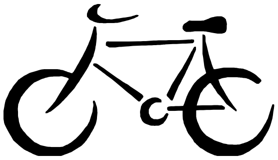
General Considerations:

- Orchestrate the class discussion so that it builds on, extends, and connects the thinking and reasoning of students
- Sequence the solution paths so that you will be able to press students to make comparisons and connections across, and between, the various solution paths.

Possible Sequence of Solution Paths	**Possible Questions and *Possible Student Responses***
1. Start with a solution using a table	**Explain your group's solution.** • *We started with one day for Bicycle City and found our how much it would cost for that day. Then we decided to continue adding up to 6 days. After that we did the same thing for Bike Town only stopping when Bike Town. We found that on day 4 they both had the same charge.* **Are either of these relationships proportional? If so, how can you tell that by looking at the table? At the context?** • *The charge for Bike Town is a proportional relationship because on day one it doesn't cost anything.* • *You can also tell that bike town is proportional because when you double the number of days you double the charge. That doesn't happen for Bike City.* • *Bike City isn't proportional because you start out with a $160 charge that gets added on.*
2. Have students share their graphical solution	**What does your graph represent?** • *We thought that it would be an easy solution and we would be able to see when the two bike companies have the same day and charge.* **How can you tell when the two bike shops charge the same charge by looking at the graph?** • *When the two lines cross that's when the two shops charge the same amount for the same number of days.* **How can you tell if either of the relationships is proportional by looking at the graph?** • *I can tell that Bike Town's charge is a proportional relationship because the graph starts at (0,0).* **From the graph can you tell if the two companies will ever have the same day and charge again?** • *The lines won't ever meet again. The lines are going at different slants because Bike Town charges more for each day than Bike City.*

3. Have students share their equations **NOTE:** For the purpose of this lesson, focus ONLY on the equations themselves during the SDA phase, not the algebraic solution of systems of linear equations.	**Explain how you arrived at the two equations.** • *Since Bike City starts with a $160 charge and then adds $80 for each day, I came up with the equation 80x + 160 = y. x is the number of days it takes to build the bike. Bike Town just charges $120 per day so their equation is 120x = y.* **How can we tell which relationship is proportional by looking at the equations?** • *We can tell that Bike Town is a proportional relationship because there's nothing added. The charge will always be 120 times the number of days.* **So what is the constant of proportionality for Bike Town?** • *The constant of proportionality is 120. The charge will always be 120 times the number of days.* **Bike City charges $80 per day. Why isn't that also a constant of proportionality?** • *You can't just multiply the number of days by 80 to find Bike City's charge. You also have to add $120. A constant of proportionality is always a multiple.*
4. Look across the different representations	**We have seen three representations – tables, graphs, and equations. How do they each help us to determine if a relationship is proportional?** • *We can see in all of them that Bike Town charges $0 for 0 days. We can see that in the first row of the table. We see that on the graph because the line goes through (0,0). We see that in the equation because when you multiply a number by zero you end up with zero, and you're not adding anything else.* • *We can also see that the charge for Bike Town is always $120 times the number of days. In the table you can multiply to check. In the equation it's 120x. It's a little harder in the graph, but you see the line always goes up the same amount. It doesn't curve.* • *You don't see any of these for Bike City.*

CLOSURE
Quickwrite: How can you tell if a relationship is proportional by looking at the context, table, graph and equation?
Possible Assessment: • .Look at different graphs and identify which are proportional and which are not with explanations.
Homework: • .Similar problem with different numbers



Bicycle Shop

Two bicycle shops build custom-made bicycles. Bicycle City charges $160 plus $80 for each day that it takes to build the bicycle. Bike Town charges $120 for each day that it takes to build the bicycle.

For what number of days will the charge be the same at each shop?

MIXING JUICE TASK
LESSON GUIDE

LESSON OVERVIEW

In the *Mixing Juice* task, students encounter an open-ended problem where they are asked to compare the "orangeyness" of four drink mixes. Students will likely approach the task using a range of different strategies, making comparisons among the mixes with ratios, percents, and fractions. Students will investigate how ratios can be formed and scaled up to find equivalent ratios. In addition, students will use proportional reasoning to decide how to use the different mixes to make juice for 240 people.

The strategies for comparing the mixes will be compared and connected during the whole-group discussion. Students should be able to see how each form, ratios, percents, and fraction, provides information needed to derive one of the other forms.

Before working on the *Mixing Juice* task, the Warm-Up task *Comparing by Using Ratios* will focus students' attention on different ways to form ratios and different notations for ratios. Allow 15 to 20 minutes for students to engage in and discuss the Warm-Up task. The *Mixing Juice* task is a challenging problem for students and at least 1.5 class periods should be allowed for students to explore and have a whole-class discussion on the task.

COMMON CORE STATE STANDARDS

- **6.RP.1** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.
- **6.RP.2** Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship.
- **6.RP.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
    - c) Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity).
- **7.RP.1** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units.
- **7.RP.3** Use proportional relationships to solve multistep ratio and percent problems. *Example: percent increase*.

ESSENTIAL QUESTIONS:

- What are different types of ratios and how are ratios used to make comparisons?
- What strategies can be used to compare ratios?
- How are ratios related to fractions?

NCTM ESSENTIAL UNDERSTANDINGS[2]:

5\. Reasoning with ratios involves attending to and coordinating two quantities.

6\. A ratio is a multiplicative comparison of two quantities, or it is a joining of two quantities in a composed unit.

7\. Forming a ratio as a measure of a real-world attribute involves isolating that attribute from other attributes and understanding the effect of changing each quantity on the attribute of interest.

8\. A number of mathematical connections link ratios and fractions:

  - Ratios are often expressed in fraction notation, although ratios and fractions do not have identical meaning.
  - Ratios are often used to make “part-part” comparisons, but fractions are not.
  - Ratios can often be meaningfully reinterpreted as fractions.

9\. Ratios can be meaningfully reinterpreted as quotients.

7\. Proportional reasoning is complex and involves understanding that:

  - Equivalent ratios can be created by iterating and/or partitioning a composed unit;
  - If one quantity in a ratio is multiplied or divided by a particular factor, then the other quantity must be multiplied or divided by the same factor to maintain the proportional relationship.

SKILLS DEVELOPED:

- Use different representations to form ratios and make comparisons with ratios.
- Use visual and numerical strategies to compare ratios.
- Form equivalent ratios and use equivalent ratios to solve problems.

MATERIALS:
Warm-Up task *Comparing by Using Ratios* and *Mixing Juice* task sheet, Calculators, Chart paper.

GROUPING:
Students will begin their work individually, but will then work in groups of three or four to discuss the task and arrive at a common solution.

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[2] NCTM (2010) *Developing Essential Understandings of Ratios, Proportions & Proportional Reasoning: Grades 6 -8*.

Warm-Up Task (*Comparing by Using Ratios)*

SET-UP

Tell students that a useful way to compare numbers is to form ratios. With students working in groups of three, give them the *Comparing by Using Ratios* task sheet and ask them to take turns reading the ratio statements to each other. Tell them they should form and interpret the ratios and think about different ways ratios can be written. They should also look for similarities and differences in the ratio statements.

EXPLORE PHASE

Monitoring Student Work:
Circulate while students are reading and interpreting the ratios. Focus students' attention on the ratio comparisons. Listen to and make note of students' debating and deciding the types of ratios in Statements A-G. Tell students you will want them to share their reasons during the whole-class discussion.

Possible Solution Paths
Each ratio is a part-to-part ratio, a part-to-whole ratio, or a ratio comparing different kinds of measures or counts (also called a rate). Statement D compares a part to a whole. Statements C and F compare two different kinds of measures; this type of ratio is called a *rate.* The remaining statements compare parts to parts. Note that statement E can be interpreted as part-to-part or part-to-whole. Make note of students who are arguing either interpretation and highlight this during the Share Discuss Analyze Phase of the Warm Up.

Ratios are often written in the form 5:6 or 5 to 6 to help students separate the ideas of ratios from fraction arithmetic.

- *How are these statements similar?*
- *How are they different?*

SHARE DISCUSS ANALYZE PHASE

General Considerations:
Start by prompting students to focus on ratio statements that they think are similar, and ask them to explain why. You might start by asking them, "How are statements A and B similar?" Once they have identified part-part, you might then ask them if D is also part-part, thus distinguishing part-whole and part-part. You might then have them decide whether G fits one of these classes. Next, ask whether C and F fit into either of these groups. Name the groups as they are formed. You could then have a discussion of E – saying that you don't know which it is. Through this discussion you should separate the statements into the three types, and introduce the terminology – part-part, part-whole, and rate. If you allow 7-10 min for small-group discussion, that will leave only 10-13 minutes for the Share, Discuss, Analyze Phase.

SET-UP (Mixing Juice Task)

Make sure students understand the context.
Suggested Questions:

- *How many of you have made juice by adding water to a mix before?*
- *What was involved in making it?*

You may want to bring in a can of frozen orange juice (thawed) and, with your class, make juice following the instructions on the can. You can discuss the fact that you have one container of concentrated juice and to this you add three containers of water (or whatever it says on the container of concentrate). Point out that the recipes given in the problem are different from the one on the can. At camp, the juice concentrate comes in a very large container without mixing proportions given.

You might let students begin to explore the juice recipes C in small-group, and then reassemble in whole-group to discuss the groups' initial ideas about the different mixes. This approach gives groups a chance to consider several representations and comparison strategies. You might discuss parts A, B and C in whole-group and then challenge groups to solve part D. Make sure that students have solved D for at least two mixes before calling them together to discuss it C in whole-group.

Remind students that they will be expected to: justify their solutions; explain their thinking and reasoning to others; make sense of other students' explanations; ask questions of the teacher or other students when they do not understand; and use correct mathematical language, and symbols.

EXPLORE PHASE: Supporting Students' Exploration of the Mathematical Ideas

Private Think Time: Allow students to work individually for 3 – 5 minutes without intervening, though you might want to circulate quickly to get an idea of the strategies that they are using.

Small-Group Work: After 3 – 5 minutes, ask students to work with their partner or in their small groups. As students are working, circulate around the room. Be persistent in:

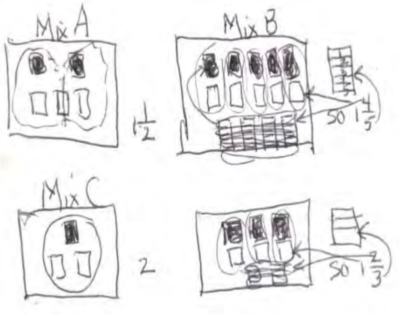
- asking questions related to the mathematical ideas, problem-solving strategies, and connections between representations.
- asking students to explain their thinking and reasoning.
- asking students to explain in their own words, and build onto, what other students have said.

As you circulate, identify solution paths that you will have groups share during the Share, Discuss, Analyze Phase, and decide on the sequence that you would like for them to be shared. Give groups a "heads up" that you will be asking them to put their solutions on chart paper to share during the whole-class discussion. Having the various strategies on chart paper will allow you to arrange the work in the room in a way that supports analyzing and making connections between and among them.

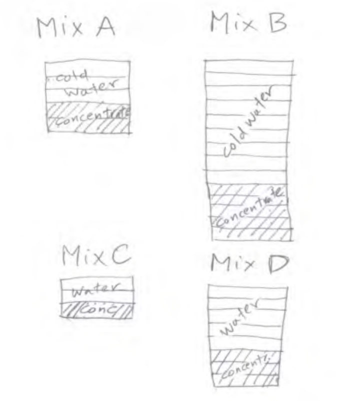
Some students will start with naïve strategies such as simply finding the difference between the number of cans of concentrate and ignore the water. Challenge this idea by asking: *Can I keep adding cans of water without making the juice less orangey?*

Questions A and B will allow misconceptions (additive strategies) as well as alternate multiplicative approaches for comparing ratios to emerge.

Question C is designed to raise the issue that the phrase "of Mix B" signals that this is a part-whole statement, thus 5/9 is not correct. Though students will discuss it during the Explore Phase, you will focus on questions A, B and D as you circulate during the Explore Phase.

<table>
<tr><th colspan="2">Possible Solution Paths for Parts A and B</th><th>Possible Assessing and Advancing Questions</th></tr>
<tr>
<td> </td>
<td>Draw pictures to show how much water there is for each cup of concentrate in each mix. The goal is to partition the water squares so that each cup of concentrate gets the same amount of water. In this way, you can see that Mix C has the most water for each cup of concentrate (least orangey) and Mix A has the least amount of water (most orangey).</td>
<td>Assessing Questions
• Tell me about your diagram. What does it show?
• How does this help to decide which mix is most or least orangey?
• What kind of ratios does this visual represent?
Advancing Questions
• If you wanted to write numerical ratios to represent what you have in this visual strategy, what would they look like? How would they help you decide which is most or least orangey?</td>
</tr>
<tr>
<td><table><tr><td>Mix
C
$\frac{1}{3}$</td><td>Mix
B
$\frac{5}{14}$</td><td>Mix
D
$\frac{3}{8}$</td><td>Mix
A
$\frac{2}{5}$</td></tr></table></td>
<td>Use part-to-whole ratios written in fraction form to express the relationships of concentrate to total liquid in a batch.
Using prior knowledge about fractions, students may represent the fractions as decimals or percents. There are a variety of strategies that can then be used to order the fractions, i.e., benchmark comparisons or common denominators.</td>
<td>Assessing Questions
• Tell me about your work.
• Tell me, for example, what $\frac{3}{8}$ means in the context of the problem. What does the 3 represent? The 8? Is there any way you can let me know that in your explanation?
Advancing Questions (if no comparisons are being made)
• How are you going to compare the mixes? Which is most orangey and how do you know?
• What are some ways that you have used in the past to make comparisons? See if you can think about some of these and then try to make comparisons, in one or two ways.
Advancing Questions (if comparisons are being made)
• How does knowing that $\frac{1}{3}$ is the ratio of concentrate to mix help you to answer Question D?</td>
</tr>
<tr>
<td>A. $\frac{1\frac{1}{2}}{1}; \frac{1\frac{4}{5}}{1}; \frac{2}{1}; \frac{1\frac{2}{3}}{1}$
B. $\frac{\frac{2}{3}}{1}; \frac{\frac{5}{9}}{1}; \frac{\frac{1}{2}}{1}; \frac{\frac{3}{5}}{1}$</td>
<td>A. Figure out how much water goes with each cup of concentrate. Notice that with these ratios we focus on most and least water.
B. Figure out how much concentrate goes with each cup of water. Notice that with these ratios we focus on most and least concentrate.</td>
<td>Assessing Questions
• Tell me about your work. What did you do and why?
• What kind of ratios have you created? What do they represent? Is there any way you can let me know that in your explanation?
Advancing Questions
• How are you using your ratios to decide which is most orangey?
• How are you going to compare the mixes? Which is most orangey and how do you know?
• What strategy are you using to order your ratios? What does the order mean in the context of the problem?</td>
</tr>
</table>

Mix A $\begin{matrix}2:3\\30:45\end{matrix}$ Mix B $\begin{matrix}5:9\\30:54\end{matrix}$ Mix C $\begin{matrix}1:2\\30:60\end{matrix}$ Mix D $\begin{matrix}3:5\\30:50\end{matrix}$	Use part-to-part ratios and make the number of cups of concentrate the same. Notice that with these ratios we focus on most and least water.	*Assessing Questions* • *Tell me about your work.* • *What kind of ratios have you created? What do they represent? Is there any way you can let me know that in your explanation?* • *How did you decide that 30 cups of concentrate would be a helpful amount?* • *How did you make the new ratios? What strategy did you use?* • *How does making the amount of concentrate the same help you to reason about the problem?* *Advancing Question* • *How are you going to compare the mixes? Which is most orangey and how do you know?* • *Tell me which is the orangeyist and how you know? Write an explanation.*

Possible Errors and Misconceptions for Parts A and B	**Possible Questions to Address Errors and Misconceptions**
Comparing the amount of water using absolute differences approach:  A. The juice that will taste most orangey is Mix C because it does not have as much water as mixes A, B, and D. B. The juice that will taste least orangey is Mix B because it has more water than mixes A, C, and D.	Assessing Question ● *Tell me about your work. Explain your thinking.* *Advancing Questions* ● *What would 2 batches of Mix C look like? How would the "orangeyness" of this new one compare to Mix A? Why?* ● *For every one cup of water in Mix A how many cups of concentrate would I have?*

Possible Solution Paths for Part D	**Possible Assessing and Advancing Questions**
Find the number of batches needed to make 120 cups of juice from each recipe. Then multiply to find the amount of water and concentrate. For example, one batch of Mix A makes 5 cups of juice and since 120 cups are needed, 120 + 5 yields 24 batches. So, 2 x 24 or 48 cups of concentrate and 3 x 24 or 72 cups of cold water are needed. 48 cups of concentrate plus 72 cups of water yields 120 cups of juice for the 240 campers.	Assessing Questions ● Why *did you decide to work with 120 cups of juice?* ● *How did you use 120 to help you solve the problem?* ● *What does the 24 mean in the context of the problem and why did you multiply the 2 and 3 by it?* Advancing Questions ● *How can you check to see that the amounts you calculated are correct?* ● *So the ratio of cups of juice in the big batch to cups of juice in the recipe is 120:5 (pointing to 120 + 5 on the paper). What is the ratio of cups of concentrate in the big batch to cups of concentrate in the recipe? What about the ratio of cups of water in the big batch to cups of water in the recipe? Do you think that will happen with the other mixes, too?* ● *Write those ratios in equation form and study the way they look. Is there a way to decide by looking at the statement that two ratios are equal*

<table>
<tr>
<td>Make a rate or ratio table to scale up:
Mix D (for example)
<table>
<tr><th>Concentrate in cups</th><th>Water in cups</th><th>Total in cups</th></tr>
<tr><td>3</td><td>5</td><td>8</td></tr>
<tr><td>6</td><td>10</td><td>16</td></tr>
<tr><td>9</td><td>15</td><td>24</td></tr>
<tr><td>12</td><td>20</td><td>32</td></tr>
<tr><td>15</td><td>25</td><td>40</td></tr>
<tr><td>45</td><td>75</td><td>120</td></tr>
</table>
</td>
<td>Students may continue to add 3:5:8 to each row until they reach 45:75:120, or they may notice that they can multiply a row by a scale factor to get to their result more quickly as is shown in the last 2 rows of this table.</td>
<td>Assessing Questions
● *Why did you decide to organize your work in a table? How is that helpful?*
● *How are you getting from one row to the next in your table?*
● *How did you know when to stop making new rows?*
● *What patterns do you see in the table?*
Advancing Questions
● *Can you show what you did in the last two rows so we don’t have to guess? Use either an numerical expression or a written explanation. Then say WHY you knew you could do what you did.*</td>
</tr>
</table>

Possible Errors and Misconceptions for Part D	**Possible Questions to Address Errors and Misconceptions**
If students are stuck on the question of making a recipe for 240 people, ask them to consider Mix A to start.	• *How much total juice does one batch of Mix A make? How can we figure out how many people one batch of this juice will serve?* • *What if each serving is one cup? What if each serving is ½ cup?* • *If you were going to serve juice to 50 people, how many cups of juice would you have to make if each person gets ½ cup of juice? How many batches of juice would this be?* • *What are different strategies you might use to answer this question?* (Students might divide 50 people by 10 servings per batch to determine that 5 batches are needed. Alternatively, some students may reason that if 1 batch makes 10 servings, then 2 batches make 20 servings, 3 batches makes 30 servings, etc. Students may make a table from which to reason.

Servings of Juice					
Number of Batches	1	2	3	4	5
Servings	10	20	30	40	50

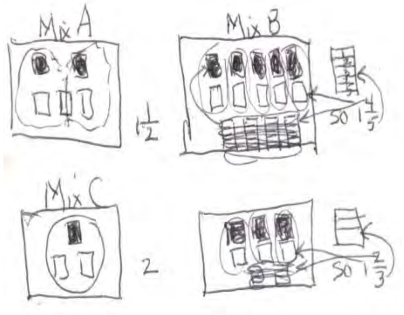
SHARE DISCUSS ANALYZE PHASE: Deepening and Connecting Mathematical Understanding

General Considerations:

- Orchestrate the class discussion so that it builds on, extends, and connects the thinking and reasoning of students.
- Sequence the solution paths so that you will be able to press students to make comparisons and connections across, and between, the various solution paths.

It is recommended that you have groups share and discuss Parts A and B, then discuss C and D. For Parts A and B, the point you want students to think about is: *What does it mean to be most orangey tasting? To be least orangey tasting? Why do we have to consider BOTH the amount of concentrate AND the amount of water?*

Possible Sequence of Solution Paths	**Possible Questions and *Possible Student Responses***
Begin with the visual strategy under Possible Misconceptions and Errors for Parts A and B.	While the reasoning that was used to decide which mix is most orangey is faulty, the visual can be used to make connections to some of the other strategies. You might decide to have the group share their drawing and come back to them after they have heard some of the other strategies described to see if they will reconsider their reasoning and their answer about which is most orangey.
Have a group who used a part-to-whole ratio share their strategy. If there are a variety of strategies that were used to reason about the part-to-whole ratios, start with reasoning from the ratio itself, then move to those that converted the ratio to a decimal, then move to percents, and finally connect to the common denominator approach.	**How did your group decide to compare the mixes?** ● *We made our comparisons using part-to-whole ratios. We compared the part of the mix that is concentrate to the whole mix. Then we made these fractions into decimals and looked for the largest because that would tell us which mix was most orangey.* **How is your strategy related to the diagrams we saw in the first strategy?** ● *The numerator in our fraction is the number of pieces that are shaded and these represent the concentrate. The denominator in our fraction is the total number of pieces in the diagram for a particular mix.* **How do your decimals relate to the diagrams?** ● *The decimal (percent) would represent the portion of the whole diagram that is shaded. In this case, the whole mix would have a value of 1 or 100%.*

Use a visual that shows how much water there is for each cup of concentrate in each mix. This is a unit rate approach.  	**Tell us what your drawing means**. - *The solid squares represent cups of concentrate and the empty squares represent cups of water. We divided the water squares so that each cup of concentrate gets the same amount of water.* **How does your drawing help you decide which mix is most orangey?** - *We can see that Mix A has the least water, 1 ½ cups, for each cup of concentrate, so it is the most orangey.* **How does your drawing help you decide which mix is least orangey?** - *We can see that Mix C has the most water, 2 cups, for each cup of concentrate, so it is the least orangey* **How does your strategy compare to the ones we saw earlier?** - *We were using part-to-part comparisons and they were using part-to-whole comparisons.* - *For them to say which mix was most orangey they had to look for the mix that had the most concentrate. For us to say which mix was most orangey, we looked for the mix that has the least water.*
Have a group that used a unit rate approach with unit ratios expressed numerically share next.	**Tell us about the ratios your group made. How did you calculate them?** - *We compared cups of water to cups of concentrate. We wanted to figure out how much water goes with each cup of concentrate, so we divided the number of cups of water by the number of cups of concentrate.* **How does this compare to the last groups' strategy?** - *They divided up the area of the squares and we used ratios, unit rates, but they both convey similar information. Like for Mix B, our ratio was* $\frac{1\frac{4}{5}}{1}$ *which is water to concentrate and that's what their picture shows.*
Have a group that used part-to-part ratios and made the number of cups of concentrate (or water) the same. *(This strategy is helpful to scaffold student thinking for Part D.)*	**Tell us about your approach. What kind of ratios did you use to compare the mixes?** - *We used part-to-part ratios. We thought if we had 4 big pots and used each pot to make many batches of each mix we could compare them. We wanted each pot to have the same amount of concentrate so we could think about how much water is in each pot.* **How will the multiple batches of juice in the pots compare to the original mix?** - *The juice in each pot will taste the same as the original mix because we kept the ratio of concentrate to water the same.* **How does your strategy compare to the other strategies we have seen?** - *Our strategy is probably most like the part-to-whole ratios where they found common denominators. Even though we used part-to-part ratios to reason, their strategy was similar because, like us, they ended up with a lot more juice than was in the original mix.*

CLOSURE

Quickwrite:

- Why is a ratio a useful way to make comparisons?
- Which of the following will taste the most orangey? 2 cups of concentrate and 3 cups of water; 4 cups of concentrate and 6 cups of water; or 10 cups of concentrate and 15 cups of water? Explain your reasoning.

Possible Assessment:

- Provide some additional contexts where they need to compare quantities. Ask them to explain their thinking in writing.

Homework:

- Find items from the current curriculum that will allow them to apply these ideas and understandings.

References

Lappan, Fey, Fitzgerald, Friel, Phillips (2009). Teacher's Guide: Connected Mathematics 2. *Comparing and Scaling: Ratio, Proportion, and Percent,* Pearson.

Comparing by Using Ratios

A useful way to compare numbers is to form ***ratios***. Talk to your classmates about what is the same and what is different about these ratio statements.

A. In taste tests, people who preferred Bolda Cola outnumbered those who preferred Cola Nola by a ratio of 3 to 2.

B. The ratio of boys to girls in our class is 12 boys to 15 girls.

C. For every four tents there are 12 scouts.

D. The ratio of boys to students in our class is 12 boys to 27 students.

E. The ratio of kittens to cats in our neighborhood is $\frac{1}{4}$.

F. The sign in the hotel lobby says:
1 dollar Canadian : 0.85 dollars U.S.

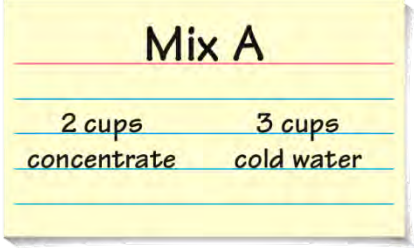
G. A paint mixture calls for 5 parts blue paint to 2 parts yellow paint.

From Lappan et al. (2002). Comparing and Scaling: Ratio, Proportion, Percent, p. 26. Connected Mathematics, Pearson.

MIXING JUICE

Julia and Mariah attend summer camp. Everyone at the camp helps with the cooking and cleanup at meal times.

One morning, Julia and Mariah make orange juice for all the campers. They plan to make the juice by mixing water and frozen orange juice concentrate. To find the mix that tastes best, they decide to test some mixes.



Mix A

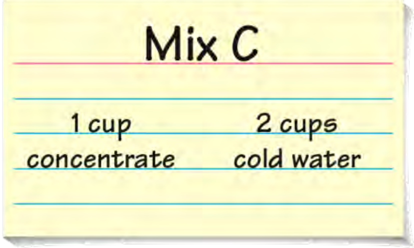
2 cups concentrate

3 cups cold water

Mix B

5 cups concentrate

9 cups cold water



Mix C

1 cup concentrate

2 cups cold water

Mix D

3 cups concentrate

5 cups cold water

Developing Comparison Strategies

A. Which mix will make juice that is the most "orangey"? Explain.

B. Which mix will make juice that is the least "orangey"? Explain.

C. Which comparison statement is correct? Explain.

a. $\frac{5}{9}$ of Mix B is concentrate

$\frac{5}{14}$ of Mix B is concentrate

D. Assume that each camper will get $\frac{1}{2}$ cup of juice.

1. For each mix, how many batches are needed to make juice for 240 campers?
2. For each mix, how much concentrate and how much water are needed to make juice for 240 campers?

From Lappan et al. (2009). *Comparing and Scaling: Ratio, Proportion, Percent*, p. 19, Lesson 2.1. Connected Mathematics 2, Pearson.

OUNCES OF COFFEE TASK
SEVENTH GRADE LESSON GUIDE

LESSON OVERVIEW:

Students will be presented with the Ounces of Coffee problem. They will be asked to determine whether or not the there is a proportional relationship between the ounces of coffee to the price. Students then will be asked to find the unit price and explain in writing what the unit price means in the context of the problem. Finally, students will explain why it is helpful to determine if the relationship between the amount of coffee and price is proportional.

Students should be able to see the proportional relationship between the ounces of coffee and the price. They should be able to find the unit price and to see that the cost per ounce is the same.

Before working on the Ounces of Coffee task, students will do a Warm-Up task identifying ratios and the appropriate representations of unit rates. Students will be allowed 15-20 minutes to engage in and discuss the Warm-Up task. At least 2-3 periods will be allotted for students to explore and share their work.

COMMON CORE STATE STANDARDS:

- **6.RP.1** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.
- **6.RP.2** Understand the concept of a unit rate *a*/*b* associated with a ratio *a*:*b* with *b* ≠ 0, and use rate language in the context of a ratio relationship.
- **6.RP.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
  a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
  b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*
  c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
  d. Use ratio reasoning to convert measurement units to manipulate and transform units appropriately when multiplying or dividing quantities.
- **7.RP.1** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour.*
- **7.RP.2** Recognize and represent proportional relationships between quantities.
  d. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
  e. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

f. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn*.

2. Explain what a point *(x, y)* on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r)* where *r* is the unit rate.

- **7.RP.3** Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*
- **8.EE.5** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.
- **8.EE.6** Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation *y* = *mx* for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at *b*.

DRIVING QUESTION:	**NCTM ESSENTIAL UNDERSTANDINGS[3]:**	**SKILLS DEVELOPED:**
• What are different types of ratios? • How can ratios be used to make comparisons? • How are ratios related to fractions?	1. Reasoning with ratios involves attending to and coordinating two quantities 2. A ratio is a multiplicative comparison of two quantities, or it is a joining of two quantities in a composed unit. 3. Forming a ratio as a measure of a real world attribute involves isolating that attribute from other attributes and understanding the effect of changing each quantity on the attribute of interest. 4. Ratios can be meaningfully reinterpreted as quotients. 5. Proportional reasoning is complex and involves understanding that: • Equivalent ratios can be created by iterating and/or partitioning a composed unit; • If one quantity in a ratio is multiplied or divided by a particular factor, then the other quantity must be multiplied or divided by the same factor to maintain the proportional relationship.	• Use different representations to form ratios and make comparisons with ratios. • Form equivalent ratios and use equivalent ratios to solve problems. • Find the Unit Rate.

MATERIALS:	**GROUPING**:
Warm up task, Ounces of Coffee sheet, calculators, Smartboard	Students will work alone and then in groups of three to four.

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[3] NCTM (2010) Developing Essential Understandings of Ratios, Proportions & Proportional Reasoning: Grades 6 -8.

SET-UP

Instructions to Students: Students will discuss their understanding of ratio, unit rate, rate and proportion. A student will be told to read the problem while others follow along silently. Explain to students that they will have to justify their solutions and explain their reasoning. Students will be told to work alone for 7 to 10 minutes and then in small groups.

EXPLORE PHASE: Supporting Students' Exploration of the Mathematical Ideas

Private Think Time: Allow students to work individually for 3-5 minutes without intervening, though you might want to circulate quickly to get an idea of the strategies that they are using.

Small-Group Work: After 3-5 minutes, ask students to work with their partner or in their small groups. As students are working, circulate around the room. Be persistent in:

- asking questions related to the mathematical ideas, problem-solving strategies, and connections between representations.
- asking students to explain their thinking and reasoning.
- asking students to explain in their own words, and build onto, what other students have said.

As you circulate, identify solution paths that you will have groups share during the Share, Discuss, Analyze Phase, and decide on the sequence that you would like for them to be shared. Give groups a "heads up" that you will be asking them to come to the front of the room. If a document reader is not available, give selected groups an OVH transparency or chart paper to write their solution on.

Possible Solution Paths	Possible Assessing and Advancing Questions
If a group is unable to start: Focus students on the table. 5. *What does the task ask us to figure out?* 6. *What is being compared?* 7. *What are the items on the table being compared?* *We are comparing ounces of coffee to price in dollars.*	Assessing • What are we trying to figure out in this problem? • What can you tell me about the ounces of coffee and the price in dollars?
oz. oz. oz. oz. oz. oz. .40 .40 .40 .40 .40 .40	Assessing Questions • Tell about your work. • How did you figure out that each ounce would cost 40¢? • How did you know that each ounce of coffee would cost forty cents? Advancing Questions • Is there another way to show the relationship between the amount of coffee and the price?

$$\begin{array}{r} \underline{0.40} \\ 6\ |2.40 \\ 2.40 \end{array} \qquad \begin{array}{r} \underline{0.40} \\ 8|3.20 \\ 3.20 \end{array} \qquad \begin{array}{r} \underline{0.40} \\ 16|6.40 \end{array}$$

Assessing Questions

- Tell us about your work.
- Why are you dividing the price by the ounces?
- What does the .40 tell us? What do we call it?
- How do you know that dividing the ounces by the dollars will give you the unit rate?

Advancing Questions (Not all of these would be asked at the same time.)

- How does knowing the unit price benefit you?
- How can you represent the quantities from the table in a ratio? Can you represent the ratio in fraction notation?
- How can you compare the ratios? Are they the same or equivalent?
- Is there a proportional relationship?

1	*.40*
2	*.80*
3	*1.20*
4	*1.60*
5	*2.00*
6	*2.40*
8	*3.20*

Assessing Questions

- Tell me about your table and what you noticed. Do you see a pattern?
- What patterns do you see in your table?

Advancing Questions

- If students have not noticed a pattern then, tell me about the pattern in the table?
- What does the pattern tell you?
- So if you have 32 ounces of coffee, how much will that cost?
- Is there a proportional relationship between the ounces of coffee and the cost? Why or why not?
- Will this method for solving for proportion always work? How do you know?
- Can you think of a proportion that this method of solving will not work with?

Possible Errors and Misconceptions	**Possible Questions to Address Errors and Misconceptions**
Students may ignore the decimal point in the $2.40, $3.20 and $6.40.	Assessing Questions • What is being compared? • Where did you get the 240, 320, and 640 from? Or devils advocate: Wow is that 240 dollars? • How is it the same as the quantities in the table? Advancing Questions • What are the quantities being compared in this problem? • How are you going to use ratios to help you see if there is a proportional relationship?

SHARE DISCUSS ANALYZE PHASE: Deepening and Connecting Mathematical Understanding

General Considerations:

- Orchestrate the class discussion so that it builds on, extends, and connects the thinking and reasoning of students
- Sequence the solution paths so that you will be able to press students to make comparisons and connections across, and between, the various solution paths.

<table>
<tr><th>Possible Sequence of Solution Paths</th><th>Possible Questions and <i>Possible Student Responses</i></th></tr>
<tr><td><i>A focus on Pattern Finding and Describing the Proportional Relationship</i>

<i>ORTIZ</i>
<table>
<tr><td><i>1</i></td><td><i>.40</i></td></tr>
<tr><td><i>2</i></td><td><i>.80</i></td></tr>
<tr><td><i>3</i></td><td><i>1.20</i></td></tr>
<tr><td><i>4</i></td><td><i>1.60</i></td></tr>
<tr><td><i>5</i></td><td><i>2.00</i></td></tr>
<tr><td><i>6</i></td><td><i>2.40</i></td></tr>
<tr><td><i>8</i></td><td><i>3.20</i></td></tr>
<tr><td><i>16</i></td><td><i>6.40</i></td></tr>
</table>
I looked at the possible ratio as a fraction and simplified it to its lowest terms. I found that the ounces of coffee to price in dollars were a ratio of 1 to .40 for each given quantity.</td>
<td>• Tell us about your work.
• Do you see a pattern?
• What patterns do you see in your table?
• What made you create this table?
• What does the pattern tell you?
• Is there a proportional relationship here? What is it?
• What was the method that this group used to figure out if there was a proportional relationship? Will this method for solving for proportion always work? How do you know?
• So, if you have 32 ounces of coffee, how much will that cost?
• Can you think of a proportion that this method of solving will not work with?</td></tr>
</table>

A) There is a proportional relationship between ounces of coffee and price in dollars because when you divide ounces of coffee to the price in dollars, it gives you 1/.40 or 1 divided by .40 and this is for each quantity given. For example, 6/2.40 = 1/.40 and 8/3.20= 1/.40 and 16/6.40 = 1/.40 so the relationship is proportional.

B) 1/.40 is the unit rate for this problem. When I simplified 6 over \$2.40; 8 over \$3.20 and 16 over \$6.40, they all resulted in 1 over .40.

C) For every ounce of coffee the price is 40¢.

D) The reason why Julia has to find out if it is proportional is the lowest price per ounce would be the best bag to buy. If it is proportional then Julia will know that it is okay to purchase any bag because the coffee price will always remain the same per ounce of coffee. If not, Julia will need to find the lowest price per ounce of coffee.

Miraj A Focus on using ratios to see if the different price and amount are proportional.

Since we have three different sized bags and three different prices, I will use ratios to compare the amount of coffee expressed in ounces to the price in dollars. Based on the table given, I will write the following ratios:

\$2.40 : 6 or	\$2.40 to 6 or	\$2.40/6
\$3.20 : 8 or	\$3.20 to 8 or	\$3.20/8
\$6.40 :16 or	\$6.40 to 16 or	\$6.40/16

I am trying to see if there is any proportional relationship between the amount of coffee and the price. To do that, I had to compare to see if the ratios I created are the same.

To see if the ratios are proportional, I cross multiplied and I found out that the first two ratios are in a proportional.
The evidence for that is the equation

$$\frac{\$2.40}{6} = \frac{\$3.20}{8} = \frac{\$6.40}{16}$$

Explain your group's solution.

- How did you find out whether or not the ratios were the same or proportional?
- How did you know that they were in a proportional relationship?
- If you had \$19.20 how many ounces of coffee can you buy?
- How can you represent this information in another way?

B) The unit rate will be determined by one of the ratios since all of the ratios are the same I will take $2.40 divided by 6 equals $.40 or forty cents. C) The unit rate means the price for coffee for each ounce in each bag is the same, forty cents. D) It is important for Julia to know that the amount of coffee and the price of the coffee is proportional so she can calculate how much money she will need for the new bag of coffee.	
Boatright: A Focus on using quotient to get the price of Ounces/Price in Dollars 6/2.40 ÷ 6/6 = 1/.40 Ounces/Price in Dollars 8/3.20 ÷ 8/8 = 1/.40 Ounces/Price in Dollars 16/6.40 ÷ 16/16 = 1/.40 I looked at the possible ratio as fractions and simplified it to its lowest terms. I found that the ounces of coffee to price in dollars were a ratio of 1 to .40 for each given quantity. The unit rate is one ounce per $ 0.40 a. Yes, the proportion is equal to 1/.40, for each quantity given. Therefore, it is proportional. b. 1/.40, is the unit rate because when simplified the ratio is one to 40 hundredths. c. Ounce of coffee/price of coffee is 1/.40. This means that one-ounce cost 40 cents. d. It is helpful for Julia to find the unit price because this way she assures herself that each coffee package has the same cost per ounce.	Explain your group's solution. • What is a ratio? • What is a proportion? • Why are you simplifying? • What is a unit rate? What does it mean in this context? • Can a unit rate be simplified? • Can a unit rate be negative?

CLOSURE
Quick Write: Choose one of the questions below depending on your students' understanding. • **WHEN SOMETHING IS CHANGING PROPORTIONALLY, WHAT INFORMATION CAN WE GET FROM THE RATIO TO DESCRIBE THE CHANGE?** • **What does it mean if there is a proportional relationship? Refer to the coffee problem in your explanation. (I wonder if this will permit you to see if they refer to the ratio.)**
Possible Assessment: • Continue to provide similar problems in which various solution paths can be used.
Homework: • Jose and Russell jogging problem, weight on the moon problem and the light bulb problem.

OUNCES OF COFFEE

Name ________________________ Date ____________________

Julia made observations about selling price of a new coffee that sold in three different sized bags. She recorded those observations in the following table:

Ounces of Coffee	**6**	**8**	**16**
Price in Dollars	**$2.40**	**$3.20**	**$6.40**

a) Is there a proportional relationship between the amount of coffee and the price? Why or why not?

b) Find the unit rates associated with the problem.

c) Explain in writing what the unit rates mean in the context of this problem.

d) Explain in writing why is it helpful for Julia to determine if the relationship between the amount of coffee and the price is proportional before she buys a new bag of coffee.

MELINDA AND AKIRA'S WALK
SEVENTH GRADE LESSON GUIDE

LESSON OVERVIEW:

Melinda and her sister Akira are walking around the track at school. Melinda and Akira walk at a steady rate and Melinda walks 5 feet in the same time that Akira walks 2 feet.

a) Set up a table and draw a graph to represent this situation. Let the x-axis represent the number of feet that Melinda walks and the y-axis represent the number of feet that Akira walks.
b) When Melinda walks 45 feet, how far will Akira walk? Explain in writing or show how you found your answer.

(Expand on this section in the future: explain the purpose of the lesson.)

COMMON CORE STATE STANDARDS:

- **6.RP.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
  - e. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
  - f. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*
  - g. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
  - h. Use ratio reasoning to convert measurement units to manipulate and transform units appropriately when multiplying or dividing quantities.
- **7.RP.1** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour.*
- **7.RP.2** Recognize and represent proportional relationships between quantities.
  - g. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
  - h. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
  - i. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.*
    - 3. Explain what a point *(x, y)* on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r)* where *r* is the unit rate.
- **7.RP.3** Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

<table>
<tr>
<td>DRIVING QUESTION:</td>
<td colspan="2">NCTM ESSENTIAL UNDERSTANDINGS[4]:
6. Reasoning with ratios involves attending to and coordinating two quantities.
7. A ratio is a multiplicative comparison of two quantities, or it is a joining of two quantities in a composed unit.
8. Forming a ratio as a measure of a real-world attribute involves isolating that attribute from other attributes and understanding the effect of changing each quantity on the attribute of interest.
9. A number of mathematical connections link rations and fractions:
a) Ratios are often expressed in fraction notation, although ratios and fractions do not have identical meaning.
b) Ratios are often used to make “part-part” comparisons, but fractions are not.
c) Ratios and fractions can be thought of as overlapping sets.
d) Ratios can often be meaningfully reinterpreted as fractions.
10. Ratios can be meaningfully reinterpreted as quotients.
11. A proportion is a relationship of equality between two ratios. In a proportion, the ratio of two quantities remains constant as the corresponding values of the quantities change.
12. Proportional reasoning is complex and involves understanding that:
a) Equivalent ratios can be created by iterating and/or partitioning a composed unit;
b) If one quantity in a ratio is multiplied or divided by a particular factor, then the other quantity must be multiplied or divided by the same factor to maintain the proportional relationship; and
c) The two types of ratios – composed units and multiplicative comparisons – are related.
13. A rate is a set of infinitely many equivalent ratios.
14. Several ways of reasoning, all grounded in sense making, can be generalized into algorithms for solving proportion problems.</td>
<td>SKILLS DEVELOPED:</td>
</tr>
<tr>
<td colspan="2">MATERIALS:</td>
<td colspan="2">GROUPING:</td>
</tr>
</table>

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[4] NCTM (2010) Developing Essential Understandings of Ratios, Proportions & Proportional Reasoning: Grades 6 -8.

SET-UP

Instructions to Students:

EXPLORE PHASE: Supporting Students' Exploration of the Mathematical Ideas

Private Think Time: Allow students to work individually for 3-5 minutes without intervening, though you might want to circulate quickly to get an idea of the strategies that they are using.

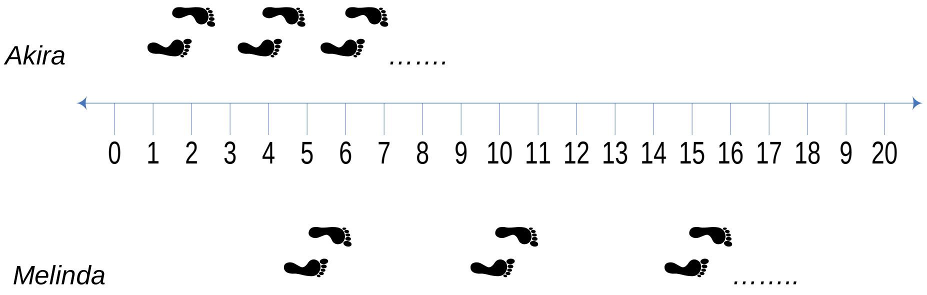
Small-Group Work: After 3-5 minutes, ask students to work with their partner or in their small groups. As students are working, circulate around the room. Be persistent in:

- asking questions related to the mathematical ideas, problem-solving strategies, and connections between representations.
- asking students to explain their thinking and reasoning.
- asking students to explain in their own words, and build onto, what other students have said.

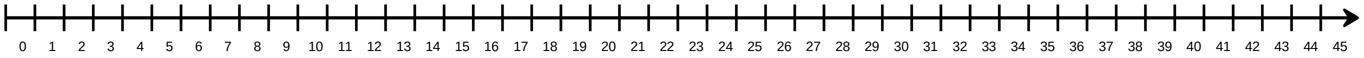
As you circulate, identify solution paths that you will have groups share during the Share, Discuss, Analyze Phase, and decide on the sequence that you would like for them to be shared. Give groups a "heads up" that you will be asking them to come to the front of the room. If a document reader is not available, give selected groups an OVH transparency or chart paper to write their solution on.

Possible Solution Paths

If a group is unable to start:
8. Make a number line picture or diagram of the problem:



Below is a number line extended to 45:



Possible Assessing and Advancing Questions

Assessing Questions

- Tell me what you are thinking.
- What is the task asking you to do?

Advancing Questions

- Are Melinda and Akira walking at the same rate? Can you make a diagram of the problem?
- Tell us what your diagram shows.

Create a Table

Melinda	0	5	10	15	20	25	30	35	40	45
Akira	0	2	4	6	8	10	12	14	16	18

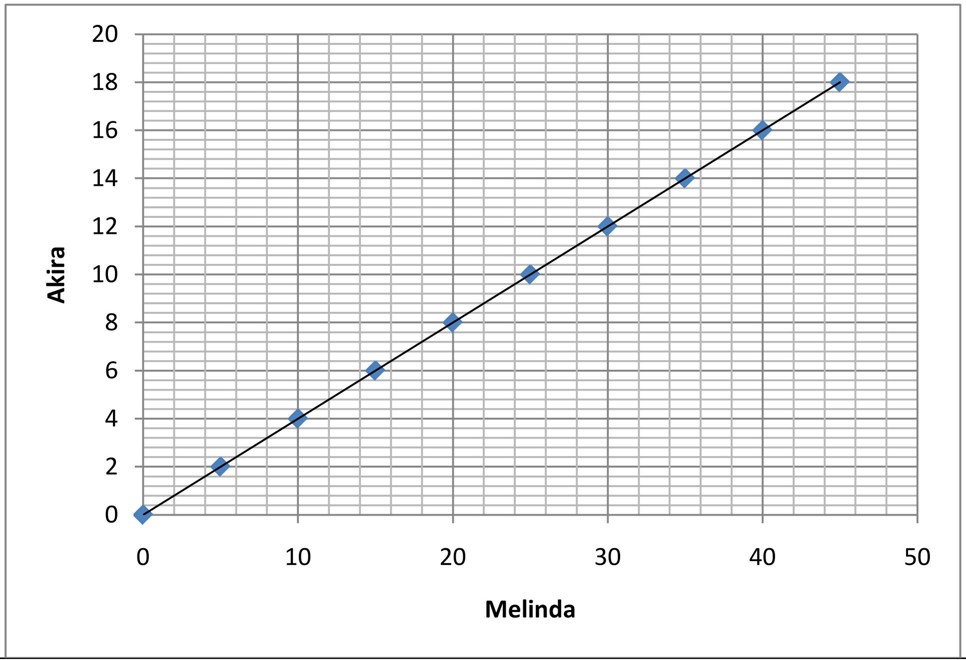
Assessing Question

- Tell me what the table tells you about Melinda and Akira's walk.

Advancing Questions

- What patterns do you see in the table? Explain the pattern.
- Can you describe a pattern in the table that uses multiplication?

9. Make a Graph
   Melinda and Akira's Walk



Assessing Question

- What does this point mean in this context? *(Pointing to the next to uppermost point on the graph.)*

Advancing Questions

- Does your graph represent a proportional relationship? Why or why not?
- Can you predict how far Akira will walk if Melinda walks 1000 feet?

10. Write and Solve a Proportion:

$$\frac{5}{2} = \frac{45}{x}$$

$$5x = 90$$

$$\frac{5x}{5} = \frac{90}{5}$$

$$x = 18$$

Assessing Question

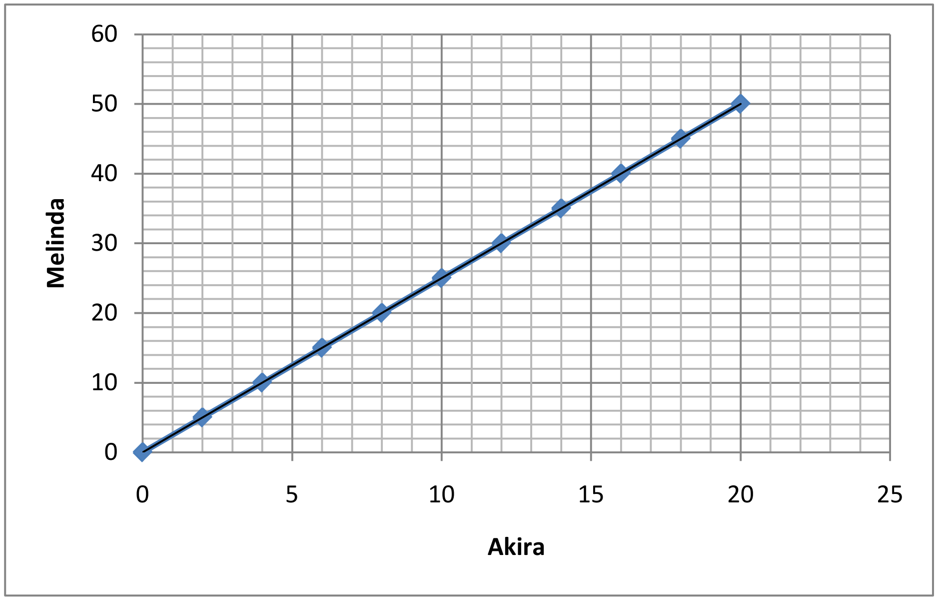
- How do you know that the relationship between Melinda and Akira's pace is proportional? Explain.

Advancing Question

- When Melinda walks 450 feet, how far will Akira walk?

<table>
<tr><th>Possible Errors and Misconceptions</th><th>Possible Questions to Address Errors and Misconceptions</th></tr>
<tr><td>

1. Reversing the Axes on the graph (Melinda is supposed to be represented by the x-axis and Akira by the y-axis:



a) focusing only on the difference in the paces – Melinda is always 3 feet ahead of Akira.

</td><td>

Assessing Question

- Can you explain what your graph means?

</td></tr>
<tr><td colspan="2">SHARE DISCUSS ANALYZE PHASE: Deepening and Connecting Mathematical Understanding</td></tr>
<tr><td colspan="2">

General Considerations:

- Orchestrate the class discussion so that it builds on, extends, and connects the thinking and reasoning of students.
- Sequence the solution paths so that you will be able to press students to make comparisons and connections across, and between, the various solution paths.

</td></tr>
</table>

Possible Sequence of Solution Paths	**Possible Questions and *Possible Student Responses***
CLOSURE	
Quick Write: ●	
Possible Assessment: ●	
Homework: ●	

MELINDA AND AKIRA’S WALK

Melinda and her sister Akira are walking around the track at school. Melinda and Akira walk at a steady rate and Melinda walks 5 feet in the same time that Akira walks 2 feet.

a) Set up a table and draw a graph to represent this situation. Let the x-axis represent the number of feet that Melinda walks and the y-axis represent the number of feet that Akira walks.

b) When Melinda walks 45 feet, how far will Akira walk? Explain in writing or show how you found your answer.